A TEXT-BOOK OF PRACTICAL PHYSICS

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A TEXT-BOOK OF PRACTICAL PHYSICS

BY

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PREFACE

In teaching Practical Physics at King's College, London, we have employed for several years manuscript books of instructions for experiments, each providing a short description of the apparatus required together with the necessary theory. But we have felt the need of permanent records easily available for large numbers of students, and it is hoped that this book, based on these laboratory instructions, will prove of service not only to our own students but also to those of other teachers.

The work is designed primarily to cover the Intermediate Pass Courses in Science, Engineering, and Medicine of the University of London, but it is also suitable for Intermediate Honours candidates and for University Scholarships. Further, it meets the requirements of the Civil Service Commissioners in connection with Junior Appointments, the Post Office (Engineers), and the Army Entrance Examination.

No student could be expected to cover in one year the whole of this course, and a selection of suitable experiments to meet the needs of particular students should be made by the teacher. The fact that Engineering Students in the University of London are required to take a practical course in Applied Mathematics explains the presence of a larger number of experiments in practical mechanics than is usual in a text-book of Practical Physics.

For completeness, the scope of the book has been extended somewhat and a limited number of more advanced experiments has been included. For example, a description of the simpler

phenomena of Surface Tension, and of the elementary methods by which it can be measured, has been incorporated though not usually studied in an Intermediate Course. In the part dealing with Electricity will be found a number of experiments suitable for students who are commencing the study of Electrical Engineering.

The volume in its present form covers the greater part of the work necessary for a Pass Degree in Science in most Universities.

The letterpress has been divided so as to separate descriptions and theoretical discussions from those parts of the nature of laboratory instructions; the latter are indicated by indenting the letterpress. The teacher should select for each meeting of the class experiments suitable for the individual students, and it is a good plan for the student to be informed at the end of the day's work what section he should read in preparation for the next practical class.

It is seldom that sufficient apparatus is available for all students to be working at the same experiment simultaneously, and it is usually impossible to arrange that each individual student should follow the exact order here given.

The student must be warned against a prevalent idea that an experiment is completed when the manipulations or observations are finished. It must be made clear to him that the results must be calculated and considered with care, and a record of the experiment be written in his own words.

Every experimental exercise should have for its object the elucidation of physical principles, and we have kept in view the theoretical aspect of the experiments throughout this work. Stress has been laid on the degree of accuracy obtainable in an experiment, and the student has been shown how to conduct the measurements so as to obtain the best results with the means at his disposal.

Most of the experiments described are such as can be carried out with simple apparatus. Many instrument makers advertise for laboratory use apparatus more adapted for lecture purposes.

PREFACE

The apparatus used in a laboratory course should be such as will develop experimental and manipulative skill. Hence, although measuring instruments of precision are essential, the use of elaborate apparatus, sometimes almost automatic in its action, is to be deprecated in a laboratory for students.

An important educational aim, to be kept in mind by the teacher of Physics, is the development of men and women capable of doing good work in adverse or unfamiliar circumstances, and of carrying out original investigation and research. For this, facility in the handling of apparatus and co-ordination of hand, eye, and ear are essential.

Several additional exercises, many of them selected from College examination papers, have been given at the end of each Part.

Most of the diagrams have been drawn specially for this book, but we are indebted to Messrs. Macmillan & Co., Ltd., for a number of illustrations, and gladly thank the Cambridge Scientific Instrument Company and Messrs. R. W. Paul for several pictures of apparatus.

Mr. F. Castle has been good enough to grant permission for the inclusion at the end of the volume of a number of mathematical tables selected from his *Logarithmic and other Tables for Schools*.

In conclusion, we desire to thank sincerely Sir Richard Gregory and Mr. A. T. Simmons for the great assistance they have rendered by invaluable suggestions and advice while the book was passing through the press.

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H. MOORE.

UNIVERSITY OF LONDON
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PART I

PROPERTIES OF MATTER

CHAPTER I

INTRODUCTORY

§ 1. General Instructions

In the practical work of any branch of science, the results aimed at may be divided into two kinds: **qualitative** and **quantitative.** In physics, the purely qualitative type of result is rarely desired, the experimental work, even in the elementary stages of the subject, being such as will give quantitative results with little more trouble than would be required to obtain mere qualitative knowledge. For this reason almost every physical experiment involves the taking of one or more measurements, so that physics has been termed, somewhat contemptuously perhaps, 'the science of accurate measurement.'

The fact that *measuring* is such an important part of the practical side of physics, sometimes leads to a student hurriedly taking certain measurements (possibly before he understands the reasons for doing so) without studying the apparatus he uses, and without taking more than one series of observations. Too much stress cannot be laid on the really *practical* side of physics, as distinct from the mere taking of readings. To obtain full benefit from a course of practical physics, not only should the purpose of an experiment be understood thoroughly, but also some time must be spent in setting up the apparatus and studying carefully the construction and working of the component parts, before a single observation is taken.

The purpose of a course of practical physics is not fulfilled

completely unless the student acquires dexterity in the manipulation of apparatus and a considerable sympathy with the instruments he uses.

Having realised the aim of an experiment and the general method to be adopted in carrying it out, the necessary apparatus must be assembled and arranged for use. It is of great importance that all observations and readings should be obtainable without necessitating awkward bodily positions; also any part of the apparatus requiring frequent manipulation or adjustment should be placed within easy reach. Careful attention to these points will react indirectly on the accuracy of the experiment, as greater care will be taken in making observations and adjustments if these can be done in comfort and with ease; there will also be less likelihood of accidentally deranging the apparatus.

It is frequently convenient to go rapidly through an experiment to see that all is in order, before proceeding to carry out any accurate observations.

§ 2. Recording Results—Note-Books

For a course of practical physics two note-books should be used. One of these is reserved for the 'fair' record; it should be a large note-book (quarto size is suitable) with alternate pages ruled in millimetre squares. The other is a smaller note-book for recording observations, and for calculations; in this should be entered also a description of any noteworthy phenomena observed during an experiment, brief notes being taken in the laboratory to be amplified later in the fair record. The taking of these rough notes is quite as important as any other part of the work done, especially if there is any delay in writing up the final account; points which are of considerable importance may be forgotten if no record is made of them at the time they are observed. Rough notes on loose sheets of paper are mislaid easily, and for his own sake the student should avoid taking notes in this manner.

In taking observations and readings, *every measurement made*

should be recorded in the rough note-book immediately, and checked after it has been written down. Each number should have written against it what it represents, and in all cases where a series of sets of observations is made, the observations should be arranged in tabular form. *All calculations required for working out the result must be done in the rough note-book, and must be shown clearly.*

In the large note-book a full record of each experiment should be given, and this should be in the student's own words. The record should be made according to a definite scheme such as that given below.

1. A description of the apparatus used, with diagrammatic illustrations drawn on the squared paper, and lettered for reference.

2. A *short* account of the theory of the experiment.

3. A detailed account of the operations carried out, and of the observations taken. Each reading or observation must be entered; and where series of observations are made, these should be tabulated.

4. The result obtained from the experiment should be entered, *but not the arithmetical working.* In general, it is convenient to enter the result as a compound fraction, followed by the calculated value expressed as a whole number or as a decimal fraction.

The result should be entered *prominently*, preferably occupying the last line of the record. The units in which the result is expressed must be stated.

Wherever possible, the results should be expressed graphically; each graph should occupy one complete page, and the names of the quantities plotted, with the units in which they are represented, must be given.

When a graphic construction forms part of an experiment, the original drawing (or a copy drawn to scale) should be inserted in the note-book.

§ 3. Accuracy of Observations—Null Methods and Deflection Methods

In general, readings should be taken to the highest degree of accuracy obtainable with the apparatus provided. To test the degree of accuracy obtainable the adjustment should be repeated, and the reading taken again with the greatest possible care; any discrepancy between the two readings is attributable to errors inherent in the apparatus, provided sufficient care has been taken in adjustment and reading.

It is good practice for a student to determine in this way the degree of accuracy obtainable in various types of measurement during the earlier stages of a practical physics course. This experience will enable him later to estimate the proportional accuracy of most types of observations, without making an actual determination; though whenever a completely new kind of measurement has to be made, the proportional accuracy should be determined once or twice in this way.

It is worthy of note that, in general, observations which depend on the balancing of two effects so as to neutralise each other are more accurate than observations depending on measurement of the magnitude of an effect. In other words, **null methods are, in general, more accurate than deflection methods.**

When using a null method in any experiment, we balance the effect of an unknown quantity against the effect of a known or standard quantity of the same type. The resulting effect is observed on an instrument which has to detect only the slight difference between the two effects. If we measured the effect of either quantity directly, we should require to use an instrument which gives only a moderate deflection when subjected to the whole effect, *i.e.* an instrument of relatively low sensitiveness. As a result of this low sensitivity, a small, unavoidable error in reading the deflection would have an appreciable effect on the result. If a null method were employed, a much more sensitive instrument could be used—an instrument of the highest

sensitiveness possible. The possible error in adjusting the effects to reduce the reading to zero would probably be the same amount *on the scale of the instrument* as was the error of reading in the deflection experiment; it would, however, indicate a very much smaller error in the quantity under measurement.

An excellent example of this principle is afforded by the measurement of mass. A spring balance measures the mass by a deflection method, the quantity observed being the extension of the spring due to the weight of the body. An ordinary balance is an apparatus which depends on a null adjustment, and gives much greater accuracy than can possibly be obtained by a spring balance designed to weigh up to the same limit.

It is important to know that, in general, observations which depend on weighing are much more accurate than determinations of either length or time. Wherever possible, experiments should be designed in such a way that the most important observations are made by means of a balance. As an example of this, the student is referred to the experiment on the expansion of liquids by means of a weight thermometer. This experiment is so arranged that the coefficient of increase of *volume* is determined without a single determination of volume being made, every observation from which the result is calculated being a 'weighing.'

With the exception of the determination of length by optical methods, which requires elaborate apparatus, no other physical determination can be made so accurately as the determination of mass or the comparison of masses, owing to the great sensitiveness which is obtainable with a well-designed balance. The only other simple physical measurement which approaches this in accuracy is the determination of electrical resistance by means of a Wheatstone's Bridge, again a *null* experiment.

§ 4. Calculation of Results

Since there is a limit to the accuracy obtainable in the determination of any physical quantity, it is obvious that there

must also be a limit to the accuracy of any result calculated from such determinations. In calculating results, therefore, it is unnecessary to work to a greater number of significant figures than the observations merit; the final result can only be trusted to a certain number of places, and any figures beyond this number are meaningless.

Much labour will be saved in arithmetical calculations if, at each stage in the calculation, the number of significant figures in the result of that stage is suitably cut down before proceeding to the next part of the calculation. For example, consider the determination of the volume of a cylinder 2·37 cm. long, and 1·13 cm. in diameter. Its volume is given by $\frac{\pi}{4} \times (1{\cdot}13)^2 \times 2{\cdot}37$ c.c.

Neither of the measured quantities is more accurate than 1 *in* 1000, and there is therefore no need to retain more than four figures after any arithmetical process; and π may be taken as 3·142, or even as 3·14.

$(1{\cdot}13)^2$ is 1·2769, and may be taken as 1·277.

1·277 × 2·37 is 3·02649, and may be taken as 3·026.

3·026 $\times \frac{\pi}{4}$ is 3·026 × 0·7854, which gives as the product 2·560404, and the final result is written down as 2·56 c.c.

Contracted methods of multiplication are of great value in simplifying the arithmetical work.

It has become customary to give results only to such a number of figures as can be claimed to be accurate. If therefore a result is stated to five figures, it is at once assumed that accuracy to five figures is claimed. To write down five figures in a result which is only accurate to 1 in 1000 is thus not only unnecessary but actually misleading, as giving an erroneous idea of the accuracy of the observations on which it is based.

If it should happen that the last significant figure is a cipher, this is included in the result even if it is after the decimal place,

its inclusion indicating that accuracy is claimed to that number of significant figures, *e.g.* 1 in. = 2·5400 cm. indicates that this is true to 1 part in 25,000.

In many cases where the numbers dealt with are very large, powers of 10 are put after a small number instead of writing a large number of ciphers after it. As an example of this method of stating the magnitudes of quantities, 28,000,000 may be written as $2{\cdot}8 \times 10^7$ if accuracy of 3 or 4 per cent only is claimed, but $2{\cdot}80 \times 10^7$, $28{\cdot}0 \times 10^6$, or 280×10^5 if the accuracy is 1 in 300. Similarly negative powers of 10 are used for extremely minute quantities instead of writing a number of ciphers after the decimal point. Thus, 0·00003500, indicating accuracy of 1 in 3000, could be written as 3500×10^{-8}, but to write it as $3{\cdot}5 \times 10^{-5}$ would be incorrect, as claiming an accuracy of 3 per cent only instead of 1 in 3000, the accuracy merited by the observations.

In calculations made by logarithms the order of the approximations made depends on the number of figures in the tables of logarithms used. Four-figure logarithms give an accuracy of about 1 in 2500 in a calculation *involving four or five numbers*, the possible *error increasing* with the number of factors to be multiplied together or divided. Five-figure logarithms are about ten times as accurate as this, while with a slide-rule (10 inch), if more than four factors are multiplied or divided, greater accuracy than 1 in 500 is not obtainable except with much care.

In most cases, before performing the accurate calculations, approximate calculations should be made in order to determine the *order of magnitude* of the result (or the position of the decimal point); *this precaution is of special importance when a slide-rule is used*, especially in inexperienced hands.

An example of this may be given, using the dimensions of the cylinder on p. 8.

$$V = \frac{\pi}{4}(1{\cdot}13)^2 \times 2{\cdot}37.$$

Roughly this may be written down as

$$\begin{aligned} V &= 1 \times 1{\cdot}0 \times 1{\cdot}2 \times 2{\cdot}5, \\ &= 3, \end{aligned}$$

i.e. the volume is of the order of 3 c.c.

The slide-rule gives the number 256 as the result, and this can be written at once as 2·56 c.c., the position of the decimal point being determined by the result of the rough approximation just given.

In calculating the results of an experiment where a certain quantity is to be determined from measurements of a number of independent quantities, it is *not* advisable, as a rule, to express that quantity explicitly in terms of the various measured quantities. This procedure generally results in a long and complicated expression, which is difficult to work out and liable to occasion arithmetical errors. More important than this, however, is the definite loss of meaning the various quantities suffer when grouped together in a complex expression. The *physical significance* of each step of the calculation should be kept in view as far as possible.

As a particular example of this, consider the expression on p. 147. The equation $mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$ indicates three definite physical quantities, and if retained in this simple form it conveys a meaning at once, and can be written down from first principles immediately. To express 'I' explicitly would be to destroy the meaning of the equation to a very great extent.

The use of 'formulae,' which often put a meaningless burden on the memory, should be avoided; and, wherever possible, an endeavour should be made to work out the solution of a problem by reasoning from first principles.

§ 5. Graphic Methods

The use of graphic methods is of great value both in theoretical and experimental physics. Whenever the observations taken in an experiment form series relating to two interdependent quantities, a graph should be drawn to illustrate the connection between them. The graph shows the way in

which the dependent variable (y) depends on the independent variable (x). It is customary to plot the values of the independent variable as the abscissae horizontally from left to right, and the values of the dependent variable as the ordinates upwards, when the paper is held in a vertical plane.

As a typical example may be cited the experiment on the simple pendulum. In this, the period t of a pendulum of length l is measured, l being varied arbitrarily, and the corresponding values of t being determined. Here l is the independent variable and should be plotted horizontally, t being plotted on a scale running from the bottom of the page to the top.

The units in which the variable is expressed, and the designation of the variable, must be marked clearly along the corresponding co-ordinate axis. Great care must be taken in choosing the scale to which each variable is plotted, so that the resulting graph may cover as large a portion of the sheet as possible.[1]

The points indicating the observations should be shown by dots with small circles drawn round them, or by small crosses. A *smooth* curve should then be drawn to represent the average distribution of the points, *i.e.* the curve should pass as evenly as possible between the points so that there are about as many on one side of the line as there are on the other. A test should first be made to see whether the graph can be represented by a straight line. A line ruled on a long strip of glass or celluloid is useful for testing this, as it is possible to see the points on *both* sides of the line. If a straight line cannot be drawn through the points, a curve should be drawn, either freehand or by means of a thin flexible strip of wood bent to fit the curve.

If the resulting graph is a straight line the relation between the variables is of the form $y = mx + c$, where m and c are constants. If the graph is not a straight line the form of the curve may suggest the relation between the variables. Familiarity with curves corresponding with equations such as

[1] It is assumed here that a sheet such as is found in the student's note-book is employed. On a large sheet, too large a scale would tend to exaggerate accidental errors of observation.

$$y = x,\ y = x^2,\ y^2 = x^3,\ y = x^3, \text{ and } y = \log x,$$

will guide the student as to the *type* of curve most likely to fit the observations plotted. Then by plotting *powers* of one of the quantities against the other, a straight line may result. Or a straight line may be obtained by plotting the logarithm of one quantity against the other quantity, or against the logarithm of the other quantity.[1] When a straight line graph has been obtained, the connection between the two physical quantities concerned can be expressed by means of an algebraic equation.

Results can often be obtained by means of graphic methods with much less labour than arithmetical calculations would entail. Reference should be made to the instances considered in the text (*e.g.* pp. 83-89, 238, 241, 276).

§ 6. Units employed in Physical Measurement

The measurement of any quantity is expressed in a phrase of two parts—the **number** and the **unit.** Thus '12 seconds' contains the number 12, and the unit of time, the second. Each of the various quantities dealt with in physical measurements requires a unit. It is, however, possible to express some quantities *in terms of other quantities*; we can, for example, express speed in terms of the distance travelled in a certain time, and it is obviously advantageous to measure speed in units which bear a *simple* relation to the units of length and time. All the physical quantities that are met with in mechanics may be expressed in terms of *three selected* quantities. The three independent units for these quantities are said to be the **fundamental units of the system of units,** the other units of the system being called **derived units.**

In scientific work the fundamental quantities chosen are **length, mass, and time.** The units employed for these quantities are the **centimetre,** the **gram,** and the **second,** so that the system is known as the **C.G.S. system of units.**

The **centimetre** is one-hundredth part of the metre, which is

[1] See Expt. 43.—Friction of rope over a fixed pulley (p. 95).

defined as the distance between the ends of a certain rod of platinum preserved in Paris.

The **gram** is one-thousandth part of the kilogram, which is the mass of a certain cylinder of platinum preserved in Paris. The kilogram was intended to have the same mass as one cubic decimetre (1000 c.c., or 1 litre) of distilled water at the temperature of its maximum density. Consequently the mass of 1 c.c. of water at 4° C. is almost exactly 1 gram.

The **second** is the mean solar second, defined as 1/86400 of the mean solar day, which is determined by the time of rotation of the earth on its axis.

CHAPTER II

MEASUREMENT OF FUNDAMENTAL QUANTITIES

§ 1. Measurement of Mass

THE BALANCE

The measurement of mass by means of an ordinary **balance** consists in balancing two forces against each other so that their turning moments on a lever are equal and opposite. When this is achieved, the forces themselves, if parallel to each other, are inversely proportional to the distances of their points of application from the fulcrum of the lever. **The forces which act on the beam of a balance are the weights of the masses suspended from the beam**, and thus the ratio of the *weights* of these masses is determined. As, however, the weight of a body is proportional to its mass, the ratio of the *masses* is the same as the ratio of the *weights*, *i.e.* **the ratio of the masses suspended from the beam of a balance when in equilibrium, is the reciprocal of the ratio of the 'arms' from which they are suspended.**

In an ordinary balance, the beam is a stiff rod sometimes of girder construction, which is supported at some point on **knife-edges** resting on flat plates at the top of the pillar of the balance. At the two ends of the beam are mounted knife-edges from which the scale-pans are suspended; the two parts of the beam are called the **arms** of the balance. Knife-edges must be used for the fulcrum and for the points of suspension of the scale-pans in order that the arms of the balance shall be of a definite length. As the ratio of these two is the reciprocal of the ratio of the masses on the

scale-pans when balanced, it is obvious that this ratio must be accurately known, hence the arms themselves must have exactly defined lengths. In general this ratio is one of equality, but occasionally a ratio of 10 to 1 is used.

The knife-edges have to support a considerable weight, and hence must be made of very hard material, so that they will not be deformed when the balance is loaded. Hardened steel is used for the knife-edges of balances of moderate accuracy, but agate is used for the more delicate balances for scientific work. In order to reduce wear of the knife-edges, a lever is generally fitted whereby the beam can be raised from the knife-edge supports, when the

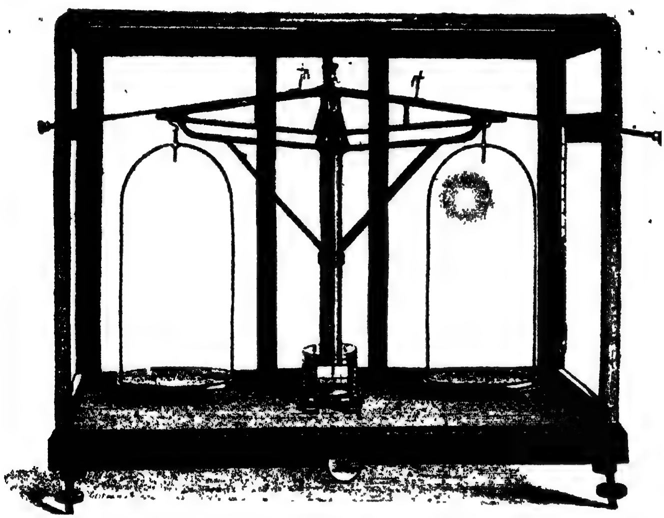


FIG. 1.—Sensitive Balance.

balance is not in use, and allowed to rest on a brass bar supported in a fork of brass. This same lever raises the scale-pans, so that their weight does not rest on the knife-edges at the ends of the beam. The arrangement is called the **Arrestment** of the balance.

In order to avoid chipping or otherwise deforming the knife-edges, it is essential that the beam should be raised or lowered so as to rest in the brass fork, before moving the balance, or altering the weights on the scale-pans. The beam must be raised and lowered *gently* for the same reason.

For most purposes the arms of an ordinary balance may be assumed to be exactly equal, and therefore the mass of the body being 'weighed' may be taken as equal to the mass of the 'weights'

which are required to balance it. Even if the arms are *not* exactly equal, this need not affect the accuracy of most experiments in the slightest, provided that the 'weights' are always used on one pan and the unknown mass on the other. If this is done, the 'weights' used, though not equal to the unknown masses, will bear a constant *ratio* to these, and since in most experiments the *ratio* of the various masses used is required, the actual result will be unaffected.

A good rule is to place the weights always in the right-hand pan, and the unknown mass in the left.

In using a balance for comparing masses, it is essential that the beam and scale-pans should be balanced accurately when unloaded. Then, when the masses have been adjusted till the beam is in equilibrium again, the masses in the two scale-pans can be taken as equal. The beam will rest horizontally or oscillate about a horizontal position when in equilibrium. In order that this may be tested, the beam is furnished with a long pointer rigidly fixed to it, the end of the pointer moving over a small scale fitted to the pillar supporting the beam. When the beam is horizontal, the end of this pointer oscillates about the middle of the small scale, and thus a sensitive method of testing the horizontality of the beam is provided. Before loading the balance, the balance case must be levelled by means of the levelling screws, so that the base may be horizontal as tested by the plumb-line or spirit-level attached to the apparatus. The beam should then be released so as to rest on the knife-edges, and the motion of the end of the pointer be observed. Usually these oscillations will not be exactly about the middle of the scale, but provided the mean position is not far from the centre, the balance can be used without further adjustment, *the weights being always adjusted until the pointer oscillates about the same position as when the beam was unloaded.* This is called working to a **false zero.**

If the pointer has a mean position several divisions from the centre when the balance is unloaded, it is advisable to correct this before commencing to weigh. This can be done usually by moving a small nut along a screw fitted to one end of the beam, or by altering the position of a 'flag' mounted on the beam. *This should not be attempted until the student has become familiar with the handling of balances, and care must be used in doing it, so as to avoid damaging any part of the balance, particularly the knife-edges.*

Having seen that the balance when unloaded oscillates about the zero position, or having determined the false zero if this is not the case, the beam should be lowered and the unknown mass placed gently on the left scale-pan. Weights from the box of weights should then be placed on the right-hand pan, commencing with the

larger weights and proceeding downwards. *The beam should be lowered before touching the scale-pan either to add or remove weights.* This rule must be observed even for the *smallest* weights. In testing for balance at first, it is unnecessary to raise the beam completely, the want of balance being obvious as soon as the beam begins to rise. The beam need not be raised to the full extent until the centigram weights are being used.

In some cases weights smaller than 1 centigram are not supplied, and a 'rider' is used for getting the weight to milligrams or less. This rider is a wire bent so as to 'ride' on the top of the balance beam, the mass of the wire being usually 1 centigram. The beam is divided into parts equal to one-tenth of the length of the arm, and the position of the rider is noted when adjusted till it gives the exact balance required. Obviously, a centigram rider at a point one-tenth of the distance along the arm is equivalent to 1 milligram in the scale-pan at the end, and so on. Thus with a centigram rider the weight of a body can be determined to within 1 milligram or less, provided the arm of the balance is subdivided in this way and the balance is sufficiently sensitive to detect a difference of this order.

A box of weights requires as much care as the balance with which the weights are used. Any corrosion or oxidation will alter the mass of a 'weight'; great care should be taken therefore to keep the weights from contact with acids, mercury, or water. It is obviously absurd to weigh to 1 milligram if one of the larger weights is wrong through corrosion to an amount greater than this. *All* weights of any accurate box should be lifted with the forceps provided in the box, this rule being applicable to the largest weights as well as to the smaller. Care should be taken not to bend the smaller weights; they should be held by the corner or side bent up for this purpose. To facilitate handling the smaller weights, they can be put on the top of the larger weights when on the scale-pan.

The weights used in any experiment should as far as possible all come from one box. If two boxes must be used, the weights should be returned to their respective boxes.

In reckoning up the weights, having 'weighed' a body, the weights should be counted whilst on the scale-pan and the total recorded in the note-book. They should be removed one at a

time, the total being checked as they are returned to their places in the box; in this way any error will be observed and corrected. Failure to observe this precaution may frequently cause a weighing to have to be repeated, or may render a whole experiment useless.

The weight of a body cannot be determined accurately when there is an appreciable difference between the temperature of the body and that of the room, on account of the convection currents set up in the air. If the body is colder than the atmosphere, moisture may condense upon it and make the observed weight too large.

No corrosive liquid should be allowed inside the balance case except in a securely stoppered vessel, and all vessels containing liquid should be wiped clean on the outside before being put on the scale-pan.

EXPT. 1. **Determination of the Mass of a Body by Means of the Balance.**—Level the balance case by means of the levelling screws. Turn the handle of the arrestment so as to release the beam, and see that the beam rests without constraint on the knife-edges. If the beam does not begin to swing, start a gentle current of air by a rapid movement of the hand above one of the pans. Observe the mean position of the pointer on the scale as the beam swings from side to side. Use the arrestment to stop the swinging of the balance when the pointer is near the mean position. Place the unknown mass on the left-hand pan, and place in the middle of the right-hand pan a weight estimated to be large enough to counterbalance the load on the left. Release the beam and note whether the weight is too large or too small. Continue the process of weighing, passing from the larger to the smaller weights in regular order. Remember always to arrest the motion of the balance before adding or removing weights. When the pointer swings about the same mean position as at first, stop the motion by means of the arrestment, count the weights as they lie on the scale-pan, and record the result. Count the weights again as they are removed one at a time to their places in the box. Determine in this way the masses of two bodies A and B. Check the result by determining the mass of the two bodies together and seeing that this is the sum of the two separate masses.

§ 2. Measurement of Length

The measurement of length is possibly the simplest exercise required of a student of physics, and the use of scales of length is familiar to every one before commencing any precise scientific work at all. The accuracy required in various kinds of length measurement is, however, widely different, and we must consider the methods of obtaining these various degrees of accuracy in typical cases.

It should be noted here that in all measurements of length *two* observations must be made, one at each end of the length measured, and that therefore the possible error in the value of the length obtained is *double* the error of each observation.

The accuracy of observations made with an ordinary scale is limited, because the dividing lines have a finite thickness, and because the eye cannot estimate fractions of divisions to nearer than 0·1 mm. In any estimation of length made with an ordinary scale, therefore, the accuracy is not greater than about 0·2 mm. If accuracy of a higher order than this is demanded, it is essential that apparatus should be used to assist the eye, and also that the divisions on the scale should be marked with fine regular lines. The error in using an ordinary scale may be even greater than 0·2 mm. if the scale is used 'flat,' for an appreciable **parallax error**[1] is then possible owing to the thickness of the scale (Fig. 2).

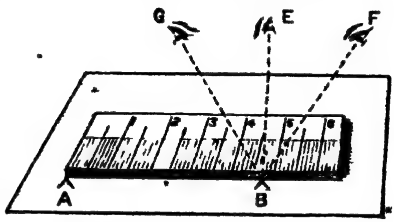


FIG. 2.—Error due to Parallax.

The *graduated edge* of the scale must always be placed in contact with the points whose distance apart is to be measured, the scale being stood on its side if necessary, as would, for example, be the case when measuring the distance between two marks on a sheet of paper (Fig. 3).

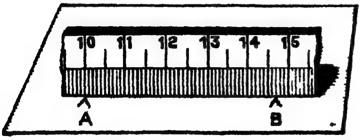


FIG. 3.—Correct Method of using Scale.

[1] Parallax means an apparent change in the position of an object due to a change in the position of the observer (p. 229).

When it is not possible to measure the size of an object by direct application of a rule, a pair of dividers, or *inside* or *outside* callipers, may be employed. In some cases a beam compass is useful; this is a rigid bar provided with two sliding pieces to which are fixed, at right angles to the bar, the points of the compass.

PRINCIPLE OF THE VERNIER

A very ingenious device for obtaining accuracy of a greater order than that obtainable by eye-estimation was invented by P. Vernier (1580–1637), and is known by his name. In this device a small auxiliary scale is provided, which slides along the ordinary scale, the divisions of this **vernier scale** being either a little longer or a little shorter than the divisions of the ordinary scale.

The great value of this device lies in its simplicity, and in the fact that it can be used to measure to any fraction of a division required, if the auxiliary scale is divided suitably.

The form most generally used is that in which the vernier divisions are slightly shorter than true scale divisions, and therefore this type only will be described, though the principle underlying both forms is the same.

The auxiliary scale is graduated from a division which may be called the zero of the vernier, this division being indicated by an arrow or some other distinguishing mark. The scale consists of n equal divisions on one side of the vernier zero, and in some cases it is continued one or two divisions on the other side of the zero. These n vernier divisions are exactly equal to $n - 1$ scale divisions. Consequently one vernier division is equal to $\frac{n-1}{n}$ or $1 - \frac{1}{n}$ of a scale division. Thus, **each vernier division is shorter than a scale division by $1/n$ of a scale division.** This quantity $1/n$ of a scale division is called the **Least Count** of the vernier; as we shall see, the vernier can be used to measure to the nth part of a scale division.

Suppose the vernier scale is moved along the main scale till the zero of the vernier is exactly opposite one of the divisions of the main scale, then the distance between the zero of the main scale and the zero of the vernier (which is the distance we want to find) is an exact number of scale divisions. The other vernier divisions will not exactly correspond with scale divisions, being respectively $1/n$, $2/n$, $3/n$, etc. of a scale division on the *zero side* of the consecutive scale divisions. Now suppose the vernier is moved a little further along the main scale until the zero of the vernier has moved through $1/n$ of a scale division. It is clear that the first division of

the vernier will have moved till it is exactly in line with a certain division on the main scale. If the vernier is again moved so that the zero passes over $1/n$ of a scale division the *second* division of the vernier will be exactly in line with a division on the main scale. If the total movement of the zero of the vernier is $3/n$ of a scale division, the *third* division of the vernier is brought opposite a division on the main scale, and so on. In general, if the mth division of the vernier comes into line with one of the divisions of the main scale, it indicates that the zero of the vernier has moved through m/n of a scale division from the division immediately before it.

In using a vernier scale, therefore, the least count must first be determined. The reading is then taken according to the following **Rule: Read the scale division next before the zero of the vernier scale; find the number of the vernier division which is in line with a scale division, and add this number of nths of a scale division to the scale reading.**—The result gives the distance from the zero of the main scale to the zero of the vernier scale.

Consider the two following examples of the way in which a vernier should be examined and used:

(1) A vernier scale has 10 divisions, the ordinary scale being a scale of millimetres, and the 10 vernier divisions are equal in length to 9 millimetres. The vernier is placed with its zero between the divisions 26 and 27 mm. on the ordinary scale, and the 7th division along the vernier is exactly in line with a division along the mm. scale. The reading is required.

The vernier reads to decimals of 1 mm., or the least count is

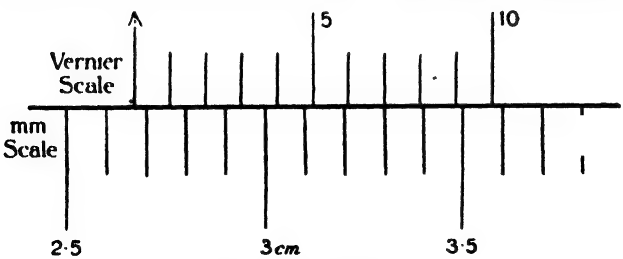


FIG. 4.—Vernier scale.

0·1 mm., because 10 vernier divisions are equal to 9 scale divisions, and the scale divisions are millimetres.

The reading of the scale division *next before the vernier zero* is 26 mm. The 7th vernier division is in line with a scale division (and the vernier reads to 10ths of scale divisions). *Therefore the reading is* 26·7 *mm.*

The scale division which is in line with the 7th vernier division has nothing at all to do with the reading.

(2) A circular scale is divided into angles of 1°, and each degree is divided into three equal parts, so that the scale may be *said to consist of large* divisions of 1° each, and *small* divisions each equal to $\frac{1}{3}°$.

A vernier scale of 20 divisions moves over this, the 20 vernier divisions being equal to 19 *small* scale divisions. The zero of the vernier is between the large divisions marked 8° and 9°, and is in the *last* section of this large division. The coincidence between a vernier division and a scale division occurs at the 4th vernier division, and the angle reading is required.

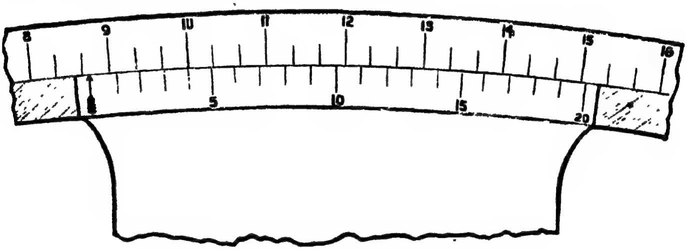


FIG. 5.—Angular Scale and Vernier.

The vernier reads to *twentieths* of the *small* scale divisions. The reading must therefore be made to the small scale division next before the zero of the vernier: in the example given this is $8\frac{2}{3}°$.

The vernier reading is 4, *i.e.* we must add to the scale reading an amount equal to $\frac{4}{20}$ of a small scale division.

The reading thus will be $8\frac{2}{3}° + \frac{4}{20}$ of $\frac{1}{3}°$. But $\frac{1}{3}°$ is equal to 20′, and therefore we can write the *scale* reading as 8° 40′ and the vernier reading as 4′, so that the full reading is 8° 44′.

Thus the vernier and scale can be used to measure to one minute of arc, the scale being graduated to 20′ divisions, and the vernier reading to $\frac{1}{20}$th of these small divisions, *i.e.* to 1′.

The method of reading any type of vernier can be worked out in a similar way.

An instrument called the **Vernier Callipers** is used for measuring the linear dimensions of bodies. It consists of a metal rule furnished with two jaws, A and B, projecting at right angles to

the rule. Of these, one is fixed, whilst the other can slide backwards and forwards. On the rule is engraved a scale divided into millimetres. The sliding jaw is also provided with a short scale V called a vernier.

Expt. 2. **Measurement of the Length of a Rod by Means of the Vernier Callipers.**—If the instrument is adjusted correctly, the zero of the vernier will coincide with the zero of the millimetre scale, when the sliding jaw is brought into contact with the fixed one. If this is not the case, the instrument possesses a 'zero error,' which must be read and allowed for in making measurements. Determine the least count of the vernier.

To measure the length of an object, it is placed between the fixed and sliding jaws, and the latter is adjusted till it makes

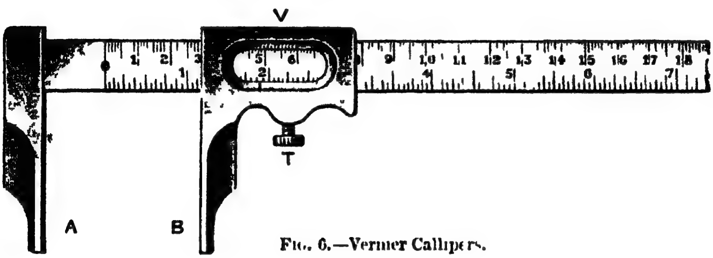


Fig. 6.—Vernier Callipers.

contact with one end of the object when the other is in contact with the fixed jaw. In dealing with small bodies it is convenient to adjust the pressure till it is just sufficient to hold the object between the jaws. The reading on the millimetre scale, which is just before the zero of the vernier, is then taken.

This reading is the distance between the zero of the vernier and the zero of the millimetre scale, and since these should coincide when the jaws are closed, it should be the distance between the jaws, *i.e.* the length of the object.

In general, the zero of the vernier is not exactly opposite a division on the rule, and it is necessary to determine the value of the fraction of a millimetre. This is done by means of the vernier scale. Look along the vernier scale until a graduation is seen which is exactly in line with one of the marks on the millimetre scale. If the correspondence is at the third graduation of the vernier scale, and the least count is 0·1 mm., the fraction required is 0·3 mm.; if it is at the fourth graduation the fraction is 0·4 mm., and so on. The

reason for this has already been discussed (p. 20): the actual distance required is obtained by taking the scale reading *next before* the zero of the vernier and adding to this the vernier reading as above described.

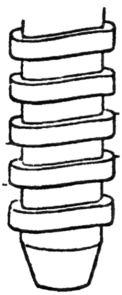
Measure carefully the lengths of two rods of glass or metal cut from a long uniform rod. Find the ratio of these two lengths. Weigh the two rods on a balance, and find the ratio of the two weights. Assuming the original rod to be uniform, these ratios will be the same.

PRINCIPLE OF THE MICROMETER SCREW

Another form of apparatus which enables determinations of length to be made with considerable accuracy is an accurately cut screw thread working in a close-fitting nut.

In general, there is a **circular head** of a large diameter fitted to the screw and moving past a scale fixed parallel to the axis. The head is subdivided into a definite number of equal divisions, so that the screw can be turned through fractions of a revolution and these fractions read on the micrometer head.

In one complete revolution the point of the screw advances a distance equal to the **pitch** of the screw, this being the distance between *similar* points on consecutive turns of the thread. If, therefore, we turn the head of the screw through *one-hundredth of a revolution*, the point of the screw will advance by *one hundredth of the pitch* and so on; hence the screw point can be moved forward by minute *known* amounts, provided the pitch is known. The accuracy obtainable by the use of micrometer screws is limited only by the accuracy with which the screw is cut and fitted to the nut. Where extreme care has been taken, as in grinding the screw for the ruling of Diffraction Gratings, it is possible to set off small distances accurate to the hundred-thousandth part of a centimetre.



p = pitch of screw

FIG. 7.—Screw.

It is worthy of note that, owing to wear between the screw and the nut, there may be an appreciable amount of slackness in the fit

of these. This causes what is called **back-lash,** *i.e.* if the screw has been adjusted by turning it in one direction, and is then turned *back*, the head may be rotated through an appreciable angle before the screw begins to move along its axis. Error due to this may be avoided to a considerable extent by always turning the screw in the *same* direction when making the *final* adjustment to any particular position. Even this will not prevent error if the screw thread has worn unevenly in different parts, and a badly-worn screw should therefore be replaced by a new screw *and nut* if accuracy is desired.

The Micrometer Screw Gauge is an instrument for measuring the linear dimensions of small objects. It depends on the fact that when a perfect screw works in a fixed nut, the motion of translation of the screw is directly proportional to the amount of rotation that is given to it. By using a screw of fairly fine pitch, and by arranging for the measurement of small fractions of a turn, very small distances can be measured with accuracy, as already described.

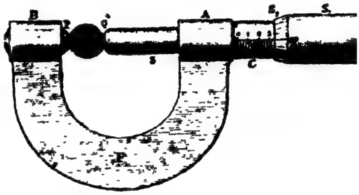


FIG. 8.—Micrometer Screw Gauge.

For scientific work, a screw the pitch of which is $\frac{1}{2}$ mm. or 1 mm. is frequently used. In engineering work the pitch is frequently $\frac{1}{50}$th inch (nearly but not quite the same as $\frac{1}{2}$ mm., since a metre is nearly 40 inches).

EXPT. 3. **Measurement of the Thickness of a Plate by Means of a Micrometer Screw Gauge.**—In using the micrometer screw gauge it is necessary to find first of all the pitch of the screw, that is, the distance through which it travels for one complete turn. The screw itself is concealed in the nut A, but if the divided head is screwed out a little way a scale C will be found engraved on the nut, from which the pitch of the screw can be found readily. Determine the length of each division of this scale by comparing it with an inch or centimetre rule and notice how many complete turns of the screw are required to carry it from one division to the next.

Notice next the number of divisions on the cylindrical divided head S_2, and determine the travel corresponding to rotation through one division.

For example, if the pitch is $\frac{1}{2}$ mm. and the head is divided into 100 parts, each division corresponds to a movement of

$\frac{1}{200}$ mm. or 0·005 mm., and two divisions correspond to $\frac{1}{100}$ mm. or 0·01 mm.

In the micrometer screw gauge there is a butting point P rigidly attached to the nut by means of a bent arm F. When the point Q of the screw is brought into contact with it by turning the head with a gentle pressure of the fingers, the zero of the scale on the divided head should correspond with the zero of the scale on the nut. If this is not the case, the instrument possesses a zero error which must be observed and allowed for.

Care must be taken to avoid screwing the point of the screw against the butting point with pressure. This treatment would damage the threads and distort the frame of the instrument. In some instruments there is a 'free wheel' device which allows the head to turn freely in the fingers when the pressure exceeds a definite limit. This arrangement tends to eliminate uncertainties in reading caused by differences in pressure.

To measure the linear dimensions of an object, the screw is turned back until the body can be inserted between the point of the screw and the butting point. The point is then screwed forward till the object is held gently between the two, the pressure of the fingers on the head of the screw being as nearly as possible the same as that used in the first observation. It is convenient to make the adjustment by holding the smooth part S_1 of the head, not using the milled ridge H at all. The screw is turned till the fingers *slip* on S_1 when *lightly* gripping it.

Take the reading of the gauge in this position and add or subtract the zero error as the case may require, *i.e.* subtract *algebraically* the zero reading from the reading obtained. Repeat the observations several times and take the mean of the results.

Measure in this way the thickness of a metal plate, repeating the observations at different points of the plate so as to obtain the mean thickness. Measure also the mean thickness of a second plate of the same metal having the same outline and, consequently, the same area. Find the ratio of these two thicknesses. Weigh the two plates, and find the ratio of the two weights. Assuming the plates to be uniform and of the same density, the ratio of the thicknesses will be the same as the ratio of the weights.

MICROMETER MICROSCOPE

There are many optical methods of making accurate measurements of length, among them being the microscope with a **Micrometer Eye-piece.** A fine transparent scale is fixed near the focus of the eye-piece; in some instruments a spider line can be moved across the scale by means of a micrometer screw to measure fractions of a division. The microscope is used to give a magnified image of the object to be measured (see p. 282). A real image is formed near the focus of the eye-piece and is compared with the fine scale placed there, the scale being seen at the same time as, and superposed on, the image of the object viewed. A body of known size is then viewed with the microscope in order to find the magnification produced by the objective, and thus the size of the small object can be determined. It is essential that the adjustment of the microscope should remain unaltered for the two observations.

If, for example, the magnified image of the small object occupies 52·4 micrometer divisions, and a millimetre scale seen through the microscope, when in the same adjustment, covers 40·3 micrometer divisions per mm., it is obvious that the small object is 1·300 mm. across. The chief use of the microscope with a micrometer eye-piece is, however, for accurately comparing small distances, not for their actual determination in mm or cm. It is largely used in some forms of investigation, for observing and measuring the minute motions of the gold leaf of an electroscope.

TRAVELLING MICROSCOPE

In the **Travelling Microscope** or **Vernier Microscope** a compound microscope is mounted so that it may be moved in a direction at right angles to its axis by means of a screw or a rack and pinion. The distance through which the microscope is moved can be read on a fixed scale with the aid of a vernier that moves with the microscope. In the instrument illustrated in Fig. 9 the microscope has both a vertical and a horizontal traverse. It has also an angular motion so that it may be used

with the axis vertical or horizontal, or inclined to the horizontal at any angle. The eye-piece should be provided with cross-wires, and in focussing on any object the intersection of the cross-wires should be brought *into coincidence with the point of the* object to be observed. To measure the distance between two points the microscope is focussed first on one and then on the

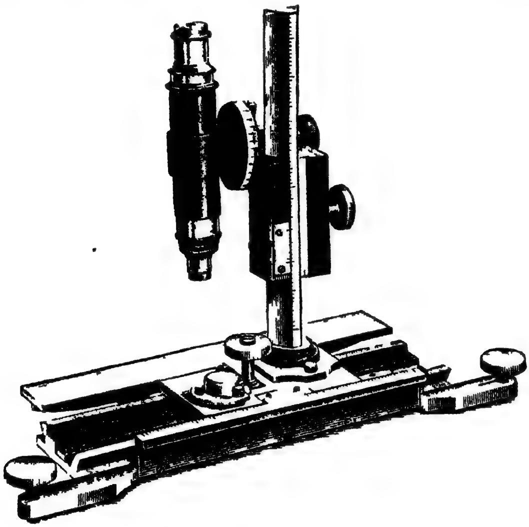


FIG. 9.—Travelling Microscope.

other. It is necessary that the line joining the points should be parallel to the direction of traverse of the microscope. The difference between the readings in the two cases gives the distance required. Examples of this method will be found in Expts. 77 and 78.

The comparison of two lengths may be carried out by a **substitution method** using two vernier microscopes.

EXPT. 4. **Comparison of the Yard and the Metre.**—Set up securely two vernier microscopes so that the line joining them may be parallel to the direction of travel of each microscope. Arrange supports for the two scales so as to

raise their engraved faces to the *same* height above the table, adjusting this height so that the divisions may be focussed by the microscopes. Focus one microscope on a division at one end of the yard scale, and the second microscope on a division near the other end. Bring the centre of the cross-wires over the centre of the division in each case. Note the distance in inches between the cross-wires of the microscopes. Now remove the yard scale and substitute the metre scale. If the preliminary adjustments were made correctly, the divisions on this scale should come into focus at both ends. Move the scale until a division near one end exactly coincides with the centre of the cross-wires at that end; then the centre of the cross-wires at the other end will fall between two scale divisions. Move the microscope at this end by means of the slow adjustment *towards* the microscope at the other end till the centre of the cross-wires coincides with a scale division. Observe the distance through which it moves by means of the vernier and scale of the instrument, and note also the distance between the divisions of the metre scale. Then the number of inches in the first observation is equal to the number of millimetres in the second *plus* the distance measured on the scale of the microscope. From this result may be calculated the length of the inch or of the yard in centimetres.

§ 3. Measurement of Time

Of all the measurements with which we have to deal in elementary physics, that of Time is the most difficult. The scientific unit of time, the **mean solar second**, depends, as we have said, on the period of rotation of the earth on its axis. This period is determined by astronomical observations. To obtain multiples or submultiples, we employ a mechanism—a clock or watch—designed on the assumption that the oscillations of some body—a pendulum or balance-wheel—are **isochronous**, that is, of equal duration, and consequently mark equal intervals of time. This vibrating body is the essential part of the apparatus, the rest being merely an arrangement for counting the oscillations. No mechanism which is absolutely trustworthy and regular has yet been devised for the measurement of time. The **clock rate** can, however, be determined by astronomical methods. In

order to measure an interval of time, the period is observed by means of a clock or watch, and this period is then corrected by the proper factor depending on the clock rate.

In all but the most exact determinations it may be assumed that the time intervals given by a well-regulated clock or watch, keeping civil time, correspond accurately with mean solar time.

Even if the clock or watch does keep correct time, there are unavoidable errors in time observations which are a direct consequence of the usual mechanism. In most cases the seconds-hand does not move uniformly but in a series of jerks, receiving an impulse each time the balance-wheel or pendulum passes through its position of rest. When therefore a stop-clock or stop-watch is started, there is a possible error equal to half the period of vibration, and a similar error exists when it is stopped again.

Suppose, for example, that the watch ticks every one-fifth of a second, then if it is just approaching the position of rest when it is started, the seconds-hand will jump forward one-fifth of a second immediately the watch starts. Or again, if the watch is stopped just as it is about to tick, the final one-fifth of a second will not be recorded, whereas the slightest possible delay in stopping the watch would have recorded it.

It will be seen, therefore, that stop-watch and stop-clock determinations of time intervals cannot be relied upon to closer than one 'tick' of the watch or clock, even if the clock rate is quite accurate.

For accuracy of a given order it is therefore essential that the time observations shall be prolonged over a certain length of time determined by the duration of the 'ticks' and the accuracy required: accuracy of 1 in 1000 demands a period of more than three minutes if a watch ticking fifths of seconds is used, and so on.

EYE AND EAR ESTIMATIONS

If an ordinary clock or watch is used instead of a stop-watch the possible error is even greater, owing to the difficulty of

estimating exactly the position of the moving seconds-hand. This may be got over to some extent by combining eye and ear observations, and as this method is used frequently in certain types of work, it will be described here briefly.

Suppose that observations are being made on the motion of a pendulum. In commencing the time observations the observer starts counting ticks as the seconds-hand commences a fresh minute or passes some other convenient point. This counting is then continued by *ear*, the *eye* being turned to observe the pendulum. If the pendulum passes the middle of its swing between the 17th and 18th ticks, it is easy to work out the exact moment when this passage took place, and hence the commencement of the set of swings to be observed is known to the nearest tick of the watch. When the last vibration of the pendulum is completed, *i.e.* when the pendulum is moving through the middle of the last swing to be observed, the observer begins to count watch ticks again, and continues to do so until he can look at the watch face and observe the time corresponding to the ticks he is counting. An example will illustrate the method:

Counting started at 2 h. 31 m. 0 s.
Pendulum passed middle point at 17th tick after this.

Counting started at completion of 100th complete vibration.
Watch indicated 2 h. 32 m. 20 s. at 31st count.

Each watch tick $=\frac{1}{5}$th sec.
∴ First swing commenced at 2 h. 31 m. 3·4 s. and 100th swing was completed at 2 h. 32 m. 13·8 s.
Hence 100 complete swings take 1 m. 10·4 secs., or the period of one swing $=0{\cdot}704$ sec.
The possible error is 0·2 sec. at each observation.
The period $=0{\cdot}704 \pm 0{\cdot}004$ sec.

It will be observed that even using precautions of this type and taking a large number of swings, the possible error is more than $\frac{1}{2}$ per cent in this case. As in most cases the time has to be squared, this error usually is doubled. With a slower-ticking watch the error is correspondingly greater; an experienced observer, however, using a clock, or chronometer, ticking half-seconds is able to estimate to one-tenth of a second.

It is also worthy of note that the percentage error depends on the total time observed, and not on the number of oscillations taken, so that the same accuracy is obtainable with a smaller number of slow swings as with a large number of quick swings,

provided the times occupied by the swings observed are approximately equal.

It is impossible accurately to estimate fractions of a vibration, and therefore the student must invariably find the time taken for a given number of swings, not the number of swings in a given time.

NOTES ON ACCURATE WEIGHING

Weighing by Oscillations.—The pointer of a balance moves over a scale which usually has 20 divisions. Imagine these to be numbered from the left-hand end, subdividing each division mentally into 10 parts, so that the central mark is called 100, and the mark at the right-hand end is 200.

We must first determine the **zero-point**, or the position of rest of the beam when unloaded. To do this, the beam is allowed to swing freely without any load in the scale-pans. Five consecutive readings of the "turning-points" are taken: three of these will be successive maximum swings to one side, and the other two will be the maximum swings to the other side occurring between them. The mean of the three swings to one side is taken, and also the mean of the other two swings. The zero-point lies midway between the two means thus obtained.

The object is placed in the left-hand pan, and weighed as already described (pp. 16-18). The centigram rider is placed on the divided beam, and moved until when it is on one division the pointer is to the right of the zero-point, while when it is on the next division the pointer lies to the left of the zero-point. The resting-point of the balance is determined for each of these positions of the rider by taking five readings of the turning-points as in finding the zero. The weight of the body can then be found to a fraction of a milligram by the method of "proportional parts."

As an example, suppose the resting-points are:

Empty balance	112
Loaded with 47·634 gm.	114
Loaded with 47·635 gm.	98

The last two give the **sensibility** of the balance, *i.e.* the deflection of the pointer for 1 mgm. as $114 - 98 = 16$ divisions. When the balance is loaded with 47·634 gm., the resting-point is 2 divisions from the zero-point. This difference corresponds to $2/16 = 0{\cdot}125$ mgm., and if this weight were added in the right-hand pan, the loaded balance would swing about the zero-point.

The weight of the object is thus obtained as 47·6341(2) gm.

To facilitate reading the positions of the turning-points, a low-power microscope is sometimes fitted to the front of the balance case.

Weighing by Substitution.—Borda employed a method in which the body is counter-balanced as accurately as possible. It is then removed and replaced by standard weights. When the balance again swings about its zero position, the weights in the pan must evidently have the same mass as the body.

Double Weighing.—In the method of Gauss, the body is first weighed in the left-hand pan, and then in the right-hand pan. If A and B denote the weights thus determined, the true weight W is given as $W = \sqrt{AB}$.

These two methods eliminate any error arising from inequality of the arms of the balance.

CHAPTER III

MEASUREMENT OF QUANTITIES IN DERIVED UNITS

OF the quantities which are measured in 'derived' units, among the simplest to measure are areas, volumes, and densities.

§ 1. MEASUREMENT OF AREA

MEASUREMENTS OF AREAS BOUNDED BY STRAIGHT LINES

The unit of area used in scientific work is the square centimetre, *i.e.* is the area of a square each side of which is 1 cm. long.

For the measurement of areas bounded by straight lines the ordinary rules of mensuration are applied, the lengths required being measured by means of a scale. It is possible to subdivide any figure of this type into triangles, and the total area is found by adding together the areas of the individual triangles, the area of a triangle being half the product of the base and the perpendicular height.

If the area of a sheet of metal with straight edges is required, vernier callipers (p. 22) may be used to obtain greater accuracy than is possible with an ordinary scale by eye estimation. Where convenient, part of the area may be subdivided into rectangular figures, and only the corners treated as triangles.

EXPT. 5. **Measurement of the Area of a Rectilinear Figure.**—Find, by measuring the base and the perpendicular height, the area of a triangle cut out of a thin metal sheet.

The area is one-half the product of these quantities. Since each side in turn may be chosen as the base, three independent determinations can be made. The three results should agree within the limits of experimental error. Take the mean of the three values as the area of the triangle.

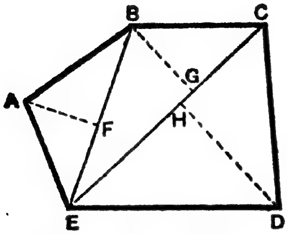


FIG. 10.—Pentagon divided into Triangles.

Determine also the area of other figures, such as a quadrilateral, a pentagon and a hexagon, by subdividing them into triangles as in Fig. 10.

MEASUREMENT OF AREAS WITH CURVED BOUNDARIES

In the case of certain figures with curved boundaries the relation between the area and the linear dimensions is known. Thus for a circle of radius r, the area is πr^2, while for an ellipse of semi-major axis a, and semi-minor axis b, the area is πab.

I. The method of subdividing the area into triangles and rectangles can be adopted even with irregular areas in order to obtain approximate values for the area. The accuracy depends on the degree of subdivision to a certain extent, but if carried too far, the total possible error due to minute errors in determining the various small areas may more than discount the additional accuracy obtained by increased subdivision.

This method is the basis of that used in surveying.

II. If the figure is drawn on squared paper, the area can be found by counting the number of squares. It is clear that the accuracy obtained depends on the fineness of the ruling—the smaller the elementary squares, the more closely can the outline of the given figure be followed (Fig. 11).

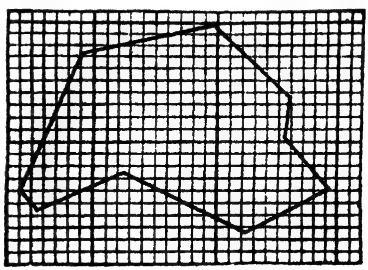


FIG. 11.—Measurement of Area.

This method is a particular case of I., a small square being chosen as the unit by which the figure is built up.

III. Areas can be determined with considerable accuracy by making use of the balance. The figure is drawn on a sheet of cardboard or thin metal whose thickness should be as uniform as possible. The area is then cut out and weighed. From the same sheet is cut an area, of which the shape may conveniently be a rectangle or a triangle, and its weight is found. The area in this case can be determined from the linear dimensions. The unknown area is then calculated by simple proportion, assuming that the first area is to the second as the weight of the first figure is to the weight of the second.

EXPT. 6. **Measurement of the Area of a Circle.**—Draw a circle of convenient radius (5 to 10 cm.) and determine its area by each of the three methods I., II., and III., and deduce a value of π in each case.

IV. The area can be found by means of Simpson's rules. These rules serve to determine approximately the area included between any regularly curving line and two ordinates drawn at the extremities of the curve perpendicular to some base line.

Divide the base line into a number of equal parts, and draw the corresponding ordinates, dividing the area into a number of strips.

First Rule.—Add together the halves of the extreme ordinates and the whole of the intermediate ordinates, and multiply the result by the common interval (the distance between consecutive ordinates).

Second Rule.—Add together the extreme ordinates, twice the sum of all the odd ordinates (omitting the first and last) four times the sum of all the even ordinates, and multiply by one-third of the common interval. In this case the number of strips must be even.

The first rule is easier to apply than the second, but is slightly less accurate. These rules are frequently employed by engineers in the measurement of indicator diagrams.

EXPT. 7. **Measurement of the Area of a Semicircle.**—Draw a semicircle of convenient radius and determine its area by the above rules. Compare the results with the area found by calculation.

V. *The area can be measured with a* **Planimeter.** This method is of great importance to the engineer or surveyor, but is of a more advanced character than the methods so far considered.

THE PLANIMETER

A number of instruments have been devised for the direct determination of plane areas of any contour, the general name **Planimeter** being applied to instruments of this class. Of these probably the most elegant and simple is that due to Professor Amsler of Schaffhausen, and as this is the type in general use, we shall confine ourselves to describing the construction and the method of using this planimeter.

The instrument (Fig. 12) consists of two rods OA and AB hinged together at A; the rod OA is fixed at the end O, so that A can move

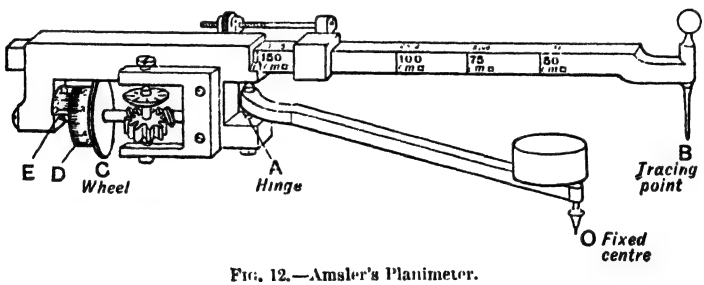


FIG. 12.—Amsler's Planimeter.

only in the path of a circle about O. At B is the **tracing point,** and the hinge at A is so arranged that the point B can move in any direction in the plane OAB, the motion being limited only by the lengths of the arms and the mechanical construction of the instrument. Somewhere along the arm AB is mounted a wheel C, whose axis is parallel to the arm AB, this wheel being usually on the side of A remote from B, although this is not necessary for the working of the instrument. This wheel is fitted with a circular or cylindrical scale D subdivided into 100 equal parts, and a vernier scale E fixed

to the frame enables the position of the wheel to be read to $\frac{1}{1000}$ of a revolution. Whole revolutions are registered on a small revolution counter connected to the wheel by a worm gearing.

The instrument rests on the **fixed centre** O (Figs. 12 and 13), on the edge of the wheel C, and on the tracing point B. If the point B is moved, the whole arm AB will move. Any motion of AB along its own direction will cause mere *sliding* of the wheel, no rotation being produced whatever. On the other hand, if AB moves *perpendicular* to its length, the wheel will *roll* a distance equal to the distance moved through by the arm AB perpendicular to its own length. **However AB moves, the component of its motion perpendicular to its length will be registered by the rolling of the wheel, and therefore the distance rolled through by the wheel due to any motion of AB, is the total distance through which AB has moved perpendicular to its own direction.**

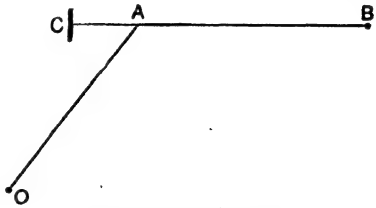


FIG. 13.—Plan of Planimeter.

From this distance it is possible to determine the area of the figure round which B has been taken.

There is an important difference between the case where O is outside the figure round which B moves, and the case when O is included within this contour; and we shall first consider the case when O is outside the area to be determined.

Area not enclosing the Fixed Centre

Consider the arm AB to move from the position A_1B_1 to the position A_2B_2 (Fig. 14). A would move along the circle about O from A_1 to A_2, and B might traverse the path B_1B_2. The same position would have been reached if AB had moved parallel to itself into the position A_2B_3 and then had rotated about A_2 into the final position A_2B_2.

If the perpendicular distance between A_1B_1 and A_2B_3 is a small amount δs, and the angle between A_2B_3 and A_2B_2 is a small angle $\delta\phi$, the area swept out by the arm AB in this motion would be equal to $b\delta s + \frac{1}{2}b^2\delta\phi$, where b = length of arm AB.

The actual area swept out, $A_1A_2B_2B_1$ will differ from this by the small area $B_1B_2B_3$, which will be negligible if δs and $\delta\phi$ are small, and hence we have that

$$\text{Small area swept out by } AB = b\delta s + \tfrac{1}{2}b^2\delta\phi.$$

Any motion of AB can be taken as the sum of a number of

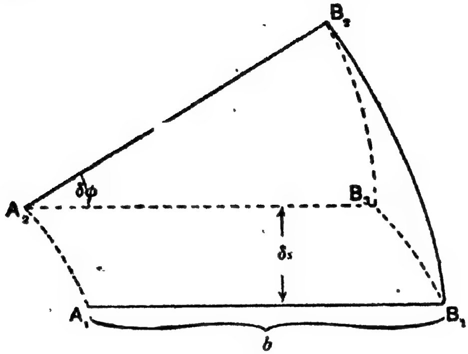


FIG. 14.—Elementary Area traced by Planimeter.

elementary motions of this type, and therefore, in any motion of the arm AB,

$$\text{Total area swept out by } AB = \Sigma(b\delta s + \tfrac{1}{2}b^2\delta\phi).$$

The symbol Σ here, and elsewhere, is used to denote the sum of a series of terms which are all of the same type.*

Suppose that the point B moves completely round an area not enclosing the fixed centre O.

Let the extreme positions of the arm be A_1B_1 and A_2B_2 (Fig. 15), B circulating round the area in the positive direction, *i.e.* so that the area always lies on the right hand side of an observer moving round with the point B.

In traversing the path B_1EB_2, the arm AB sweeps out an area $A_1A_2B_2EB_1$, while on the return journey via F, it sweeps out an area $A_1A_2B_2FB_1$. Thus, **the net area swept out by the arm AB when B moves round the given contour, is the area enclosed by the contour.**

Thus $\qquad \text{Area } B_1EB_2F = \Sigma(b\delta s + \tfrac{1}{2}b^2\delta\phi),$

or $\qquad \text{Area required} = b\Sigma\delta s + \tfrac{1}{2}b^2\Sigma\delta\phi.$

Now the arm AB returns to the same position as at first, when B goes completely round the area, and therefore $\Sigma\delta\phi = 0$.

Hence $\qquad \text{Area } B_1EB_2F = b\Sigma\delta s = bS,$

where S = total distance rolled through by the wheel.

For areas not enclosing the fixed centre O, the area round which the tracing point is taken is equal to the distance through

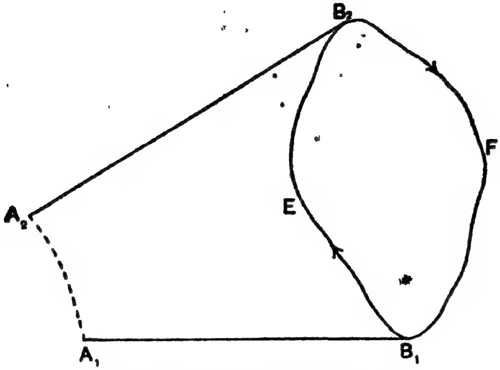


FIG. 15.—Finite Area traced by Planimeter.

which the wheel rolls, multiplied by the length of the arm from the hinge A to the tracing point B.

Areas enclosing the Fixed Centre O

The Zero Circle.—*Before considering the general case of any area enclosing the fixed centre, it is essential to consider the special* case of the 'zero circle.' If we clamp the two arms of the planimeter together in such a position that the plane of the wheel passes through the fixed centre, the point B can move only in a circle of radius OB about O (Fig. 17, p. 42).

If we cause B to trace out this circle, the wheel C will not roll any distance whatever, because it is moving perpendicular to its own plane the whole time. Thus, when this circle is traced out by B, the reading of the wheel is unaltered or the registration of the wheel is zero, hence the term 'zero' circle which is applied to this particular circle.

By placing the planimeter in this position (it is unnecessary to *clamp* it) the length OB can be measured, and hence the area of the zero circle can be determined.

General Case of any Area enclosing the Fixed Centre.—Consider the area ABCDEF (Fig. 16) enclosing the fixed centre O, and let the dotted line indicate the zero circle. If we take the tracing point from A to B along the curve, and then return to A along the zero circle, we shall have traced out the area AGBH in the positive

direction, and therefore the reading on the wheel will correspond with this area AGBH, *since this does not include O.*

Now this reading of the wheel was all registered during the motion from A to B along the curve, since there is no rolling of the wheel when the tracing point travels along the zero circle. Hence the area AGBH is registered by the wheel while it is moving from A to B along the curve.

If now we start at B and go round BKCL we shall have traversed the contour of the area BKCL in the *negative* direction, *i.e.* the wheel will have rolled *backwards* a distance corresponding with the area BKCL. Again, the whole of this movement was recorded while the tracing point was moving along BKC, and hence in moving along BKC, the area BKCL was recorded *negatively.*

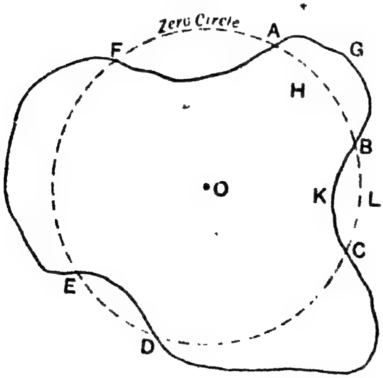


FIG. 16.—Area enclosing the Fixed Centre.

It is obvious from the foregoing that the wheel automatically reverses its direction of rotation as the tracing point crosses the zero circle, and therefore the zero circle need not be drawn. It is therefore apparent that when the tracing point is taken round an area which includes the fixed centre, **the total distance rolled through by the wheel corresponds with the algebraic sum of the areas outside the zero circle.**

The area required is given by the area corresponding with the distance recorded by the wheel, *plus* the area of the zero circle; hence the zero circle must first be determined as described above.

Great care must be used to notice in what direction the instrument is recording when using it with the centre inside the area to be measured, and of course the positive or negative sign prefixed as required: the tracing point must always be caused to go round the contour in the positive direction.

EXPT. 8. **Calibration of the Planimeter.**—The first thing to be done in using a planimeter is to find out what area corresponds with one whole revolution of the rolling wheel. This of course depends on the length of the tracing arm from the hinge to the tracing point, and also on the diameter of the wheel.

(i.) **Determination of the Length of the Tracing Arm.**—Set the hinge carrier until the index on it is in line with one of the lines on the side of the tracing arm—the line marked 100 cm. □ is a convenient one, or if the planimeter gives inches it may be adjusted to the line marked 10 in. □. We have now to find the length from the hinge to the tracing point. This is no easy matter, as usually the hinge is enclosed almost entirely by the hinge carrier and adjustments. The best way is to lay the instrument on its side on a sheet of squared paper, placing the tracing point on some definite line in the paper, and estimating to the nearest 0·1 mm. where the axis of the pivots lies. The tracing bar must be placed parallel to one side of the squared paper in order that the length may be obtained accurately.

In some forms of planimeter two points are carried on the *top* of the tracing arm; one of these is fixed near the end of the arm, and the other moves with the hinge carrier. They are so placed by the maker of the instrument that the distance between them is exactly equal to the distance between the tracing point and the axis of the hinge. Consequently this distance may be measured with a scale—a much simpler and more accurate observation than that already described. The distance thus obtained is the length b already referred to in the description of the instrument.

(ii.) **Determination of the Circumference of the Wheel.**—The circumference of the tracing wheel is determined by measuring the diameter d with a micrometer screw gauge and multiplying by π, care being taken to adjust the micrometer screw till it touches the wheel edge very lightly only, otherwise the edge of the wheel may be deformed and the accuracy of the instrument destroyed.

The product of the length of the arm b and the circumference of the wheel πd is the area corresponding with one revolution of the wheel when the instrument is in this adjustment.

This product will be found to coincide very nearly with the indication 100 cm. □ or 10 in. □ on the side of the tracing arm to which the hinge carrier was adjusted. These graduations are made by the maker of the instrument, and are the areas corresponding with one revolution of the wheel with the instrument adjusted in this manner. It is obvious that the above methods for getting b and d are somewhat crude; the instrument maker has more accurate means of measuring these quantities, so that, unless the instrument is old, or has been handled severely and distorted, the values indicated on the tracing arm should be used.

EXPT. 9. **Determination of Small Areas with the Planimeter.**—The area considered is small enough for the fixed centre O to be taken outside the figure.

(i.) **Percentage Error with Planimeter.**—Trace out a square of side 10 cm. and, having taken the tracing point round the contour, convert the reading of the planimeter into sq. cm.; express the difference between this and 100 sq. cm. as a percentage of the total area. This gives the percentage error of observation with this form of instrument.

(ii.) **Area of a Circle with Planimeter.**—Draw a circle of 10 cm. radius and find its area by the planimeter; hence determine the value of π.

EXPT. 10. **Determination of the Area of the Zero Circle of the Planimeter.**—The use of the instrument for small areas, and the method of translating its indications being now thoroughly understood, it is essential that the zero circle should be determined in order that large areas *enclosing the fixed point* may be measured.

(i.) **Calculation of the Area of the Zero Circle.**—Place the wheel on a piece of squared paper so that its point of contact with the paper is exactly on one corner of the squares, and its plane lies along one of the sides of the squares. Place the tracing point on one of the edges, and the fixed centre on the edge in the plane of the wheel, pricking both points into the paper.

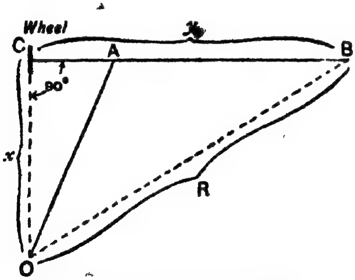


FIG. 17.—Position for Zero Circle.

The instrument is now fixed with the plane of the wheel passing through the fixed point, and therefore, as described in the foregoing investigation, the distance between the fixed point and the tracing point is the radius of the zero circle. This distance can be measured, and hence the area of the zero circle found, or, since in Fig. 17

$$x^2 + y^2 = R^2,$$

we can write

$$\text{Area of Zero circle} = \pi(x^2 + y^2),$$

and R need not be measured.

If the top of the tracing bar is examined, it will be found that there are various numbers marked on it at different points, each number being above one of the graduations on the side of the bar. These are the areas of the zero circles corresponding with the various positions of the hinge carrier, and *are usually expressed in revolutions of the wheel* in that position; *e.g.* if the index on the hinge carrier is set to the line marked 100 cm. □, and the number on the top of the bar opposite to this line is 20·731, this means that the zero circle has an area equivalent to 20·731 revolutions of the wheel, *i.e.* an area of 2073·1 sq. cm.

Express the indicated zero circle area in sq. cm., and compare this with the area calculated from your observations, as described in the preceding paragraph. As before, the methods available to the instrument maker are probably much more accurate than the crude method described, and the indicated area should be used unless the instrument is obviously badly worn or distorted.

(ii.) **Experimental Determination of the Zero Circle Area.**—The zero circle area can be determined quite simply by the indications of the planimeter itself. Take some area (regular or irregular) of such a size that it can be contoured without difficulty with the fixed point *outside*: a figure about 20 cm. across is suitable for most instruments when set to the 100 cm. □ graduation.

Go round this area with the tracing point, having the fixed centre *outside*, and write down the area as indicated by the planimeter: let this be A.

Next place the fixed centre *inside* the area and go round the figure again. This must be done slowly, as the wheel revolves very rapidly, and may slip or jump from off the paper if care is not taken. Be careful to notice also that the reading is *diminishing* all the time, *i.e.* the area is giving a *negative* registration on the wheel: the area is of course being contoured in the positive direction.

When this registration has been obtained, we may use the equation already proved for the case where the fixed centre is inside:

$$\text{Area } A = \text{registered area} + \text{zero circle area.}$$

In general, this equation would be used to find the area A, but we can apply it in this special case to find the zero circle area. We have determined already the area A by means of the planimeter, using it with the fixed point outside, and we have just obtained the area, registered with this point inside, so that the zero

circle area is equal to the *algebraic* difference of these two quantities, or, since one is negative, to their arithmetic sum.

As an example of this :—

Fixed point outside

First reading . . .	2·139 revolutions.
Second reading . .	5·713 revolutions.
Registration . . .	3·574 revolutions.

Area = 357·4 sq. cm.

Fixed point inside

First reading . . .	(2) 5·781 revolutions.
Second reading . . .	8·633 revolutions.
Registration . . .	− 17·148 revolutions.

The registration was negative all the time, and the wheel counting whole revolutions of the rolling wheel passed through its zero twice, hence the (2) in front of the first reading.

Thus

$$357{\cdot}4 = (-1714{\cdot}8) + \text{zero circle area},$$

or

$$\text{Area of Zero circle} = 2072{\cdot}2 \text{ sq. cm.}$$

The indicated value was 2073·1 sq. cm.

Find the area of the zero circle by these two methods, and compare their results with the area indicated on the top of the bar.

EXPT. 11. **Determination of Large Areas with the Planimeter.**—In this case the area is supposed to be so large that the fixed centre O must be taken inside it.

Draw a large ellipse about 40 cm. × 70 cm., using a loop of thread and two pins, and find its area, using the planimeter. Show that the area is $\frac{\pi}{4}$ times the area of the circumscribing rectangle, the sides of which are parallel to the major and minor axes of the ellipse.

§ 2. DETERMINATION OF VOLUME AND DENSITY

The **density** of a substance is defined as the mass per unit volume or the mass of 1 c.c. (in the C.G.S. system). If M be

the mass and V the volume, the density is given by the quotient M/V; this will be expressed as gm. per c.c. if M is in gm. and V in c.c.

Expt. 12. **Determination of the Density of a Regular Solid by use of the Vernier Callipers.**—Measure carefully the linear dimensions of a number of regular solids with the vernier callipers, reading to 0·1 mm., deducting (algebraically) the 'zero reading' of the callipers from each of the readings taken.

Repeat each observation at least three times, and if possible at different points of the object, and take the mean of the results, *e.g.* in getting the diameter of a cylinder it should be measured at each end, and in the middle as well, to correct for any 'taper.' At each place, two diameters at right angles to each other should be measured to correct for any ellipticity, the mean of these six observations being taken as the true diameter.

Calculate from your observations the volume of the body, expressing the results in cubic centimetres. (Formulæ for this are given on p. 595 for various regular shapes.)

Find, by means of the balance, the mass of the body, and calculate its density.

Expt. 13. **Determination of the Density of a Regular Solid by means of a Micrometer Screw Gauge.**—Determine the volume of a number of regular solids by measuring their linear dimensions with the micrometer screw gauge.

Find the mass of each solid by weighing it on a balance, and calculate the density, *i.e.* the mass of unit volume, as in the preceding experiment.

§ 3. The Spherometer

The **Spherometer** is an instrument used for measuring the radius of curvature of a spherical surface. In many cases—as, for example, when dealing with a lens—the surface is only a small portion of a sphere. In such a case the radius of curvature is the radius of the (imaginary) sphere of which the surface forms a part.

The instrument (Fig. 18) consists of a small table supported by three legs, A, B, C, placed as nearly as possible at the corners of an equilateral triangle. Through the centre of the table passes a

screw of fine pitch (usually 0·5 mm. or 1 mm.) forming a fourth leg O. The position of this leg can be read by means of a scale

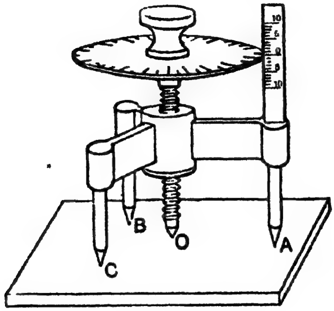


FIG. 18.—A Spherometer.

fixed at right angles to the table and a circular scale attached to the head of the screw.

EXPT. 14. **Measurement of the Thickness of a Plate by Means of the Spherometer.**—In using a spherometer the first thing to be done is to determine the value of the graduations of the two scales, for all instruments are not graduated in the same way. Find out how far the screw advances when the head is turned through one complete revolution. This distance will probably be 0·5 mm. or 1·0 mm. Notice next the number of divisions on the circular disk and determine how far the screw advances when the disk is turned through one division. For example, if the pitch of the screw is 0·5 mm. and there are fifty divisions on the graduated head, each division corresponds with a movement of 0·01 mm. of the point of the screw (see p. 24).

When the zero on the graduated disk is opposite the zero on the linear scale, the point of the screw is supposed to be in the same plane as the three fixed feet. In general this is not exactly the case, and it is necessary therefore to determine the amount of 'zero error' of the instrument, or the 'zero reading.' To do this, place the spherometer on a plane surface such as an optically-worked glass plate. Turn the head of the instrument till the point of the screw just touches the surface. The exact position can be determined by touching one of the outside legs with the finger tip or a pencil, and observing if the instrument will rotate about the screw point. The screw

must be turned until the instrument rotates when touched in this manner, but will no longer do so if the screw is raised the slightest degree. Repeat the observation several times and take the mean of the readings as the zero reading. This quantity must be subtracted algebraically from all subsequent readings.

The thickness of a plate of glass can be measured as a preliminary experiment, by determining the zero reading on a plane surface, and then finding the reading when the screw point is resting on the top of a small plate, while the other feet still stand on the plane surface.

It is often convenient not to take any account of the vertical scale except to find the distance the point rises for one turn. Instead of the vertical scale reading, the number of whole turns of the divided head should be counted, each turn being reckoned as 50 or 100 of the graduated head divisions as the case may be, the travel of the point being stated in terms of these.

EXAMPLE.—Zero reading is 23 divisions on the circular head.

In order that the screw point should rest on the top of a small plate, four whole turns and part of a fifth were made and the reading on the circular head when correctly adjusted was 65.

There were 100 divisions on the circular head, therefore the number of divisions turned through

$$= 4 \text{ whole turns} + 65 - 23,$$
$$= 442 \text{ divisions.}$$

The pitch of the screw is 0·5 mm., therefore each division is equivalent to $\frac{1}{200}$ mm.

Thickness of plate = 0·221 cm.

EXPT. 15. **Measurement of the Radius of Curvature of the Surface of a Lens or Mirror.**—Place the spherometer with the fixed feet resting on the surface, and adjust the central foot till it just touches the surface. Read the circular scale. Replace the instrument on the plane surface and find how many whole turns have to be made to bring the central foot back to the plane of the other three feet. From this and the readings of the circular head in the two adjustments find, as above, the distance through which the screw was moved. Take the mean of several adjustments and let the height be h cm.

We also require to know the distance between two fixed feet. Measure this carefully to 0·1 mm. with a millimetre

scale for each side of the triangle and take the mean of the results: let it be a cm.

Then the radius of curvature is given by the expression

$$R=\frac{a^2}{6h}+\frac{h}{2}.$$

N.B.—I. Since R depends on the square of a, a small percentage error in a means an error of twice this magnitude (per cent) in R.

II. If h is in cm., a must be in cm., and the value of R will be found in cm.

III. The term $h/2$ can often be neglected in comparison with $a^2/6h$.

The results should be entered as follows:

Reading on plane.	Reading on lens.
22 divisions	48 divisions
24 divisions	47 divisions
23 divisions	46 divisions
Mean 23 divisions	Mean 47 divisions.

Difference = 24 divisions.

The screw head was turned through two complete turns.

h = 2 turns and 24 divisions = 0·112 cm.

Distance between feet.

3·01 cm.
3·03 cm.
2·99 cm.
Mean a = 3·01 cm.

$$R=\frac{(3{\cdot}01)^2}{6\times 0{\cdot}112}+\frac{0{\cdot}112}{2}\text{ cm.}$$
$$=13{\cdot}5\text{ cm.}$$

It is useful to express the curvature of the surface in **dioptres**, this being the unit of curvature employed by opticians. **A surface whose radius of curvature is one metre has a curvature of one dioptre,** and thus **the curvature in dioptres is the reciprocal of the radius of curvature in metres.** The curvature in the example given is 100/13·5 = 7·41 dioptres.

Proof of the formula $R=\frac{a^2}{6h}+\frac{h}{2}$.

In this formula a is the length of the side of the equilateral triangle formed by the three feet (*Fig.* 19).

Let x denote OB, the radius of the circumscribing circle.

Then, if OD be at right angles to BC,

$$BD=\frac{a}{2},$$

and

$$OD=\frac{x}{2},$$

for ΔBOD is one half of an equilateral triangle.

Now $$OB^2 = OD^2 + BD^2,$$

or $$x^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{a}{2}\right)^2,$$

$$x^2 - \frac{x^2}{4} = \frac{a^2}{4},$$

$$\frac{3x^2}{4} = \frac{a^2}{4},$$

$$x^2 = \frac{a^2}{3}.$$

To find the radius of curvature R, we consider a section of the sphere by a plane through its centre and through the line BO in Fig. 19. Thus we

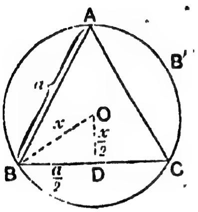


FIG. 19.—Plan of Spherometer.

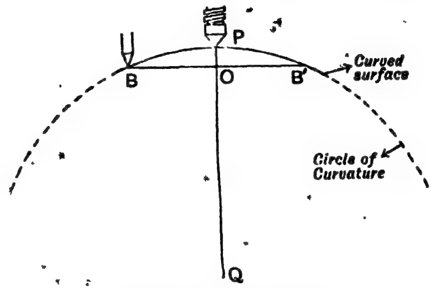


FIG. 20.—Section of Sphere.

obtain Fig. 20, in which only a portion of the circle of curvature is shown. If the diameter PQ meet this circle again in S (not shown in Fig. 20),

$$QS = QP = R,$$
$$OB = OB' = x \text{ and } OP = h.$$

We know that
$$OS \times OP = OB \times OB'.$$

Hence,
$$(2R - h)h = x^2,$$

$$2Rh = x^2 + h^2,$$

$$R = \frac{x^2}{2h} + \frac{h}{2}.$$

or, finally,
$$R = \frac{a^2}{6h} + \frac{h}{2}.$$

CHAPTER IV

DETERMINATION OF SPECIFIC GRAVITIES

§ 1. Definition of Specific Gravity

The specific gravity, or relative density, of a substance is defined as the ratio of the weight of any volume of that substance to the weight of an equal volume of some standard substance. The standard substance usually chosen is water. For exact work it is necessary to specify the temperatures at which the measurements are made. Thus water is chosen at the temperature of 4° C., that is, at the point of maximum density. For ordinary purposes, sufficient accuracy is attained by making the measurements at the temperature of the room.

§ 2. The Specific Gravity Bottle

The Specific Gravity Bottle is a bottle constructed so as to contain a definite volume of a liquid. In a common form the bottle is fitted with a ground-in stopper pierced by a small hole. The bottle is filled completely, and when the stopper is inserted the excess liquid escapes and can be wiped away.

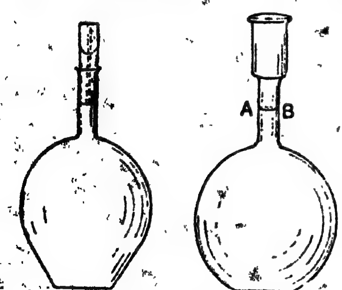


Fig. 21.—Specific Gravity Bottles.

A simpler and more accurate form has a narrow neck on which a mark, AB, is made. The bottle is filled so that the bottom of the meniscus is level with the mark, any surplus liquid being removed by filter paper,

or by a small pipette. The reason for the greater accuracy of this form of bottle is that the stopper in the first case, being a finely tapered cone, can be wedged in farther sometimes than others, particularly if the bottle is slightly warmer than the stopper.

Expt. 16. Determination of the Specific Gravity of a Liquid by using the Specific Gravity Bottle.—The bottle must first be dried thoroughly. Attach a glass tube, narrow enough to enter the bottle, to the nozzle of the foot-bellows by means of rubber tubing, and blow air into the interior of the bottle. At the same time warm the bottle very gently by holding it *above* (not in) the flame of a spirit-lamp or Bunsen burner. Hold the bottle by the neck and keep it constantly rotating so as to prevent unequal heating, which would be liable to crack the glass.

When the bottle is dry and cold, place it on the left-hand pan of the balance and determine its weight correct to the nearest centigram. Let the weight be B gm.

Then fill the bottle to the mark with water, being careful to avoid an error due to parallax by holding the bottle with the mark on the same level as the eye. Weigh again and let the weight be W gm.

The weight of water filling the bottle is therefore W − B gm.

Record (1) the weight of the bottle, (2) the weight of the bottle and water, (3) the weight of water filling the bottle.

Fill the bottle to the mark with the liquid the specific gravity of which is to be determined.

Weigh the bottle full of liquid and let the weight be w gm. The weight of liquid filling the bottle is $w - \mathrm{B}$ gm.

Therefore the specific gravity of the liquid is

$$\frac{w - \mathrm{B}}{\mathrm{W} - \mathrm{B}}.$$

Record (1) the weight of the bottle, (2) the weight of the bottle and the liquid, (3) the weight of liquid filling the bottle, and lastly the specific gravity of the liquid.

SPECIFIC GRAVITY OF A GRANULAR SOLID

The method with the specific gravity bottle is applicable to solid substances that are heavier than water and insoluble in it. The specific gravity of sand or small shot may be found by this method.

In the case of a substance like glass, the solid must be broken up into fragments small enough to go into the specific gravity bottle.

If the substance is lighter than water, or is soluble in it, we can use some other liquid in the determination, but in this case a separate experiment is necessary to determine the specific gravity of the liquid employed.

In determining the specific gravity of sand, it is not sufficient to fill the bottle with sand and proceed as though we were dealing with a liquid. By so doing we should find the specific gravity, not of sand, but of a mixture of sand and air; for a large amount of air is imprisoned between the sand grains.

EXPT. 17. **Determination of the Specific Gravity of a Granular Solid, by using the Specific Gravity Bottle.**—First determine the weight, B, of the empty bottle, as in the previous experiment.

Fill the bottle about one-third full with sand, being careful to use dry sand in a dry bottle. Weigh the bottle and sand and let the weight be W_1 gm.

The weight of sand alone is $W_1 - B$ gm. Fill up the space above the sand and between the grains with water, shaking the bottle *round and round* to get rid of air bubbles from among the sand. If great accuracy is required, the air must be removed by connecting the neck of the bottle with a vacuum pump by rubber tubing. Adjust the level of the water to the mark. Let the weight now be W_2 gm. Then the weight of water added is $W_2 - W_1$ gm.

Empty the bottle; fill it to the mark with water, and weigh again. Let the weight be W gm. Then the weight of water required to fill the bottle to the mark is $W - B$ gm. The difference between these two quantities of water (viz. $W - B$ and $W_2 - W_1$ gm.) represents the quantity of water required to fill the space occupied by the sand. Calculate this quantity and record the result.

Now the specific gravity of sand

$$= \frac{\text{Weight of sand}}{\text{Weight of equal volume of water}},$$

$$= \frac{\text{Weight of sand}}{\text{Quantity of water required to fill the space occupied by the sand}}.$$

Calculate the specific gravity of sand and record the result.

§ 3. The Hydrostatic Balance

FORCES BETWEEN BODIES IN CONTACT

When two bodies are in contact, the action and reaction constitute a pair of forces which, in accordance with Newton's Third Law, are equal and opposite.

If the force be at right angles to the surface of contact of the two bodies, it is called a **thrust. A thrust is measured in dynes** or in gm.-weight.

In any actual case the bodies must be in contact over a finite area. We speak of the **pressure** between two bodies in contact, when considering the forces as distributed over the surfaces in contact.

The **pressure at a point** is found by dividing the thrust on an element of area round that point by the number of units of area in the element. **Pressure is measured in dynes per square centimetre.**

When one of the substances in question is a fluid and the other a solid, the resultant force due to the fluid pressures on the solid can be determined in a simple way by a principle which we shall now consider.

PRINCIPLE OF ARCHIMEDES

The Principle of Archimedes is usually stated as follows:—**When a body is immersed in a fluid, its weight is apparently diminished by the weight of the fluid displaced.** A simpler and more direct statement of the principle is: **When a body is wholly or partially immersed in a fluid at rest, it experiences an upward thrust equal to the weight of the fluid displaced.**

The truth of this principle may be anticipated theoretically on certain assumptions as to the action of fluid pressure, or by a simple type of experiment it may be verified practically.

If we consider the equilibrium of any portion of a fluid at rest, it is obvious that this portion must be supported by the

fluid surrounding it, otherwise its weight would cause it to sink. It is also obvious that this supporting force must be exactly *equal* to the weight of the portion considered.

The support is given by pressures exerted by the surrounding fluid (indicated by the small arrows in Fig. 22), the resultant of which is equal to the weight of the portion considered but acts vertically upwards. The resultant supporting force may be termed the **upthrust**.

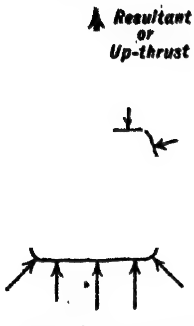


FIG. 22.—Principle of Archimedes.

If now a portion of the fluid be imagined removed, and a *solid* of exactly the same shape be put in its place, the surrounding fluid will still exert the same pressures as before; therefore the solid will be supported with a force *equal* to that which was exerted on the fluid which has been displaced.

But this supporting force is equal to the weight of the displaced fluid, therefore **the solid will be supported by a force—the upthrust—equal to the weight of the fluid it has displaced,** and its weight will apparently be diminished by this amount.

EXPT. 18. **Experimental Verification of the Principle of Archimedes.**—For the experimental demonstration of the truth of this principle two cylinders are used, one solid and the other hollow, the latter being made just large enough to allow the solid cylinder to be placed inside it.

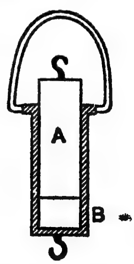


FIG. 23.—Hollow and Solid Cylinder.

The cylinders are provided with hooks so that the solid cylinder (A) may be suspended underneath that which is hollow (B): Fig. 23 gives a sectional view of B with A partly withdrawn.

A is suspended below B, and both are hung from the beam of a balance; in the hydrostatic balances generally used, they are hung beneath the short pan as shown on the left in Fig. 24, a short piece of thread or very fine brass wire being used to let A swing well clear of B.

A and B should then be counterpoised by weights placed in the other pan. If now a vessel of water is placed beneath A and raised (or A lowered) until A is immersed completely, it will be seen that the beam is no longer horizontal, the counterpoising weights being too great.

Equilibrium is restored completely, however, if B is filled to the brim with water, thus showing that the weight lost when A is immersed, is just made up by adding a quantity of water equal in volume to A; *i.e.* when A is immersed in *water*, it loses a weight equal to the weight of its own volume of *water*.

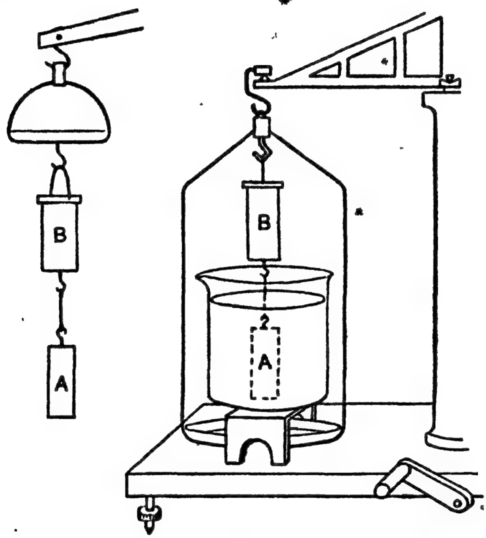


FIG. 24.—Verification of the Principle of Archimedes.

Remove the water from B and add weights in the small scale-pan until balance is restored again. Evidently the weight thus added is equal to the weight of water which fills B, or to the weight of water displaced by A.

Repeat the whole experiment, using methylated spirit, paraffin oil, or any other liquid convenient. Note that after immersing A equilibrium is restored, when B is filled with the *same kind of liquid* as that in which A is immersed, thus verifying the Principle of Archimedes.

It is important to note that the *apparent* loss of weight is not the same in the various cases. This must be so since the upthrusts

are the weights of equal volumes of different liquids. If one of the liquids is water, we can at once deduce the specific gravity of any of the other liquids, for

$$\text{Specific gravity} = \frac{\text{Weight of any volume of liquid}}{\text{Weight of an equal volume of water}},$$

$$= \frac{\text{Upthrust due to liquid}}{\text{Upthrust due to water}}.$$

APPLICATIONS OF THE PRINCIPLE OF ARCHIMEDES

The Principle of Archimedes affords a most important method of determining the specific gravities of both solids and liquids. When a solid is completely immersed in a liquid, the upthrust on the solid is equal to the weight of a quantity of liquid which has the same volume as the solid. Hence by comparing the weight of the solid with the upthrust, we can compare the specific gravity of the solid with that of the liquid, for the volumes considered are the same in each case.

In particular, if we find the upthrust on a solid of known weight completely immersed in water, we know the weight of water having a volume equal to that of the solid, and can at once deduce the specific gravity of the solid.

EXPT. 19. **Determination of Specific Gravities, using the Principle of Archimedes.**—The specific gravity of a solid or of a liquid may be determined with the hydrostatic balance.

(*a*) **Specific Gravity of a Solid (insoluble in water).**—Suspend the solid by a fine thread or wire from the short pan of a hydrostatic balance, or from the hook carrying the pan of an ordinary balance. Counterpoise the body when hanging freely in air, and again when immersed in water, supporting the beaker of water on a specific gravity stool if an ordinary balance is used. Hence determine the upthrust, and calculate the specific gravity, which is the ratio of the weight of the body in air to the upthrust.

(*b*) **Specific Gravity of a Liquid.**—Weigh a solid in air and in water as described in (*a*), and find the upthrust due to the water. Then weigh the solid in the liquid whose specific gravity is required, and find the upthrust due to the liquid. Since the volume considered is the same in each case, the specific gravity of the liquid is the ratio of the upthrust due to the liquid to the upthrust due to water.

(*c*) **Specific Gravity of a Solid (soluble in water).**—

Choose some liquid in which the solid is insoluble. Determine the specific gravity s of the liquid by method (b), using another solid (such as glass) which is insoluble both in water and in the liquid. Weigh the solid in air and in the liquid, and find the upthrust due to the liquid. This is the weight of liquid having the same volume as the solid. To find the weight of an equal volume of water we must divide this by s. The specific gravity of the solid can then be deduced.

(d) **Specific Gravity of a Solid less dense than Water.**—Weigh the body (say wax) in air.

As the body is less dense than water it would float on the surface of water. In order to find the upthrust when the body is completely immersed in water, we employ a 'sinker,' *i.e.* a piece of metal of high specific gravity, sufficiently large to sink the wax.

First find the weight of the metal sinker alone in water. By adding these two results we find the weight of the wax in air and the sinker in water. Then attach the wax to the sinker and find the weight of the wax and sinker together, when both are in water. The difference between this result and the former is due to *the upthrust on the wax*, for the sinker was in water on each occasion. Knowing the weight of the wax in air, and the upthrust on the wax when it is immersed in water, we can at once deduce the specific gravity of the wax.

The following example illustrates the method of working out and recording the observations:—

Weight of wax in air	= 3·235 gm.
Weight of brass sinker in water	= 6·925 gm.
By addition we obtain—	
Weight of wax in air and sinker in water	= 10·160 gm.
Weight of wax in water and sinker in water	= 6·310 gm.
Upthrust on wax in water	= 10·160 − 6·310 gm.
	= 3·850 gm.
Weight of an equal volume of water	= 3·850 gm.

$$\text{S.G. of wax} = \frac{\text{Wt. of wax in air}}{\text{Wt. of equal vol. of water}} = \frac{3{\cdot}235}{3{\cdot}850},$$

$$= 0{\cdot}846.$$

The Principle of Archimedes may be applied conveniently in other experimental determinations; some typical applications will now be given.

Expt. 20. **Determination of Volume by the Hydrostatic Balance.**—Suspend the body by means of a fine wire or thread from one arm of the balance and find its weight in air. Let this be W.

Then find the weight of the body when it is completely immersed in water. Let this weight be W_1.

The difference between W and W_1 is due to the upthrust of the fluid on the body. By the Principle of Archimedes this is equal to the weight of the fluid displaced by the body, that is, if W and W_1 are in grams weight,

$$W - W_1 = VD,$$

where V denotes the volume of the body and D the density (mass per unit volume) of the fluid.

In the C.G.S. system of units the mass of 1 c.c. of water is 1 gram, so that D is unity. In this way the volume is determined in cubic centimetres. If any other liquid of known density were used, the volume of the immersed solid could be obtained equally simply since

$$V = \frac{W - W_1}{D}.$$

Expt. 21. **Determination of the Thickness of a Plate.**—If the body is a flat plate of area A and mean thickness t, we have $V = At$, and therefore $t = V/A$.

Weigh a plate in air and in water and thus find V. Measure the length and breadth of the plate, assuming it to be rectangular, and determine A.

Calculate the mean thickness of the plate, and confirm the result by measurements with the micrometer screw. As the thickness may not be uniform, measurements should be made at several different points and the mean found.

Expt. 22. **Determination of the Diameter of a Wire.**—If the mean diameter is d, the area of cross section is $\pi d^2/4$.

If the length of the wire is l, the volume is

$$V = \frac{\pi d^2}{4} \times l,$$

$$d^2 = \frac{4V}{\pi l},$$

$$= \sqrt{\frac{4V}{\pi l}}.$$

Weigh a known length l of wire in air and in water, and thus determine its volume V.

Calculate the mean diameter of the wire, and confirm the result by measurements with the micrometer screw.

EXPT. 23. **Determination of the Length of a tangled Piece of Wire.**—Weigh the wire in air and in water. Calculate its volume from the result; measure the diameter of the wire with a micrometer screw, and hence calculate its length l.

§ 4. HYDROMETERS

The **Common Hydrometer** is an instrument for determining the specific gravity of a liquid by a flotation method. It consists of a weighted bulb provided with a vertical stem. When placed in a liquid of suitable density, the hydrometer floats with part of the stem above the surface, the condition of equilibrium being that the weight of the instrument should equal the weight of liquid displaced. The stem is graduated so as to indicate the specific gravity of the liquid.

This is a simple method of determining rapidly the specific gravity of a liquid, when only approximate results are required. Owing to the effects of surface tension, many precautions must be taken if accurate results are required.

NICHOLSON'S HYDROMETER

Nicholson's Hydrometer consists of a cylindrical float, preferably with conical ends (Fig. 25). Above this rises a stiff brass stem carrying a small scale-pan (A), while underneath the float is fixed a small basket (B). The basket is usually loaded with lead so that the instrument floats vertically, with part of the upper cone projecting out of the water. A perforated cap is sometimes fitted so that the basket can be covered over at will.

EXPT. 24. **Determination of the Specific Gravity of a Solid by using Nicholson's Hydrometer.**—In using the instrument it is floated in water, and weights are placed on the scale-pan A, the weights being adjusted until the hydrometer is immersed as far as the mark filed on the upper brass stem. It is advisable to place a slotted sheet of metal across the

top of the jar or cylinder containing the water, the stem supporting the scale-pan being in the slot, which should be wide enough for the stem to move quite freely up and down. This sheet of metal prevents the hydrometer from sinking into the water if too heavily loaded, thereby avoiding wetting the scale-pan and the weights (see Care of Weights, p. 17). It also prevents the hydrometer from coming into contact with the sides of the jar. The weight required to sink the hydrometer to the mark having been determined (say W_0 gms.), the solid of which the specific gravity is required is placed on the scale-pan and the weight *now* required to sink the hydrometer to the mark is determined. Let this be W_1; then the weight of the body in air is $W_0 - W_1$.

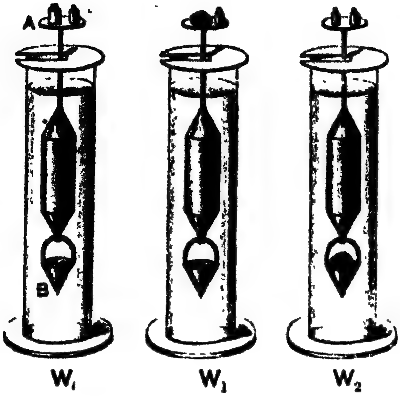


FIG. —Nicholson's Hydrometer.

The body is now placed in the basket B, and the hydrometer is sunk again to the mark by placing weights W_2 in the pan A.

The difference between the last two cases is due simply to the body being in air in the one determination, and in water in the other; that is, the difference is due to the *upthrust* of the water on the body. Hence

$$W_2 - W_1 = \text{the upthrust,}$$
$$= \text{weight of water having a volume equal to that of the wax.}$$

And

$$\text{S.G.} = \frac{\text{weight in air}}{\text{weight of this equal volume of water}},$$
$$= \frac{W_0 - W_1}{W_2 - W_1}.$$

Determine the specific gravity of each of two bodies, one more dense, the other less dense than water. In the second case it will be necessary to tie the body into the basket if no cover is used, otherwise it will float to the top of the liquid.

It is important to see that no air bubbles are entangled with the hydrometer in all these experiments.

The determinations of specific gravity made with this form of apparatus are not nearly so accurate as those obtained by the methods already described, considerable errors being introduced by forces due to surface tension acting round the stem where it leaves the liquid; to minimise this action the stem should be as thin as possible.

EXPT. 25. **Determination of the Specific Gravity of a Liquid by using Nicholson's Hydrometer.**—Float the hydrometer in water, and determine what weight must be added to sink it to the mark: let this be W_0.

Float the hydrometer in the liquid, the specific gravity of which is required, and determine the weight now needed to sink it to the mark: let this be W_1.

Weigh the hydrometer; if its weight is W, then $W+W_0$ is the weight of water displaced when the hydrometer is sunk to the mark in water, and $W+W_1$ is the weight of the liquid displaced when sunk to the mark in the liquid. But the volume displaced is the same in each case, hence

$$\text{Specific gravity of liquid} = \frac{W+W_1}{W+W_0}.$$

§ 5. DETERMINATION OF SPECIFIC GRAVITIES OF LIQUIDS BY COLUMNS EXERTING EQUAL PRESSURES

The pressure exerted by a column of liquid is independent of the shape of the containing vessel, if any effect due to surface tension be neglected; the pressure then depends solely on the vertical height of the column and its density.

The pressure in dynes per sq. cm., exerted by a column of liquid of height h cm. and density D gm. per c.c., is equal to hDg, g being the acceleration due to gravity.

Thus, if two columns of different liquids exert equal pressures, the relation between their heights and densities can be expressed as $h_1 D_1 = h_2 D_2$, when h_1 and h_2 are the heights, and D_1 and D_2 the corresponding densities of the two columns of liquid respectively. Hence $D_1/D_2 = h_2/h_1$. If the second liquid be water, the ratio of the densities is the specific gravity; for D_2 is then the density of water. Thus, the specific gravity of the liquid $= h_2/h_1$ where h_1 is the height of a liquid column which exerts the same pressure

as a column of water h_2 cm. high. This property may be used for the determination of the specific gravity of a liquid in the following manner.

Expt. 26. Determination of the Specific Gravity of a Liquid by the U-tube Method.—If two liquids do not mix, their specific gravities may be compared by balancing two columns, one of each liquid, in the sides of a U-tube, the heights of the columns being measured vertically from the level of the surface of contact C, to the free surfaces A and B (Fig. 26). If the liquids to be compared would mix together, they must be separated by some other liquid with which neither mixes, such as oil or mercury, placed in the bend of the U. In this case the quantity of the liquid used in each tube must be adjusted until the level of the intervening liquid is the same in each of the side tubes.

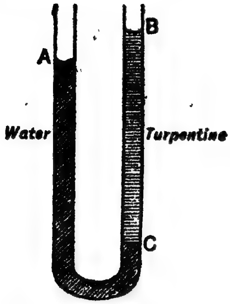


Fig. 26.

The heights of the columns are measured from the top of the intervening liquid to the free surfaces. The ratio of the specific gravities of the liquids is the inverse ratio of the heights of the columns.

The U-tube method just described has several disadvantages. The adjustment of the two surfaces of the intervening liquid to the same level demands the use of a 'spirit level' if any accuracy is to be obtained, and as this has to be used *outside* the tube, the accuracy even then is not very great. If the intervening liquid is mercury, a slight error in this adjustment leads to considerable errors in the results obtained, on account of the high density of mercury. In addition to this difficulty, there is also an actual error due to capillarity, the surface tension of the intervening liquid being different on the two sides, because its surfaces are in contact with different liquids.

These objections are, however, got over completely in a simple form of apparatus known as **Hare's Apparatus.**

Expt. 27. Determination of the Specific Gravity of a Liquid by means of Hare's Apparatus.—In this apparatus the U-tube is inverted, the open ends of the U dipping,

one into water, the other into the liquid whose specific gravity is to be found. In the bend of the U is fitted a branch tube, and by means of this a little air is withdrawn from the bend of the U so that a column of each liquid rises up the corresponding side of the tube. These columns communicate with the same air space at their top surfaces, while the free surfaces of the liquids outside the tubes are both open to the atmosphere. Hence the column in each tube adjusts its height until the column *above the level of the outside liquid* exerts a pressure equal to the difference between the external and internal pressures.

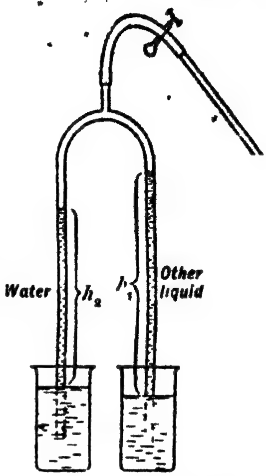


FIG. 27.—Hare's Apparatus.

Measure the heights of the two columns above the corresponding external surfaces of the liquids, for several different adjustments of the internal pressure.

Show that the ratio of the heights is the same in each case.

Tabulate the observations so as to show in parallel columns of the table, the corresponding heights of the liquids in the tubes, and the ratios of these heights, using the height of the water column as numerator in each case. The specific gravity of the liquid used is the mean value of this ratio, since the densities of the liquids are *inversely* proportional to the heights.

CHAPTER V

STATICS

§ 1. Composition of Vectors

Physical quantities can be divided into two groups, **Scalars and Vectors.**

A Vector quantity differs from a Scalar quantity in that it possesses direction as well as magnitude; a Scalar quantity has magnitude only. A vector quantity can be represented by a straight line of a definite length drawn in a definite direction. Two scalar quantities of the same kind are added together by simple addition. Two vector quantities of the same kind cannot, in general, be added in this way, but must be 'compounded' by means of the Parallelogram Law. By the **Resultant** of two vectors we understand **the single vector which would produce the same effect as the two vectors considered.**

The **Principle of the Parallelogram of Vectors** may be stated as follows :—

If two vectors be represented in magnitude and direction by the two adjacent sides of a parallelogram, the Resultant of those two vectors is represented in magnitude and direction by the diagonal of the parallelogram drawn through the point where those sides meet. This principle applies to all vectors, such as displacement, velocity, acceleration, force, etc. In the following discussion, the word *force* will generally be used, but it should be understood that wherever *force* is printed, the word *vector* could be read, or any particular type of vector, such as displacement or velocity.

In the case of forces, any unbalanced force acting on a body will cause it to move. Hence we have a simple method of detecting whether the forces acting on a body balance each other or not: if the body is at rest, the forces acting on the body constitute a balanced system. We may state the following proposition: **When a body is under the action of two forces which are equal and opposite, the body is in equilibrium.**

Suppose three forces are acting on a small body at O and the body is at rest under their united action (Fig. 28).

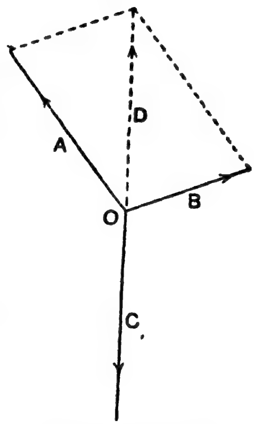


FIG. 28.—Three Forces in Equilibrium.

Represent the three forces by lines drawn from O, drawing the lines along the directions of the forces, and of lengths proportional to their respective magnitudes: let A, B, and C be these lines. By the above proposition, the body O would be at rest if we removed A and B and replaced them by a *single* force D (shown dotted in Fig. 28) which is equal and opposite to C, *i.e.* the action of A and B together is equivalent to that of a single force D equal and opposite to C. This imaginary force D is the **Resultant** of A and B; C, the force which keeps O at rest when A and B are acting also, is called the **Equilibrant** of A and B.

It is evident that the equilibrant and the resultant are equal and opposite forces.

Method of proving the Principle of the Parallelogram of Forces.—The principle of the parallelogram of forces states that if a parallelogram be constructed with the lines representing forces A and B as adjacent sides, the resultant of those two forces will be represented in magnitude and direction by the diagonal of that parallelogram drawn from O.

If this diagonal represents the resultant of A and B, it must be equal and opposite to the line representing C since the forces D and C are equal and opposite. If, therefore, the diagonal of this parallelo-

gram is *found to be equal and opposite to the line representing C*, the principle of the parallelogram of forces is demonstrated.

Method of proving the Principle of the Triangle of Forces.—The construction of the complete parallelogram is not necessary to determine the magnitude of the resultant D. If we draw *ob* representing B in magnitude and direction, and from *b* draw *bc* parallel to A and of length proportional to A, we have now drawn one-half the parallelogram. Joining *o* and *c* gives us the diagonal representing D without further construction; thus by means of a triangle the required resultant can be obtained.

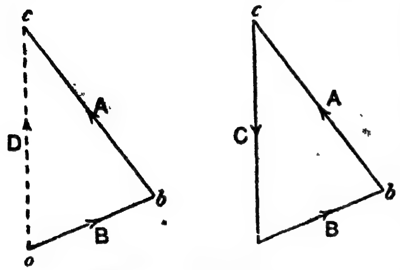


FIG 29.—Triangle of Forces.

The **principle of the triangle of forces** is more usually expressed in the following way: **If three forces having magnitudes proportional to the lengths of, and directions parallel to, the sides of a triangle, act on a body, the body will be at rest under their action, provided that the arrows indicating their directions are in cyclic order round the triangle.**

Consider the triangle *obc*; the arrow indicating the direction of action of the force A in the side *bc* is in the same cyclic order as that indicating the direction of action of B. The resultant is represented by the line completing the triangle with its arrow *opposite* to this cyclic order.

Now the same line *oc* which represents the resultant of A and B will represent the force C if the arrow is pointed towards *o*. Under the action of these three forces A, B and C the body is at rest.

This principle is proved by the experimental verification of the Principle of the Parallelogram of Forces.

The principle just discussed can readily be extended to the case of any number of forces acting on a small body. This general principle, known as the **Principle of the Polygon of Vectors,** is stated as follows: **If a number of forces A, B, C, D, E acting on a body keep the body at rest, the lines representing them in magnitude and direction, drawn end to end, with the arrows indicating the direction of the forces following each other in cylic order, will form a closed polygon.**

This can be shown from the principle of the triangle of forces in the following way. Draw the triangle *oab* representing the forces A, B, and their resultant. Construct on the line *ob* a triangle *obc* such that *bc* represents the force C. The line *oc* is the resultant of *ob* and C; hence *oc* is the resultant of the *three* forces A, B, and C.

The line *ob* is evidently unnecessary; for by drawing *oa*, *ab*, and

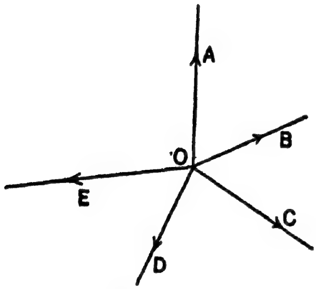


FIG. 30.—Five Forces in Equilibrium.

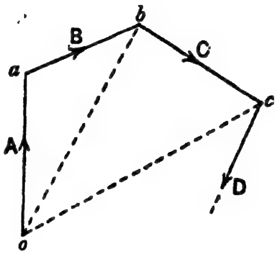


FIG. 31.—Polygon of Forces.

bc as directed, *oc* is found, being the line closing the quadrilateral figure thus obtained.

The fourth force D can be added to A, B, and C by adding another line to the polygon, starting from C; and similarly any number of forces can be compounded by this polygon method. Thus to obtain the resultant of any number of forces acting together at a point, a figure is constructed by drawing lines representing the forces, end to end, with the arrows indicating the direction of the forces following each other in cyclic order.

The line closing the figure with its arrow *opposite* to the cyclic order represents the *resultant* of all these forces.

The same line, with the arrow *following* the cyclic order, represents the *equilibrant* of all these forces.

This is an alternative statement of the principle of the Polygon of Forces.

If the figure is already closed, the resultant force is zero, *i.e.* the body is in equilibrium under the action of the forces given.

The order in which the forces are taken is entirely immaterial.

APPARATUS FOR EXPERIMENTS ON FORCES

To verify these various principles relating to forces, a convenient apparatus can be constructed in the following way :—

Round the edges of a blackboard arrange a number of light, frictionless pulleys, over which light strong cords (fishing-line) are passed (Fig. 32). These should have loops at one end, and, if possible, watch-chain clips at the other. The board is mounted

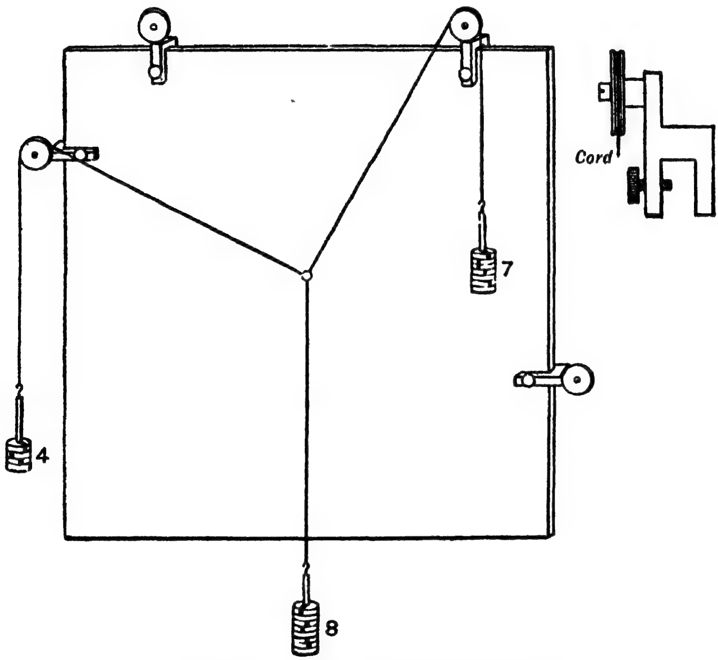


Fig. 32.—Apparatus for Experiments on Forces.

vertically on the wall or on a rigid stand, or if the pulleys are arranged with their planes perpendicular to the board, the board can lie horizontally on a table, the ends of the strings hanging well clear of the edges.

Sometimes the pulleys, instead of being fixed to the edges of the board, are attached to capstan blocks mounted on tripod stands.

A light ring is used as the small body, being clipped to the cords by the watch-clips.

When any forces whatever are applied through the cords, the ring

will move into a position of equilibrium, and the directions of the various strings can be marked on the blackboard with finely pointed chalk, or on a sheet of drawing-paper fixed to the board with a pencil. These lines give the directions of the forces acting, and by measuring lengths along them proportional to the weights hanging from the other ends of the cords, we obtain lines representing the forces in both direction and magnitude.

From these force lines the various propositions already stated can be proved experimentally by using two, three, and larger numbers of forces.

It may be noted that, on account of friction, the positi n of the ring when in equilibrium under a given set of forces may vary over a small area. To obtain the true position, the position taken up should be marked, the ring disturbed, and the new position of rest observed. The middle of the area covered by the positions taken up after several disturbances may be taken as the true position of equilibrium.

Also, if the pulleys are mounted with their planes perpendicular to the board, they must be free to swivel about a vertical axis so as to follow the various directions of the cord passing over them. This introduces friction in addition to the friction due to turning about the axle, and hence the vertically mounted board is preferable.

EXPT. 28. **Determination of the Conditions of Equilibrium under the Action of Two Forces.**—Attach two cords to the ring, and hang on to the cords various weights. Note that the two cords always draw out into one straight line, and that the ring only comes to rest if the weights hanging from the two cords are equal.

EXPT. 29. **Verification of the Principle of the Parallelogram of Forces and the Principle of the Triangle of Forces.**—Fasten to the ring three cords with weights attached. Draw lines representing the forces acting on the ring when in the position of equilibrium as already described.

Construct a parallelogram with any two of the force lines as adjacent sides and show that its diagonal is equal in length to the third line, and that the diagonal and this third line are in continuation of one another.

Draw a triangle at the side of the board with two sides parallel to two of the forces, their lengths being proportional to these forces respectively, and the arrows indicating their directions following each other in cyclic order. Complete the triangle and show that the third side is parallel to the third force, and represents the magnitude of the third force on the

same scale as the other two forces are represented by the other lines.

EXPT. 30. **Determination of the Weight of a Body by Means of the Parallelogram of Forces.**—Using two known weights and one unknown, proceed as in Experiment 29, constructing the parallelogram with the lines representing the known forces as adjacent sides. From the length of the diagonal, determine the weight of the unknown body. Verify this with an ordinary balance.

EXPT. 31. **Verification of the Principle of the Polygon of Forces.**—Use four or five weights and proceed as in Experiment 29, but construct at the side of the board a figure consisting of lines parallel and proportional to the forces, the arrows indicating the directions of the forces following each other in cyclic order. When all the forces but one have been represented, close the polygon and show that the closing line represents the remaining force in both magnitude and direction, provided the arrow follows the same cyclic order as the others. Do this for the same set of forces two or three times, taking the forces in different orders in each case so as to get polygons of different shapes. Show that the length of the closing line and its direction is entirely independent of the order in which the other lines are taken, and that in every case it represents the remaining force.

§ 2. RESOLUTION OF VECTORS

We have seen how two vectors acting on one body can be compounded into one equivalent vector. It is necessary now to consider the **resolution of one vector into two other vectors which together shall be equivalent to the original vector.**

If B and C represent two such vectors it is evident that they are equivalent to A, if A is represented by the diagonal of the parallelogram constructed on B and C (Fig. 33).

If *B were perpendicular to C*, B would have no effect whatever along the line of action of C, nor would C have any effect along the direction of B, hence we could say that *C represents the total effect of A along the direction of C*, while *B represents the total effect of A along the direction of B.*

B and C would in such a case be called the **Resolved Parts**

of A along their respective directions, or B and C are the **Components** of A along these directions (Fig. 34).

Now $\quad B = A \cos \phi$, and $C = A \cos \theta$,

therefore we may say that the **Resolved part of any force**

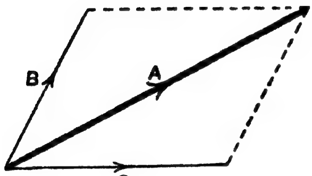


FIG. 33.—Resolution of Vectors.

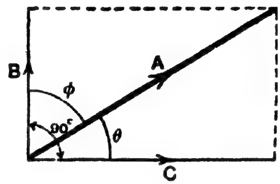


FIG. 34.—Resolved Parts of a Vector.

acting along a given direction, is equal to the magnitude of the force multiplied by the cosine of the angle between the direction of the force and the given direction.

Direct verification of this principle is given in the experiment called the Static Inclined Plane described on p. 72 (Fig. 36). In this a mass of weight W is maintained at rest, on a plane inclined to the horizontal at an angle θ by a force P acting along the plane.

Evidently the *effect of the weight* W *along the plane* is only equal to P, since P keeps the mass from moving down the plane under the action of its weight W. That is, P is equal to the resolved part of the weight acting parallel to the plane.

But P is shown to be equal to $W \sin \theta$, therefore the resolved part of the weight W acting along the plane must be equal to $W \sin \theta$.

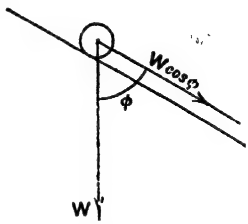


FIG. 35.—Inclined Plane.

Now the angle between the plane and vertical is an angle ϕ (Fig. 35), such that $\phi = 90° - \theta$, and $\cos \phi = \sin \theta$.

$$\therefore W \sin \theta = W \cos \phi.$$

Thus the resolved part of W acting along the plane at an angle ϕ to the direction of the weight W, is equal to $W \cos \phi$.

STATIC INCLINED PLANE

If a load W is resting on an inclined plane, it may be maintained in equilibrium on the plane, or pulled without acceleration up the plane, by a force P acting parallel to the surface of the plane. The force P is much smaller than the weight of the body W, the value of P diminishing as the inclination of the plane diminishes.

Consider the forces acting on a body of weight W, which is just maintained in equilibrium on a plane inclined to the horizontal, at an angle θ, by a force P acting along the plane.

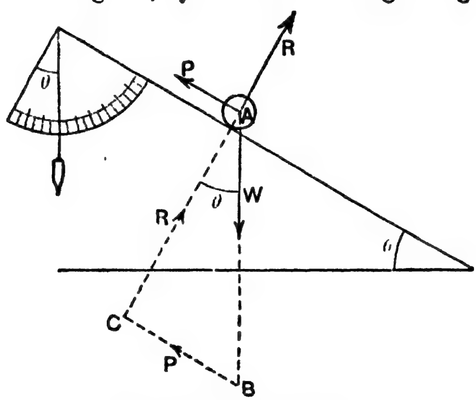


FIG. 36.—Static Inclined Plane.

We have first its weight W acting vertically downwards, secondly the force P acting along the plane, and in addition to these there is a force exerted by the plane called the **reaction R of the plane**. This force acts perpendicular to the surface of the plane, if the plane be smooth (Fig. 36).

These three forces together keep the body at rest; their directions are all known, and hence if the magnitude of one of them is known, the other two can be determined by applying the principle of the triangle of forces.

Let the line AB denote the weight W. Draw AC perpendicular to the plane, *i.e.* parallel to the reaction R, and BC parallel to the plane, *i.e.* parallel to P.

They intersect at C; hence AC and BC represent R and P respectively.

The angle CAB is equal to the angle θ; since AC is perpendicular to the plane and AB is perpendicular to the base. Thus

$$\frac{BC}{AB} = \sin \theta$$

But BC and AB represent P and W,

therefore $$\frac{P}{W} = \sin \theta,$$

or $$\mathbf{P = W \sin \theta.}$$

A somewhat simpler proof is obtained if we consider the **energy gained** by the weight when pulled up the plane, and the **work done** by P in pulling it up the plane.

The height of the top of the plane above the bottom is h, hence the weight gains an amount of potential energy $= Wh$ when raised to the top.

The force P acts through a distance l in its own direction when pulling the weight up the plane, if l is the length of the plane, hence the work done by the force is Pl.

By the principle of the conservation of energy

energy gained = work done,

i.e. $$Wh = Pl,$$

or $$P = \frac{Wh}{l} = W \sin \theta.$$

The apparatus used in this experiment consists of a plane board hinged at its lower end, and supplied with some means of adjusting its inclination to several different values. At the upper edge is usually fixed a pulley, over which a cord passes. To the end on the plane the load W is attached, while from the hanging end weights can be suspended to exert the force P on the cord. In order to eliminate friction between the load W and the plane, W is usually a small roller, supported on an axle carried by a suitable framework, to which framework the cord is attached. In some forms of apparatus the pulley and hanging weight are replaced by a spring balance which automatically adjusts itself to exert the required force P at any inclination, and P can be read off directly.

The angle θ can be measured by means of a protractor placed with its edge along the horizontal base and its centre at the centre of the hinge, but it is better to have a graduated quadrant fitted near the top of the plane, with a plumb-line hanging from its centre. If the zero line of the quadrant stands out at right angles to the plane, the angle θ is read off as the angle between the plumb-line and this zero line. This method is preferable to the first, as it is necessary to level the base by means of a spirit-level if the first method is used, and other errors easily creep in when measuring the angle θ.

EXPT. 32. **Static Inclined Plane.**—Apply different forces P over the pulley, using different inclinations of the plane. Adjust P in each case *until the roller moves equally freely up or down the plane* when given a slight push. This effectively gets rid of the effect of any friction at the pulley and enables very accurate readings to be taken. Note corresponding values of P and θ, taking five or six different inclinations. Tabulate your observations thus :—

P.	θ.	$\sin\theta$.	$\frac{P}{\sin\theta}$.

Mean $\frac{P}{\sin\theta} =$

Since the same roller is used in each case, W is constant. But $P = W \sin\theta$, therefore the values of $P/\sin\theta$, the quotients tabulated in the last column, should be constant and equal to W. The mean value of $P/\sin\theta$ should be equal to the weight of the roller quite accurately; verify this by actually weighing the roller.

If the quadrant for measuring θ is not fitted to the apparatus employed, measure h and l directly, taking the same point on the plane each time and measuring its height above the base and its distance *along the plane* from the centre of the hinge. The table of observations would then be arranged under the headings $P, h, l, \frac{h}{l}, P/\frac{h}{l}$.

Find the mean value of Pl/h. This should be equal to W measured directly.

If the weights exerting the force P are put in a scale-pan

suspended from the cord, the weight of the scale-pan must be included in the force P.

§ 3. General Conditions for the Equilibrium of any Body under any Forces

MOMENT OF A FORCE

The turning effect of a force about any axis is called the moment of the force about the given axis, and is defined as the product of the force and the perpendicular distance from the axis to the line of action of the force.

The direction of rotation, 'clockwise' or 'anti-clockwise,' is termed the **sense** of the moment: it is immaterial which sense is chosen as positive, provided the convention made is retained throughout.

If a body is acted upon by any system of forces, the body will remain at rest only if the two conditions given below are satisfied.

1. **The Resultant Force in any direction must be zero.**
2. **The Resultant Moment of all the forces about any axis must be zero.**

Condition 1 is included implicitly in condition 2, but it is so important that an explicit statement is of value.

PRINCIPLE OF THE LEVER

The first of the above conditions has been proved experimentally in considering the composition of forces; it is purposed now to demonstrate the truth of the second statement by experiment. The simplest way to do this is by pivoting the body on which the forces are to act about a suitable axis called the **fulcrum**. Then the forces required to prevent the body from moving *as a whole* are introduced at the fulcrum, and condition 1 is satisfied without further trouble: a body pivoted in this way is called a **lever**.

Since we cannot determine easily the force brought into action

at the fulcrum, we can only find its *moment* about *the fulcrum itself*. **The moment of any force about its point of application is zero,** hence the forces brought into action at the fulcrum have no moment about the fulcrum itself. Thus in considering the equilibrium of the lever, we can leave out of consideration the forces brought into action at the fulcrum, so long as we consider the moments about the fulcrum alone.

The condition of equilibrium of a lever may therefore be *stated thus*: **A lever will be in equilibrium, if the moments of all the applied forces about the fulcrum have a zero resultant.**

LEVERS

A convenient apparatus for demonstrating the principle of the lever, and therefore verifying the law of moments, may be constructed in the following way. A rigid framework of wood carries

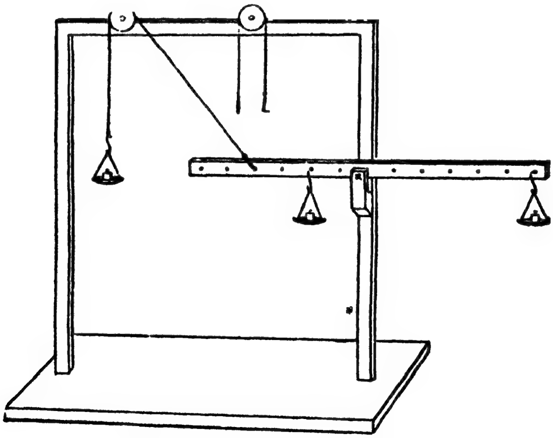


FIG. 37.—Experiments on Levers.

at a point near the middle of one of the uprights, a round rod of brass projecting horizontally. On this rod is placed a metre scale with holes drilled about every 2 cm. along its length, the rod passing through the hole at the *middle* of the scale. By this means the centre of gravity of the rod is at the point of support, and the

effect of the weight of the rod is eliminated (*see* Centres of Gravity, p. 80).

Pulleys are mounted on the top bar of the framework, with cords passing over them. The cords can be attached to the metre scale at any point by means of brass hooks tied to the ends. The other ends of the cords carry scale-pans, so that various forces can be applied to the lever by weights placed in the scale-pans. Other scale-pans can be hung from the holes in the metre scale directly, so that the forces can be applied vertically upwards or downwards, or inclined at different angles to the horizontal.

Since friction of the pulleys and at the fulcrum cannot be eliminated, the apparatus should be given a slight initial velocity when approximately adjusted, the weights being adjusted finally until the lever turns equally freely in either direction. If one of the forces acts obliquely, this slight motion given to the lever should not deflect it to any great extent from the horizontal position, otherwise the angle at which the force acts will be altered, and its moment about the fulcrum will thereby be changed.

When the lever turns equally freely in either direction, the moments of the various forces about the fulcrum must be calculated, and their senses noted. This is done by multiplying each force by the perpendicular distance from the fulcrum to the line of action of the force, positive or negative signs being prefixed according as the sense of the moment is clockwise or anti-clockwise. The moments of all the forces must then be added together *algebraically*; the sum of the moments in each case should be zero.

NOTE.—The weights of the scale-pans must be included in each of the forces considered.

EXPT. 33. **Levers.**—Perform the experiment for each of the following cases, applying forces as indicated in Fig. 38:—

Case I., frequently referred to as a **lever of the first order.**

The force F causes clockwise rotation, *i.e.* its moment Fd about the fulcrum is *positive*.

The force F′ causes anti-clockwise rotation; its moment $F'd'$ about the fulcrum is therefore *negative*.

Show (algebraically) $Fd + F'd' = 0$.

EXAMPLE. $F = 350$ gm. $d = 48$ cm

$$Fd = +16800,$$

$$F' = 750 \text{ gm.} \quad d' = 22 \text{ cm.}$$

$$F'd' = -16500,$$

$$Fd + F'd' = 16800 - 16500 = 300,$$

Error 2%.

Case II., frequently referred to as a lever of the second order.

F causes anti-clockwise rotation, *i.e.* its moment Fd about the fulcrum is negative.
F′ causes clockwise rotation, *i.e.* its moment $F'd'$ about the fulcrum is positive.
Show as before $Fd+F'd'=0$.

Case III., generally known as a lever of the third order.

Fd is again negative, and $F'd'$ positive.
Show $Fd+F'd'=0$.

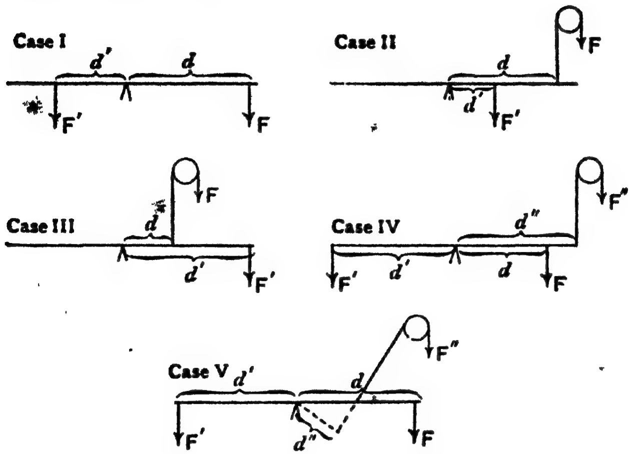


FIG. 38.—Levers.

The general principle is applicable to any number of forces, acting at any angle to the lever, as is shown in the two following cases. Any other sets of forces can be substituted for these, and the total moment about the fulcrum will in all cases be found to be zero.

Case IV.

Fd is positive.
$F'd'$ is negative.
$F''d''$ is negative.
Show algebraically $Fd+F'd'+F''d'=0$.

Case V.

Fd is positive.

$F'd'$ and $F''d''$ are both negative.

Show $Fd+F'd'+F''d''=0$ as before.

Enter all results for each of the five cases as indicated in Case I., stating any observed error as a percentage of the total moment in one direction. It is advisable to use fairly large forces—200 to 300 gm. at the ends of the scale and up to 1 kgm. nearer the middle. By this means the effect of the friction at the fulcrum is made a comparatively small proportion of the actual turning moments applied, and greater accuracy is obtained.

Expt. 34. Weighing a Metre Scale by Application of the Principle of the Lever.—Support the metre scale at a point about 10 cm. from one end. Hang a scale-pan from the end hole in the short side of the scale, and adjust weights in the scale-pan until the scale is just balanced. The weight of the scale acts downwards at its Centre of Gravity, *i.e.* at the middle of the scale. Let the weight of the metre scale be W gm. and the weight in the scale-pan (including the scale-pan itself) w gm. The distance of the fulcrum from the centre of the scale being D, and from the scale-pan d, we have

$$WD = wd.$$

Measure D and d and observe w. Calculate W, the weight of the metre scale.

Repeat this for two or three positions of the fulcrum and check the result by weighing the metre scale on a balance.

§ 4. Centres of Gravity

When two parallel forces act upon a rigid body they can, in general, be replaced by a single resultant force. Thus the two forces P and Q acting at the points A and B are equivalent to a single force $R = P + Q$.

The line of action of this force cuts AB in a point C such that

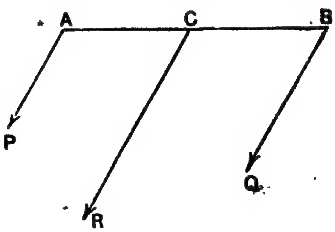


Fig. 39.—Resultant of Parallel Forces.

$$P \times AC = Q \times CB.$$

Thus the position of the point C is independent of the

direction of the forces. It is called the **centre of the parallel forces.**

Similarly, when any number of parallel forces act upon a rigid body, their resultant passes through a certain point whose position does not depend on the direction of the forces. The position of this point—the centre of the parallel forces—is fixed when the magnitudes and points of application of the forces are known.

All bodies are attracted towards the centre of the earth by the force of gravity. We may suppose a rigid body made up of particles each one of which is attracted by the earth. We thus obtain a system of approximately parallel forces acting on the body. The centre of these parallel forces is called the **Centre of Gravity** of the body.

Accordingly the **Centre of Gravity of a body may be defined as that point, fixed with relation to the body, through which the resultant of the earth's attraction on all the particles passes, whatever be the position of the body.**

When a heavy body is supported at a single point, the only forces acting on the body are its weight and the reaction at the point of support. If the body remain at rest, these two forces must be in equilibrium, and consequently must have the same line of action. Hence the point of support must be in the same vertical line as the Centre of Gravity.

EXPT. 35. **Experimental Determination of the Centre of Gravity.**—To find the Centre of Gravity of a body, suspend it by a string attached to any point A and mark the position of the vertical line AB (Fig. 40). The vertical may be determined by means of a plumb-line. Then suspend the body from another point C and mark the vertical CD. The Centre of Gravity must

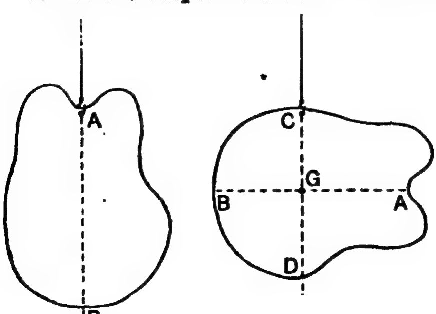


FIG. 40.—Determination of Centre of Gravity.

be in AB, and also in CD, and so must be at the point of intersection G.

Confirm this by suspending from a third point.

The Centre of Gravity of a body depends on the distribution mass throughout the body, as can be shown in the following nner. The body used for this experiment consists of a iform plate of wood, to which may be attached a load formed by a brass nut and screw P.

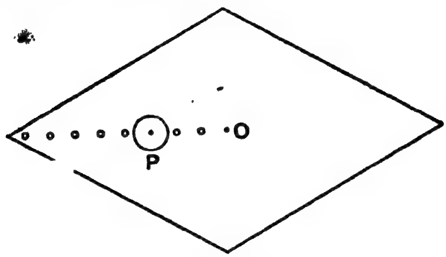


FIG. 41.—Loaded Plate.

First find the Centre of Gravity of the plate alone, and then the Centre of Gravity with the load fixed to the plate. Repeat the latter determination for several positions of the load.

Plot a graph showing how the position of the Centre of Gravity varies as the load is moved along the diagonal of the plate, taking as abscissae the distances (OP) of the load from the centre of the plate, and as ordinates the distance of the Centre of Gravity from the centre of the plate.

It will be noticed that the Centre of Gravity of the compound body moves from the position of the Centre of Gravity of the plate of wood, through a distance proportional to the displacement of the brass weight from that point. The ratio of the weights of the brass nut and the plate of wood is inversely proportional to the ratio of the distances of their centres from the Centre of Gravity of the compound body. Verify this.

§ 5. Graphic Methods of Computation

A considerable number of quantities can be determined by purely graphical, as distinct from numerical, methods. It is

also possible to investigate by such methods the conditions of equilibrium of a body or the force required to keep it in equilibrium. These are the methods of **Graphic Statics.**

Two properties of a flat uniform body which are readily found by a simple graphical construction are the position of the **Centre of Gravity of a uniform plate,** and the value of the **Moment of Inertia** of that plate about any axis. These methods not only enable the Moment of Inertia of the cross-section of a beam to be found, but also give a useful exercise on the use of the planimeter.

GRAPHIC METHOD OF DETERMINING THE CENTRE OF GRAVITY OF A UNIFORM PLATE

Consider a plate of any contour; draw at one side of it a line XX′ and another line YY′ parallel to the first, tangential with the curve at the extreme point on the opposite side (Fig. 42).

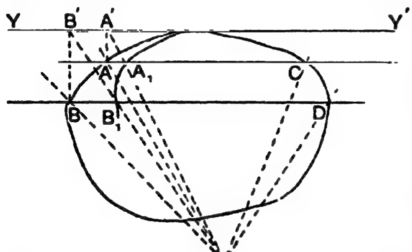


FIG. 42.—Graphic Method for Centre of Gravity.

From any point O on the first line draw a number of lines radiating in different directions. It is convenient to arrange the radiating lines so that pairs, such as OA, OC, intersect the contour at equal distances from the original line XOX′.

Through each of the points of intersection, A, B, C, etc., draw a line parallel to the base line XOX′. From each point A, B, C draw a line perpendicular to XOX′, meeting YY′ in points A′, B′, C′, etc., respectively.

Join OA′, OB′, OC′. Each of these lines cuts its corresponding parallel in some point A_1, B_1, C_1. Draw a curve passing through all the points A_1, B_1, C_1, etc., and find the area of this figure and of the original plate.

The Centre of Gravity of the plate is at a distance h from XOX′ such that

$$h = \frac{\text{Area of figure } A_1B_1C_1 \text{ etc.}}{\text{Area of original figure ABC etc.}} \times \begin{array}{l}\text{Perpendicular distance}\\ \text{between XX}' \text{ and YY}'.\end{array}$$

PROOF.—Consider a small portion of the original figure enclosed between the parallels AC and BD, imagining these to be quite close together.

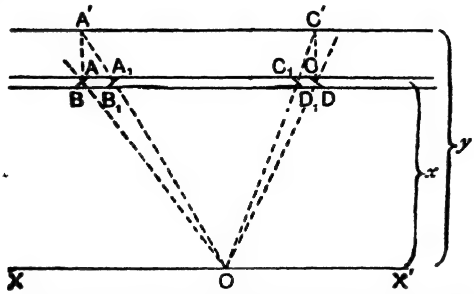


FIG. 43.—Proof of Graphic Method for Centre of Gravity.

The corresponding area of the figure $A_1B_1C_1$, etc., constructed as already described, has the same vertical dimension, but its length is reduced in the ratio of x/y,

i.e.
$$\frac{\text{Area ABCD}}{\text{Area } A_1B_1C_1D_1} = \frac{y}{x}.$$

The moment of the mass of the original figure ABCD about the axis XOX′ is equal to

$$ABCD \times x,$$

and thus is equal to $A_1B_1C_1D_1 \times y$.

Thus, for any narrow element ABCD of the original figure, the moment of its mass about the axis XOX′ is equal to the area of the corresponding element of the constructed figure multiplied by the distance between the lines XOX′ and YY′.

Now if the Centre of Gravity of the plate were at a distance h from the axis XOX′, the whole area of the plate multiplied by the height h would be equal to the sum of the products of the elemental areas, such as ABCD, multiplied by the corresponding distances (x) from the same line XOX′.

Thus $\quad$ Area of plate $\times h = \Sigma(ABCD \times x)$.

Now the value of $ABCD \times x = A_1B_1C_1D_1 \times y$,

i.e. $\Sigma(\text{ABCD} \times x) = y \times \Sigma \text{A}_1\text{B}_1\text{C}_1\text{D}_1,$
$= y \times$ Area of constructed figure.

Hence Area of plate $\times h =$ Area of constructed figure $\times y$,

i.e. $h = \dfrac{\text{Area of constructed figure}}{\text{Area of original plate}} \times$ Distance between XX′ and YY′.

By taking a different base line, say perpendicular to the line XOX′, and carrying out a similar construction, the distance h' from this second base line can be found by the same process, and therefore the exact position of the Centre of Gravity is known.

When the plate is symmetrical about any one line, the C.G. must of course be in the axis of symmetry, and thus one construction only is required for the determination of the position of the Centre of Gravity.

> EXPT. 36. **Graphic Determinations of the Centre of Gravity.**—Construct an isosceles triangle. Take its base as the axis XOX′ and show that the Centre of Gravity of the triangle is one-third of the way up the triangle, using a planimeter for measuring the necessary areas. Find also the position of the Centre of Gravity of a semicircle.

GRAPHIC DETERMINATION OF THE MOMENT OF INERTIA OF A UNIFORM PLATE

(For definition of Moment of Inertia, see p. 143.)

The construction for this determination is similar to that for the determination of the position of the Centre of Gravity of a plate (*q.v.*).

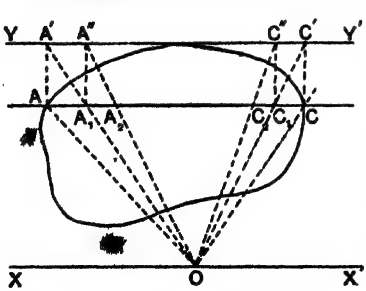


FIG. 44.—Graphic Method for Moment of Inertia.

When the points A_1, B_1, C_1, etc., have been found, lines from these points are drawn to YY′, meeting YY′ in A″, B″, C″, etc., the lines A_1A'' being drawn perpendicular to XOX′.

Join OA″, OB″, etc.

These lines cut the corresponding parallels in points A_2, B_2, C_2, etc.

Draw the contour A_2, B_2, C_2, etc., and find the area of the figure thus obtained.

The Moment of Inertia of the plate about the axis XOX′ is equal to the area of this figure multiplied by the square of the distance between the lines YY′ and XX′.

PROOF.—Consider an elemental area ABCD of the original figure enclosed between two parallels very close together.

The length of the element between A_1C_1 is equal to

$$AC \times \frac{x}{y}.$$

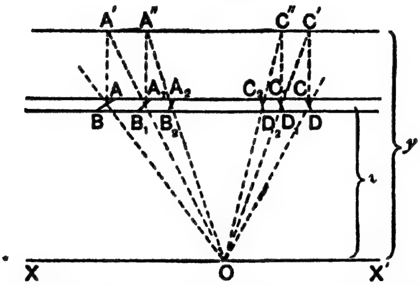


FIG 45.—Proof of Graphic Method for Moment of Inertia.

The length of the element between A_2C_2 is equal to

$$A_1C_1 \times \frac{x}{y},$$

i.e.

$$\frac{A_2C_2}{AC} = \frac{x^2}{y^2}.$$

The vertical dimension of the various parts of the element is unaltered. Hence

$$\frac{\text{Area of element of plate}}{\text{Area of corresponding element of constructed figure}} = \frac{y^2}{x^2} = \frac{ABCD}{A_2B_2C_2D_2}.$$

Now the Moment of Inertia I of the whole plate about XOX′ is the sum of the products of the mass of each element by the square of its distance from the axis XOX′, *i.e.*

$$I = \Sigma ABCD \times x^2.$$

But $$ABCD \times x^2 = A_2B_2C_2D_2 \times y^2,$$

i.e. $$I = \Sigma ABCD \times x^2 = y^2 \times \Sigma A_2B_2C_2D_2,$$

or $$I = \text{Area of constructed figure} \times y^2.$$

EXPT. 37. **Graphic Determination of the Moment of Inertia of a Circular Plate.**—Draw a semicircle, taking its diameter as the axis XOX′; find the Moment of Inertia of the semicircle about a diameter. Twice this will be the Moment of Inertia of a circular plate about a diameter. Show that this is equal to $\frac{\pi a^4}{4}$, where a is the radius of the circle (10 cm. is a convenient radius to take).

EXPT. 38. **Graphic Determination of the Moment of Inertia of a Rectangular Plate.**—Draw a rectangle of length b and breadth d (10 cm. and 15 cm. are convenient lengths to take). Using a line across the middle as the base line XOX′, find the Moment of Inertia of one half of the rectangle about this line. Twice this is evidently the Moment of Inertia of the whole rectangle about an axis through the middle. Show that this is equal to $bd^3/12$.

Do this for the same rectangle about an axis through the middle but parallel to the other pair of sides.

It will have been noted in the foregoing that the mass of the plate has not been mentioned. The contour is marked out on the paper and the construction deals merely with the area of the plate. The result obtained is what is usually known as the **Moment of Inertia of the area** about the given axis. This is the quantity generally required in engineering problems. If, however, the Moment of Inertia of an actual plate of matter is required, it may be found from the Moment of Inertia of the area in the following way:—

The Moment of Inertia of the area is numerically equal to the Moment of Inertia of a plate of that same shape whose **surface density** is unity. If, therefore, the Moment of Inertia of the area is found graphically in the foregoing manner, the **Moment of Inertia of a plate of the same shape about a similar axis is found by multiplying the result thus obtained by the surface density of the plate, i.e. by its mass divided by its area.** This form of result is very rarely required.

§ 6. Graphic Statics

Whether a body will be in equilibrium or not under the action of a given system of forces can be tested by purely graphic construction. The forces can produce motion of two kinds: (*a*) translation, (*b*) rotation. The first of these will be zero if there is no resultant force in any direction, the second will be zero if there is no resultant moment about any axis.

The graphic construction to test for no motion of translation is to construct the polygon of forces. If the polygon is closed, there is no resultant force in any direction, and the body will have no motion of translation.

If now we can devise a graphic construction to test whether the resultant moment about any axis is zero, we shall have a complete graphic method for testing the equilibrium of a body under any system of forces.

The graphic method for testing for zero moment is known as the **Link Polygon** or the **Funicular Polygon.**

Let A, B, C, D and E (Fig. 46) represent a system of forces acting on a body in such a way as to produce neither translation nor rotation. The force polygon will be of the shape shown in full lines in Fig. 47, and will be a closed polygon.

Choose some point O and draw lines from O to the corners of the force polygon, *ab*, *bc*, *cd*, etc. From some point P on the line of action of A (Fig. 46) draw PQ parallel to the line O*ab*.

From the point Q, where this cuts the line of action of B, draw a line QR from B to C parallel to O*bc*. From C to D draw RS parallel to O*cd* and so on. By this means a figure will be obtained, shown by the dotted lines drawn between the lines of action of these forces A, B, C, D and E. This is called the **Link Polygon** or the **Funicular Polygon**, and if it is closed the forces acting on the body will have no turning moment on the body. The conditions of equilibrium of a body may therefore be expressed by saying that, **for equilibrium** of any body, **the force polygon must be closed,** and **the link or funicular polygon must be closed.**

If, when these polygons are constructed, they are not found to be closed, the line closing the force polygon must be drawn; this gives the magnitude and direction of the equilibrating force. Its line of action is found by producing the open ends of the funicular

polygon until they meet. The point of intersection is on the lin
of action of the force. The direction and magnitude of the forc

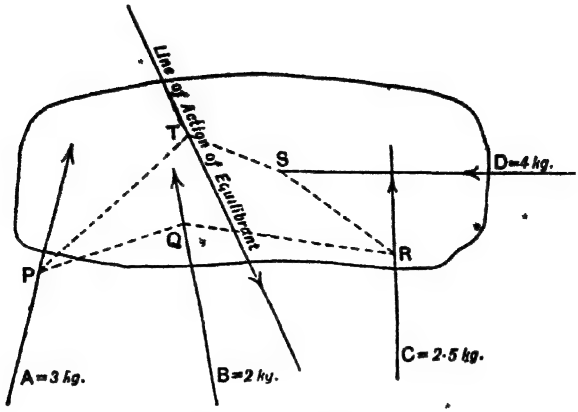


FIG. 46.—Link Polygon.

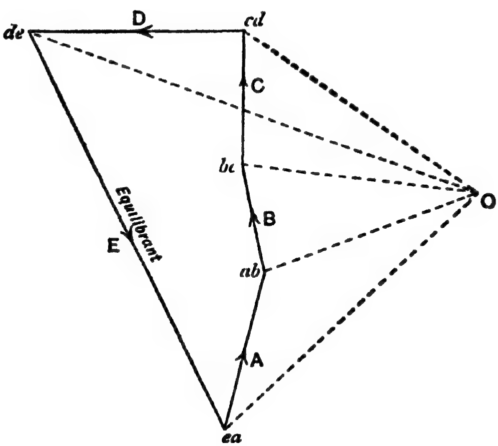


FIG. 47.—Force Polygon.

are known already, hence the force required to maintain the whol
body in equilibrium is determined completely.

The magnitude and line of action of the force E in the diagram (Fig. 46) was found by means of these methods, ABC and D being supposed known.

For the proof of the statements made in connection with the funicular polygon, and for its further applications, reference must be made to text-books of Applied Mathematics.

EXPT. 39. **Construction of the Force Polygon and the Link Polygon.**—Hang up a light cardboard or metal plate on the apparatus used for verifying the Polygon of Forces, suspending it by means of four cords applied at different points, and carrying weights at the other ends, arranging the cords over pulleys so as to act in different directions. Make a copy of the plate on drawing-paper on a drawing-board, putting in lines to represent the directions and magnitudes of *three* of the four forces acting on the body.

Construct the force polygon and the link polygon on these three forces, and find the line of action and the magnitude of the force required to keep the body at rest when under the action of these three forces. Verify that the fourth force actually acting on the plate is of this magnitude, and that its line of action coincides with that obtained by this graphic construction.

EXPT. 40. **Graphic Determination of the Weight of a Plate.**—As an additional exercise in graphic statics a heavy plate may be used, supporting it by three strings, so as to obtain three forces—acting in different directions, and applied at different points, but all in one plane. Determine the force which must be acting on the plate to keep it in equilibrium under these three forces; this will be equal to the weight of the plate. The line of action of the force obtained graphically must pass vertically through the centre of gravity of the plate. Verify these results by determining the position of the centre of gravity (p. 80) of the plate, and also by weighing it.

§ 7. FRICTION

Whenever an attempt is made to move relatively to each other two bodies which are in contact, forces are introduced which oppose the motion. Such forces are grouped together generally under the title of frictional forces, though they differ very widely in their nature. The investigation of fluid friction

is generally dealt with in experiments on viscosity and is beyond the range of this book.

SOLID FRICTION

When two solid bodies are in contact, the forces between them can be resolved, in general, into two components. The component in the direction of the common normal may be called the **thrust** between the bodies; the other component, at right angles to the normal, may be called the **friction**. When an external force is applied to one of the bodies tending to move it in a direction at right angles to the normal, the force of friction is called into play and tends to prevent sliding motion. So long as no relative motion takes place the friction just balances the applied force. If the applied force is gradually increased, a stage will be reached at which sliding motion is just on the point of commencing. The friction is then called **limiting friction.**

The limiting friction between solid surfaces pressed together by a given source is practically independent of the area of contact, provided the surfaces are not appreciably deformed by applying too large a force on too small an area.

The friction between two solid surfaces, when motion takes place, is also independent of the relative velocity of the two surfaces.

In the case of two solid surfaces in contact, the limiting friction between the surfaces depends solely on the nature and condition of the two surfaces and on the force pressing the two surfaces together. The limiting friction is proportional to the force pressing the two surfaces together, and we obtain from this relation the quantity known as the **Coefficient of Friction** between two surfaces.

COEFFICIENTS OF FRICTION

The **Coefficient of Friction** between two surfaces is defined as the **force of friction, divided by the force pressing the**

two surfaces together. Thus if the thrust, or the force acting perpendicular to the two surfaces, is F, and the force resisting

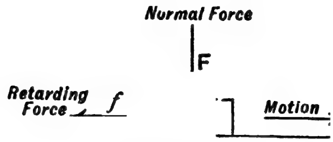


FIG. 48.—Friction.

their relative motion is f, the coefficient of friction between the two surfaces is f/F. This is usually indicated by the symbol μ, so that

$$\mu = \frac{f}{F}.$$

STATIC AND DYNAMIC FRICTION

The force required to *start* two surfaces slipping over each other when pressed together with a given force is greater than the force necessary to keep them moving over each other when once they are started. We thus have two frictional forces for a given force perpendicular to the faces. One of these is called the **static frictional force,** being equal to the force that has to be exerted to start the motion, *i.e.* **the force exerted on each other by the two surfaces when at rest.** The other force is called the **dynamic frictional force,** being equal to **the force required to keep the surfaces in steady motion over each other when once the motion has commenced.** Corresponding to these two forces there will be two coefficients of friction. The coefficient of static friction is invariably greater than the coefficient of dynamic friction.

EXPT. 41. **Determinations of the Coefficients of Friction by moving a Block over a Horizontal Table.**—Adjust the table so that the surface is horizontal. On it place a block of wood or metal whose weight has been determined. Attach a cord to a hook in the side of the block, and pass the cord over a pulley adjusted so that this part of the cord may be horizontal. To the free end of the cord attach a scale-pan on which various weights can be placed.

i. Determination of the Coefficient of Static Friction.—Place a known load on the block, and adjust the weights in the scale-pan till the block just moves; find the ratio between the force required to move the block and the force pressing the two surfaces together. This ratio is the coefficient of static friction for the pair of surfaces used. Repeat this for several different weights on the block, and show that the ratio is approximately constant.

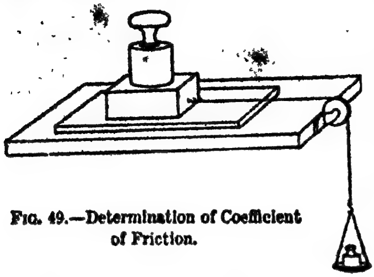


FIG. 49.—Determination of Coefficient of Friction.

The force f, applied to the scale-pan, must include the weight of the scale-pan, and the force F, pressing the surfaces together, includes the weight of the block.

Record the observations giving F, f, and μ in tabular form. The mean value of μ is the coefficient of static friction.

ii. Determination of the Coefficient of Dynamic Friction.—Place weights on the block and adjust the weights in the scale-pan until the block continues to move *without acceleration* when given a slight start. Find the ratio between the force required to keep the block in motion and the force pressing the two surfaces together. This ratio is the coefficient of dynamic friction for the two surfaces used. Do this for several different weights on the block to show that the ratio is approximately constant but smaller than the coefficient of static friction already obtained. Find the mean value of the coefficient of dynamic friction. Enter the results as in the determination of the coefficient of static friction.

By covering the table with a flat plate of brass or of zinc and using blocks of different materials the coefficients of friction between several different pairs of surfaces can be determined. A suitable selection, as giving fairly considerable differences in the values of the coefficients is: wood on wood (*a*) with, (*b*) across the grain, zinc on zinc or brass on brass, and wood on brass or wood on zinc.

It is essential for consistent results that the surface of any one plate shall be in a uniform state of polish all over. If this is not the case, the experiment must be carried out so that the portion of the lower surface moved over is always the same. This is done by marking a line on the fixed surface and starting the block always from that line.

It is also essential that the surfaces be in the same condition in all the experiments with a given pair of surfaces. If the surfaces are pressed together before applying the force, the coefficient of friction will be changed to some extent; if moisture condenses on the surfaces the coefficient will be changed entirely.

LIMITING EQUILIBRIUM ON AN INCLINED PLANE

When a body is resting on an inclined plane, and the angle θ between the plane and the horizontal is increased till the body is just on the point of sliding down the plane, the force of friction assumes its limiting value. As in the Static Inclined Plane (p. 72), the force pulling the body down the plane is equal to the force, P, required to keep the block at rest on the plane in the absence of friction. The force pressing the two surfaces together is equal to the reaction, R, of the plane. The coefficient of friction, μ, is therefore P/R.

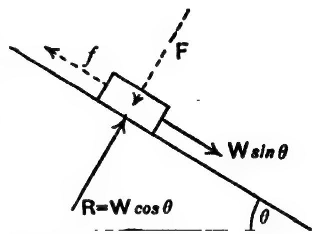


FIG. 50.—Friction on Inclined Plane.

Now $P = W \sin\theta$ and $R = W \cos\theta$.

Therefore $$\mu = \frac{P}{R} = \frac{W \sin\theta}{W \cos\theta} = \tan\theta.$$

EXPT. 42. **Determination of the Coefficients of Friction by means of an Inclined Plane.**—Place a block of material on an inclined plane and gradually increase the slope of the plane. At a certain inclination the block will slip; note the angle of inclination when slipping takes place.

Place weights on the block and raise the plane again; note that the block slips at approximately the same inclination as before. Let this angle be θ_1.

Repeat the experiment, and find the inclination at which the block will just continue to move down the plane if given a slight start—do this with the block loaded and unloaded, and show that the inclination is the same in either case. Let this be θ_2. The inclination required in this case is not so great as that required for the block to start of its own accord.

The coefficient of static friction is $\tan \theta_1$: the coefficient of dynamic friction is $\tan \theta_2$.

By facing the plane with sheets of different materials, and using different blocks, the coefficients of friction between several different sets of surfaces can be found in the same way.

THE FRICTION OF A ROPE OVER A FIXED PULLEY

When a belt or rope is stretched over a fixed cylinder equilibrium can occur with unequal tensions on the two sides on account of the friction between the surfaces in contact.

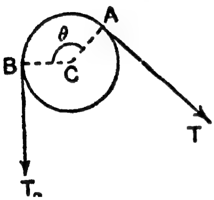


FIG. 51.—Rope over Pulley.

Suppose the string in Fig. 51 is just on the point of slipping from B towards A—the tension T being greater than the tension T_0—then it can be shown theoretically that

$$T = T_0 e^{\mu\theta},$$

where μ is the coefficient of friction, θ is the angle ACB (Fig. 51), and $e = 2{\cdot}71828 \ldots$. e is the base of the Napierian or Hyperbolic logarithms and is represented by the series

$$e = 1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \ldots$$

Taking logarithms to base e of both sides of the equation $T = T_0 e^{\mu\theta}$, we obtain

$$\log_e T = \log_e T_0 + \log_e e^{\mu\theta},$$

$$\log_e T - \log_e T_0 = \mu\theta.$$

For purposes of calculation convert to logarithms to base 10

$$(\log_{10} T - \log_{10} T_0) \log_e 10 = \mu\theta.$$

The value of $\log_e 10$ is $2{\cdot}30258 \ldots$, or with sufficient accuracy for the present purpose $2{\cdot}3$.

Hence the coefficient of friction is given by

$$\mu = \frac{\log_{10} T - \log_{10} T_0}{\theta} \times 2{\cdot}3,$$

where θ is measured in radians. Note that π radians = 180°.

Expt. 43. Determination of the Coefficient of Friction between a Rope and a Pulley.—The apparatus used to illustrate these results consists of a metal cylinder on the surface of which is stretched a string or belt. Tensions are applied to the ends of the string by attaching weights to them. It is convenient to use a known weight of say 100 gms at one end and to adjust the weight at the other end by putting weights into a scale-pan tied to the string. The weight of this scale-pan must be taken into account, and in some cases it may be necessary to take into account the weight of the suspending string.

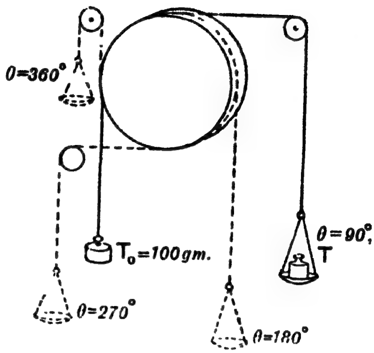


Fig. 52.—Friction between Rope and Pulley.

The 'angle of contact' for the string and the cylinder can be varied by passing the string over a small fixed pulley whose position can be adjusted as desired (Fig. 52). The friction of this pulley can be neglected.

The circumference of the cylinder may be marked off into a number of equal parts, corresponding say to 90°, 180°, 270°, 360°, 450°, 540°, etc. The string may then be wound round the cylinder so that the arc in contact corresponds with one of these angles.

For any particular angle determine what weight must be placed in the scale-pan just to start the string moving over the cylinder.

Tabulate the results as follows:

Angle.	Tension T.	$\log_{10}$ T.	$2{\cdot}3\dfrac{\log_{10}T - \log_{10}T_0}{\theta}$.
90° 180° 270° etc.			

The figures in the last column should be constant.

Plot two curves to show these results.

(1) Plot the value of $T - T_0$ as ordinate against the value of the

angle as abscissa. This will illustrate the fact that the tension increases with the angle in the same manner as a sum of money put out at compound interest.

The importance of this curve in practical and in theoretical physics is considerable, *e.g.* in damped oscillations, or in Newton's law of cooling (p. 356), a curve of exactly the same form is used to express some of the results obtained.

(2) Plot the value of $\log_{10} T - \log_{10} T_0$ as ordinate against the value of the angle as abscissa. The points should fall on a straight line.

NOTES ON THE UNITS OF FORCE, WORK AND POWER

Force

Force may be measured either in gravitational or in absolute (dynamical) units.

The **gravitational unit of force** is the attractive force of the earth on a mass of one gram, *i.e.* 1 **gm.-weight**.

The **dynamical unit of force** is the force which, acting on a mass of one gram, produces an acceleration of one cm. per sec. per sec. (Chap. VIII.). It is called 1 **dyne**.

If g denote the acceleration in cm. per sec. per sec. produced by the earth's attraction, 1 gm.-weight $= g$ dynes.

Work

Work is measured by the product of the force acting, and the distance moved through by the point of application in the direction of the force.

The **gravitational unit of work** is the work done in lifting a mass of one gram through a vertical distance of one centimetre. It is called 1 **gm.-cm.**

The **dynamical unit of work** is the work done when a force of one dyne moves its point of application through one centimetre. 1 dyne-cm. is called 1 **erg**.

A larger unit of work, the **joule**, is often used. 1 joule = 10 million ergs $= 10^7$ ergs.

Power

Power is the rate of doing work.

The **gravitational unit of power** is 1 **gm.-cm. per sec.**

The **dynamical unit of power** is 1 **erg per sec.**

A more convenient unit is 1 **joule per sec.** or 1 **watt.**

CHAPTER VI

MACHINES

§ 1. Efficiency, Force Ratio and Velocity Ratio

Any apparatus by which work may be done as a result of mechanical energy supplied is called a machine. When work is done as a result of a supply of energy other than mechanical the term 'engine' is generally used; we shall limit our study here to the consideration of machines.

EFFICIENCY

In any form of machine, a certain portion only of the energy supplied is actually employed in useful work; the remainder is lost in friction inside the machine. The more 'efficient' the machine, the greater is the proportion of energy employed usefully; we therefore say that **the efficiency of a machine is the ratio of the work usefully performed to the total energy supplied,** *i.e.*

$$\text{Efficiency} = \frac{\text{Work usefully performed}}{\text{Energy supplied}}.$$

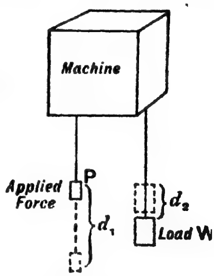


Fig. 53.—Principle of a Machine.

A **perfect machine** would utilise all the energy supplied; thus **the efficiency of a perfect machine is denoted by unity.**

In any type of machine the energy is supplied by some force P exerted *on* the machine and acting through some distance d_1

work being done, by the machine exerting some force W, through a distance d_2 (Fig. 53).

When the **Applied Force** P moves through a distance d_1, the energy supplied to the machine is Pd_1. The quantity Wd_2 gives the amount of work usefully performed in the same time.

Thus the Efficiency of the machine is given by

$$\epsilon = \frac{Wd_2}{Pd_1}.$$

MECHANICAL ADVANTAGE OR FORCE RATIO

A machine is generally designed so that by applying a smaller force P, a load W of considerably greater magnitude can be overcome.

The ratio $\dfrac{\text{Load overcome by Machine}}{\text{Force applied to the Machine}}$ is known as the **Mechanical Advantage**, since it usually represents a *gain of force*.

This is not, however, invariably the case: a very large force P, acting only through a short distance, may be used to raise a small load W through a considerable distance. In this case the ratio W/P is less than unity.

To avoid what is perhaps a misuse of the term Mechanical Advantage in cases like this, the more general name **Force Ratio,** suitable for *all* cases, is sometimes used to indicate the ratio W/P.

$$\text{Force Ratio or Mechanical Advantage} = \frac{\text{Load overcome}}{\text{Force applied to Machine}}.$$

VELOCITY RATIO

The distances moved through by the Applied Force and the Load are in general not equal. *If the machine were perfect,* we should have $Wd_2 = Pd_1$, *i.e.*

$$\frac{d_1}{d_2} = \frac{W}{P} \text{ in a perfect machine.}$$

This ideal is, however, never realised, and *always*

$$Wd_2 < Pd_1,$$

i.e.

$$\frac{W}{P} < \frac{d_1}{d_2}.$$

Now, in general, the distances d_1 and d_2 can be obtained by inspection of the mechanism, or by measurement of its parts; even if these be enclosed, the distance d_2 corresponding with any given distance d_1 can be measured. It is thus possible always to determine the ratio d_1/d_2 by simple inspection or actual measurement.

The ratio d_1/d_2 is the ratio of the distances moved through by the Applied Force and the Load in the same time; it is, therefore, the same as the ratio of the *velocities* of the Applied Force and the Load. *Rate of working* is an idea which is more important to an engineer than actual work performed, and therefore the study of *rate of motion* is more in accordance with engineering ideas than is that of the actual distance moved. The term **Velocity Ratio** therefore is applied generally to the ratio of these *distances*, since the velocities are in the same proportion as the distances, *i.e.*

$$\text{Velocity Ratio} = \frac{\text{Distance Applied Force moves}}{\text{Distance Load is overcome}}.$$

In choosing a machine for any purpose, the Mechanical Advantage, or Force Ratio, required should be estimated, and the machine chosen should have a Velocity Ratio greater than this by an amount sufficient to allow for the frictional losses in the machine itself (see note, p. 100).

RELATION BETWEEN VELOCITY RATIO, MECHANICAL ADVANTAGE (OR FORCE RATIO), AND EFFICIENCY

We have seen that the Efficiency can be expressed in the following way—

$$\epsilon = \frac{Wd_2}{Pd_1}.$$

This is conveniently re-written as—

$$\epsilon = \frac{W/P}{d_1/d_2},$$

i.e. **Efficiency** $= \dfrac{\textbf{Useful Work}}{\textbf{Energy supplied}} = \dfrac{\textbf{Mechanical Advantage}}{\textbf{Velocity Ratio}}$.

Thus if the Mechanical Advantage of a machine is determined experimentally, and its Velocity Ratio by inspection or measurement, the Efficiency is obtainable as the quotient of these two.

NOTE.—After some experience, the probable efficiency of various types of machines can be estimated with fair accuracy. If the Velocity Ratio is determined as already described, the probable Mechanical Advantage of a machine can be calculated roughly by the relation—

Mechanical Advantage = Velocity Ratio × Efficiency,

and hence its suitability for a given purpose can be judged.

§ 2. DETERMINATION OF THE EFFICIENCY, ETC., OF VARIOUS TYPES OF MACHINES

A few types of mechanism in common use will now be considered, together with the way in which their Velocity Ratios can be obtained by inspection. The method of obtaining the Mechanical Advantage is practically the same for all types of machines.

PULLEY BLOCKS

The set of pulley blocks considered is one consisting of two blocks of three pulleys each (Fig. 54). The upper block is fixed to a beam, the lower block being suspended from the upper one by a continuous cord which passes over each pulley. The one end A of the cord is attached to the framework of the upper block, while the other end hangs downward and is pulled by the applied force P. The load W is hung from the framework of the lower block.

EXPT. 44. **Efficiency of a Pair of Pulley Blocks.**—To find the efficiency two determinations must be made.

i. **Determination of the Velocity Ratio by Inspection.**—If the cord is pulled down a distance d_1, the total length of cord wrapped over the pulleys from B to A is shortened by a distance d_1. This shortening is taken up equally by all the vertical portions of the cord between B and A, since all the lower pulleys move upwards together. Hence each of the

portions of the cord between the upper and lower pulley blocks will be shortened by an amount $= d_1/6$, since there are six vertical portions.

But the centre of the lower pulley block will be raised a distance equal to the shortening of each of the vertical cords, *i.e.* C will rise a distance $d_1/6$. This is the distance through which the load W is raised, *i.e.* the distance $d_2 = d_1/6$, or the Velocity Ratio $d_1/d_2 = 6$.

In a similar way the Velocity Ratio of any system of pulleys can be determined easily.

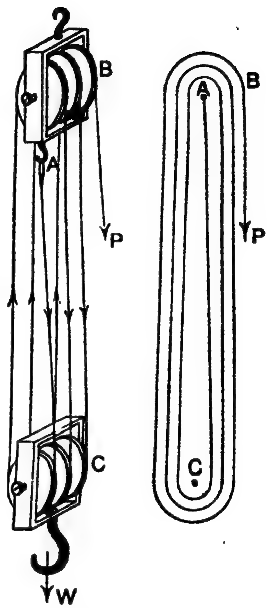


FIG. 54.—Pulley Blocks.

ii. **Experimental Determination of the Mechanical Advantage.**—In most types of apparatus designed for laboratory use, the weight of the lower pulley block is a considerable proportion of the actual loads employed. The weight of the blocks used in engineering works is, on the other hand, quite inappreciable compared with the loads lifted.

If therefore the weight of the lower pulley block is not included in the load, or the part of the applied force required to raise it is not deducted from the total force P, an erroneous idea of the practical efficiency of such a system would be given, the value obtained being too low.

We therefore deduct from P the force P_0 required to lift the pulley block alone, or include the weight of the pulley block in W when calculating out the Mechanical Advantage, although, strictly speaking, the work done in raising the block is *not* employed usefully.

If the weight of the pulley block is known, the obvious way is to include it in the load W, and to find the ratio W/P, considering the pulley block as part of the load.

If the weight of the pulley block is not known, find the force P_0 required to operate the machine when there is no load hanging from the pulley block. This force P_0 is evidently required to raise the pulley block alone. Now when a known

load W is suspended from the pulley block, a total force P_1 is required to raise the load W and the pulley block; hence the force P required to raise the *known* part of the load is given by

$$P = P_1 - P_0.$$

P_0 and P_1 should be adjusted until the machine just continues to work when given a slight start.

Determine the Mechanical Advantage of the pair of pulley blocks, using five or six different loads. Arrange the observations in one of the following ways:—

(*a*) Weight of Pulley Block known = 70 gm. (say).

Load suspended from Block, W_1 (in gm.).	Applied Force, P.	Total Load including Block, W.	$\frac{W}{P}$.
200	110	270	2·45
400	190	470	2·47
600	270	670	2·48
800	370	870	2·35
1000	450	1070	2·38

Mean of last column = Average Mechanical Advantage = 2·44.

(*b*) Weight of Pulley Block not known.

Force required to raise block alone P_0 = 30 gm. (say).

Load suspended from Block, W.	Total Applied Force, P_1.	Force required for Load W, $P_1 - P_0 = P$.	$\frac{W}{P}$.
200	110	80	2·50
400	190	160	2·50
600	270	240	2·50
800	370	340	2·35
1000	450	420	2·38

Mean of last column = Average Mechanical Advantage = 2·45.

NOTE.—If a scale-pan is used either to hold the load W or the applied force P, its weight must be included in the corresponding force.

Having determined the Mechanical Advantage and the Velocity Ratio of the set of pulley blocks, express the Efficiency as

$$\epsilon = \frac{\text{Useful Work}}{\text{Energy supplied}} = \frac{\text{Mechanical Advantage}}{\text{Velocity Ratio}}.$$

The Velocity Ratio is 6, the Mechanical Advantage is 2·45,

$$\therefore \text{ Efficiency } \epsilon = \frac{2{\cdot}45}{6} = 0{\cdot}41 = 41\%.$$

DIFFERENTIAL WHEEL AND AXLE

This apparatus is frequently met with in laboratories, and the 'differential' principle it embodies is applied not infrequently in practical forms of gearing; it forms therefore a suitable type of machine to consider in detail. The applied force P is exerted on a cord which wraps round a wheel of large diameter (Fig. 55). The wheel is fixed to an axle whose diameter is different in two parts, and round these two parts of the axle the two ends of another cord are wound in opposite directions. In the hanging loop of this cord is carried a pulley from the framework of which the load W is supported as shown in the accompanying sketch. The whole apparatus is mounted on a metal spindle supported on a suitable pair of brackets.

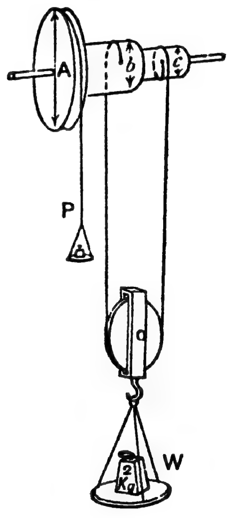


FIG. 55.—Compound Wheel and Axle.

EXPT. 45. **Compound Wheel and Axle or Differential Wheel and Axle.**—The experiment is in two parts :—

i. Calculation of the Velocity Ratio.—When the cord round the large wheel is pulled downwards so as to unwind it from the wheel, the apparatus rotates so that the other cord is wound *up* on the axle of large diameter but is wound *off* the narrower part of the axle.

Consider one complete revolution of the apparatus. Let the diameter of the wheel be A, and the diameter of the large and small portions of the axle be b and c respectively.

The applied force acts through a distance equal to the circumference of the wheel when the apparatus makes one complete revolution, *i.e.*

$$d_1 = \pi A.$$

In the same time, there is a change in the length of that part of the other cord which hangs free of the axle. An

amount equal to πb is wound up on the large part of the axle, but an amount πc is wound off the small part, so that the free length is actually shortened an amount—

$$\pi b - \pi c \text{ or } \pi(b - c).$$

This shortening is taken up equally on the two sides of the hanging loop, so that the pulley rises a distance only one-half the shortening of the loop, *i.e.* the load rises a distance $= \frac{1}{2}\pi(b - c)$, or

$$d_2 = \frac{\pi(b - c)}{2}.$$

The Velocity Ratio is thus

$$\frac{\pi A}{\pi(b - c)/2} = \frac{2A}{b - c}.$$

Note the factor 2 in the numerator of this expression.

Measure the diameters of the wheel and the two parts of the axle with a pair of large callipers, or measure the actual circumferences with string and scale or with a flexible tape measure, and calculate the Velocity Ratio.

ii. Determination of the Mechanical Advantage.—Determine the Mechanical Advantage as already described in the case of the pulley blocks, allowing for the weight of the pulley and scale-pans as detailed there (Expt. 44).

Calculate the value of the Efficiency of the apparatus; this will probably be as high as 85 or 90 per cent.

THE SCREW

The employment of a screw mechanism for various types of machines is extremely common, especially where a very large mechanical advantage is required. In practice a screw is often incorporated as part of a more complicated mechanism, though it is sometimes met with alone. Common examples are the screw lifting-jacks used for raising the axle of a motor-car when replacing a tyre, or for raising any heavy weight where hand-labour alone is available.

The experimental form of screw is usually fitted with a large diameter pulley round which a cord is wound, the cord being pulled by weights placed in small scale-pans hanging over small pulleys at the side of the apparatus as shown in Fig. 56. The large diameter pulley and the attendant cords are usually replaced by a T-shaped handle in the practical forms just referred to.

On the screw is carried a large nut fitted with a yoke which carries the load W. The screw is supported at the lower end by a bearing on which it freely turns, and near the top by a collar through which it just passes freely. A usual type of apparatus is illustrated in Fig. 56, though the framing which supports the smaller pulleys over which the cords pass is not shown. Other forms are frequently met with in laboratories. Sometimes the large pulley is only pulled by one weight, in other cases two cords are fitted and two weights are used as shown in the diagram.

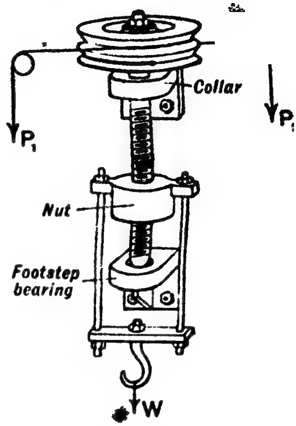


FIG. 56.—The Screw.

The second type is to be preferred, as, if P_1 and P_2 are equal, the screw is not pulled towards either side; if an unbalanced, single force is used, the screw is pulled to one side of the collar, and additional friction and wear are caused thereby.

The nut would tend to turn with the screw as it revolved, unless suitable means were taken to prevent this; a guide or a pair of guides must therefore be fitted. The usual type of guide is a rod fixed to the main framework, passing down from the collar to the footstep bearing, and fitting into slots cut in the nut. As the screw rotates, the nut cannot turn with it, being prevented by the guides, and hence it must move along the screw thread. The load is thus raised or lowered according to the direction in which the screw is turned. These guides are not shown in the diagram.

EXPT. 46. **Determination of the Efficiency of a Screw.**—If the screw is to work efficiently, it is essential to keep the two bearings and the guides well oiled, and *particularly to oil the thread of the screw itself*, as most of the friction is between the screw and the nut.

i. **Calculation of the Velocity Ratio.**—Let the diameter of the pulley at the top of the screw be D. Then in one revolution the applied force (or forces) moves downwards a distance equal to the circumference of the pulley, *i.e.* considering one revolution of the screw—

$$d_1 = \pi D.$$

In the same time, the screw advances one turn in the nut,

i.e. the nut is raised a distance equal to the 'pitch' of the screw. If the pitch of the screw is p, evidently

$$d_2 = p,$$

$$\text{the Velocity Ratio} \quad \frac{\pi D}{p}$$

Measure the diameter of the pulley with large callipers, taking care to measure the diameter across the bottom of the grooves where the cords lie, or measure the circumference with string and a scale. Measure the pitch of the screw by pressing a piece of clean paper into contact with the thread for some distance along the screw, and measure the length of, say, 20 threads. Do not forget that the pitch of the screw is the distance between two similar points on successive turns of the same thread (Fig. 7).

Calculate the Velocity Ratio.

ii. Determination of the Mechanical Advantage.—Determine the Mechanical Advantage as described in Expt. 44. In this case, the weight of the nut and yoke cannot be found, as the nut is fixed to the screw; it must therefore be allowed for by finding P_0 the force required to lift the nut and yoke alone, and deducting this from the total force required when a load is suspended from the yoke.

If two cords are fitted to the large diameter pulley, the applied force is equal to the sum of the two weights suspended from the cords.

Find the Efficiency of the screw; it will be found rarely to exceed 20 per cent even if kept well oiled, and may be as low as 7 or 8 per cent if the screw has got rusty through lack of care.

WHEEL GEARING

No account of machines would be complete without some reference to the most common of all types of gearing—gearing by means of toothed wheels. This type of gearing is of great adaptability and appears in various forms in all kinds of machinery—in watches, motor-cars, lathes, travelling-cranes, etc. We shall limit ourselves to considering a simple train of wheel gearing, and, to avoid complication, a numerical case will be taken.

Spindle A carries a large diameter drum of radius 15 cm. On

the same spindle, rigidly fixed to the drum, is a toothed wheel with 20 teeth.

This engages with the teeth of a wheel mounted on B having 80 teeth in its circumference, so that the large wheel on B rotates once for every four revolutions of A. Fixed to this large wheel on B, is a small wheel with 20 teeth which engage with the teeth on a large wheel on spindle C. This has 100 teeth in its circumference,

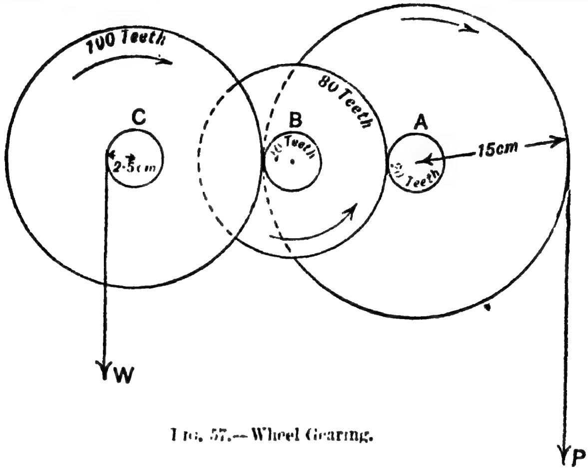


FIG. 57.—Wheel Gearing.

so that the spindle C revolves once for every five revolutions of B, or every twenty revolutions of A. On the third spindle C, is a small drum of radius 2·5 cm., round which passes the cord supporting the load W, the cord being wound so that as P falls, W is raised.

It is simplest in this case to consider the effect of one revolution of spindle C.

In one revolution of C, the load W rises a distance equal to the circumference of the small drum, *i.e.* $d_2 = 2\pi \times 2{\cdot}5$ cm.

For every revolution of C, A revolves twenty times, and hence P acts through a distance equal to twenty times the circumference of the large drum, *i.e.* $d_1 = 20 \times 2\pi \times 15$ cm.

The Velocity Ratio is therefore $\dfrac{20 \times 2\pi \times 15}{2\pi \times 2{\cdot}5} = 120.$

EXPT. 47. **Determination of the Efficiency of a System of Wheel Gearing.**—Calculate the Velocity Ratio of a system of wheel gearing, as described in the foregoing example. Determine experimentally its Mechanical Advantage as in Expt. 44 and deduce its Efficiency.

The Efficiency of a wheel gear train depends very largely on the accuracy with which the teeth are cut: a well-cut set of gear wheels may have an efficiency of 95 per cent in a simple train such as that described.

NOTE.—The terms **Weight** and **Power** are used frequently for the quantities W and P dealt with in this chapter under the names **Load** and **Applied Force**. The use of a general term Weight for a particular and special quantity is objectionable; the term 'Load' has no special scientific meaning, and its use is preferable to the use of 'Weight' in this connection. The term 'Power' has a definite and particular scientific significance, namely the 'rate of doing work.' It must not therefore be used in the sense of *force*. The term 'Effort' is used sometimes to indicate the quantity P, but it is not by any means universally employed.

CHAPTER VII

ELASTICITY

§ 1. General Theory

When a force of any kind acts on a body, the body is deformed to a greater or less extent. This deformation will, in general, disappear if the force ceases to act. The restoration takes place as a result of that property of the body itself, which is called its **elasticity.**

HOOKE'S LAW

The foundations of the subject were laid by Robert Boyle and his assistant Hooke, and the most important law connecting the force acting and the deformation produced is known as **Hooke's Law.** This may be stated as **Tension is proportional to extension,** or in more exact language, **Stress is proportional to strain.** Hooke's Law is only true up to a certain point; if the stress acting on the body exceeds a certain value, the body will not return to its original dimensions when the stress is removed. The largest deformation which does not leave permanent distortion is called the **elastic limit** of the substance. Up to the elastic limit, Hooke's Law holds good to a close degree of approximation.

DEFINITION OF MODULUS OF ELASTICITY—STRESS AND STRAIN

In order to compare the elastic properties of different materials, it is necessary to obtain quantitative knowledge of the deformations produced by various types of forces.

Stress.—So far as the amount of deformation produced is concerned, the effect of a force is found to depend on the magnitude of the force and also on the area over which it is spread, the effect being proportional to the *force per unit area.*

Stress is defined as the force exerted per unit area.

Strain.—A given stress produces a deformation depending on the size of the body on which it acts. Tensile stresses of equal magnitude acting on similar wires of different lengths will produce elongations in the wires in the same proportion as their lengths. The effect of a stress, then, is to produce a certain *deformation per unit dimension* of the deformed body.

Strain is defined generally as **the distortion per unit dimension**, or, **Strain is the fractional distortion.**

Any Modulus of Elasticity is defined as the quotient of the Stress acting and the Strain it produces.

$$\text{Modulus of Elasticity} = \frac{\text{Stress}}{\text{Strain}}.$$

Definitions of the Various Moduli.—1. **Young's Modulus** or **Coefficient of Tensile Elasticity.**—The stress considered in this case is a **linear tensile stress**, the strain being the corresponding **elongation per unit length.**

If a tensile force F acts on a fibre of length L and of cross section A, the tensile stress exerted on the wire is F/A. If the whole length is elongated an amount x, the strain is x/L. Young's Modulus is therefore expressed as

$$E = \frac{F/A}{x/L} \quad \text{or} \quad E = \frac{FL}{Ax}.$$

2. **Modulus of Rigidity, or Modulus of Shear Elasticity.**—Imagine a block of material (say indiarubber) in the form of a rectangular parallelepiped, fixed down at one side to a horizontal bed and with a plate firmly fixed to the upper horizontal face.

If this plate be pulled horizontally with a force F, the whole block will be deformed so as to have the shape shown by dotted lines in Fig. 58.

The force F is distributed through the plate so as to act

uniformly over the whole of the top area A; the stress is therefore F/A. This is called a **Shear Stress.** The top layer of particles is displaced horizontally a distance d from its original

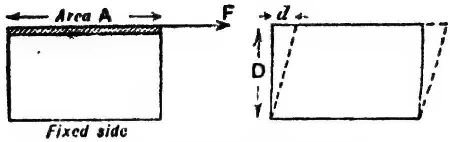


FIG. 58.—Shear Stress and Shear Strain.

position relative to the particles in the lower surface. The **lateral displacement between two surfaces unit distance apart** is called the **Shear Strain.** Thus the Shear Strain $= d/D$.

The Modulus of Shear Elasticity is therefore

$$\frac{F/A}{d/D} = \frac{FD}{dA}.$$

NOTE.—Only a solid can possess tensile and shear elasticities.

3. **Bulk Modulus, or Coefficient of Volume Elasticity.**—If a pressure p is impressed upon a body having a volume V, and the resulting change of volume is v, the force applied per unit area is p, since **pressure is force per unit area**; hence the Stress $= p$.

The **Strain** is given by v/V, since the distortion produced is v, and the dimension distorted is V.

The Bulk Modulus is therefore $= \dfrac{p}{v/V} = \dfrac{pV}{v}$.

It is not always possible to take a body initially in an unstressed condition, and hence a slight modification of the original definition of a Modulus of Elasticity is required.

If Hooke's Law is true, the Modulus of Elasticity is a constant property of a given material under fixed conditions. It is therefore true to say that if the stress is increased,

$$\frac{\textbf{Increase of Stress}}{\textbf{Increase of Strain}} = \frac{\textbf{Stress}}{\textbf{Strain}} = \textbf{Modulus of Elasticity.}$$

The first of these fractions is used frequently in measuring a Modulus of Elasticity.

In the case of a gas, where Hooke's Law does not hold good, we use the quotient **increase of pressure divided by the corresponding volume strain** to express the Bulk Modulus of the gas in any given conditions.

It is unnecessary to measure the Bulk Modulus of a gas, as it can be calculated from theoretical considerations. To measure it for solids or liquids presents great difficulties. We shall limit ourselves therefore to considering the experimental methods of determining Young's Modulus and the Modulus of Rigidity.

NOTE.—**A Stress is always a force per unit area** and must be expressed in **dynes per sq. cm.** or other units of similar dimensions. **A Strain is a ratio and has no dimensions.**

A Modulus of Elasticity $= \dfrac{\text{Stress}}{\text{Strain}}$, **and is therefore expressed in the same units as those used for the Stress considered,** *i.e.* **in dynes per sq. cm.** if C.G.S. units are used.

§ 2. YOUNG'S MODULUS

YOUNG'S MODULUS FOR A MATERIAL IN THE FORM OF A WIRE

The apparatus required for the experiment consists of two vertical wires with their upper ends fixed close together on the same support. One is stretched by a constant load which need not be known, while the other carries a scale-pan A, in which any desired load may be placed. The first wire carries a short scale C, while on the second wire is mounted a vernier B which slides freely over the scale C.[1] The two wires should be of the same material and size.

By this method of using two exactly similar wires, serious sources of error are avoided:—

Any yielding of the point of support when the load A is increased, will depress the scale C to the same extent as the vernier B is depressed, and hence will not be recorded. Similarly any change

[1] We believe that this arrangement of the scale and vernier on two parallel wires originated in the Wheatstone Laboratory of King's College, London.

in the length of the wire due to temperature variations is not registered, as both wires are affected equally.

If we maintain a steady force on one wire, and apply different forces to the other, any elongation due to these additional forces will be registered by a motion of the vernier B along the scale C.

The length of the wire and its radius may be measured by ordinary methods; hence the elongation strain produced by a known stress can be found, and the value of Young's Modulus for the material determined.

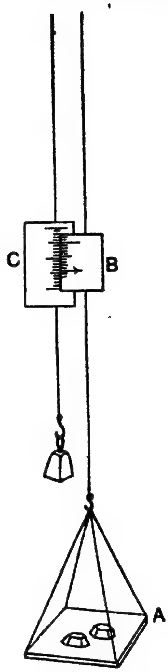


Fig. 59.—Young's Modulus for a Wire.

Expt. 48. **Determination of Young's Modulus for a Wire.**—Place a load of 2 kgm. on the scale-pan attached to the wire under test, to take out any slight bends in the wire. Note the reading of the scale C and the vernier B.

Increase the load by steps of 2 kgm. to a maximum of 12, noting the reading for each load. Diminish the load by steps of 2 kgm. till the original load of 2 kgm. is reached, taking the reading for each load again before it is diminished. Take the mean of the observations for each load as the actual reading corresponding to that load.

If the reading at the end of the experiment differs appreciably from the initial reading (with 2 kgm. suspended), it is possible that the wire has been stretched beyond the elastic limit, though the change may be due merely to straightening. Repeat the observations in this case: if a further elongation remains permanently after removing the load, the experiment must be again repeated with a new wire, applying a maximum load of not more than 8 kgm.

Measure the diameter with great care at several points on the wire, using a micrometer screw gauge. The accuracy of this measurement is of great importance. An error of 0·01 mm. in the diameter is of the order of 1 per cent, and will affect the final result with an error of 2 per cent, since the radius occurs to the second power. An error of 1 or 2 cm. in measuring the length of the wire is of less importance than 0·01 mm. in measuring the radius.

Measure the length of the wire from the point of support to the zero of the vernier.

Tabulate the observations as follows :—

Load.	Readings.		Mean Reading.	Extension for an increase of 6 kgm. in mm.
	Load increasing.	Load diminishing.		
2 kgm.	1·13 mm.	1·15 mm.	1·14 mm.	
4 ,,	1·33 ,,	1·35 ,,	1·34 ,,	(2 to 8) = 0·53
6 ,,	1·50 ,,	1·55 ,,	1·53 ,,	(4 to 10) = 0·51
8 ,,	1·67 ,,	1·67 ,,	1·67 ,,	(6 to 12) = 0·57
10 ,,	1·83 ,,	1·87 ,,	1·85 ,,	
12 ,,	2·10 ,,		2·10 ,,	

Mean extension for 6 kgm. = 0·537 mm.
= 0·0537 cm.

Radius of wire (mean of four determinations) = 0·675 mm.
= 0·0675 cm.

Length of wire from support to vernier = 250 cm.

The Stress produced by a load of **6 kgm.** suspended from this wire

$$= \frac{\text{Force}}{\text{Area of cross section of wire}}.$$

The force is the weight of 6 kgm., *i.e.* = 6000 × 981 dynes.
The area of cross section is $\pi \times (0{\cdot}0675)^2$ sq. cm.,

i.e. Stress due to 6 kgm. $= \frac{6000 \times 981}{\pi \times (0{\cdot}0675)^2} = 404{,}000{,}000$ dynes per sq. cm.,

i.e. Stress $= 4{\cdot}04 \times 10^8$ dynes per sq. cm.

The Strain produced by the addition of a load of **6 kgm.**

$$= \frac{\text{Mean elongation for 6 kgm.}}{\text{Length of wire from support to vernier}}$$

$$= \frac{0{\cdot}0537}{250}$$

$$= 0{\cdot}000215 \text{ or } 2{\cdot}15 \times 10^{-4}.$$

Young's Modulus for the wire is therefore

$$\frac{\text{Stress}}{\text{Corresponding strain}} = \frac{4{\cdot}04 \times 10^8}{2{\cdot}15 \times 10^{-4}} \text{ dynes per sq. cm.}$$

$$= 1{\cdot}88 \times 10^{12} \text{ dynes per sq. cm.}$$

Plot a curve showing how the extension of the wire varies with the weight suspended from it. Show that the plotted points follow approximately a straight line.

Attention must be directed to two points in the foregoing. First, note the way in which the extension is worked out in the last column of the table.

It is said frequently that the average elongation for 2 kgm. should be taken, and this value used for determining the Modulus of Elasticity. This is done by taking successive differences between the numbers in the last column but one, adding together these successive differences and taking their mean. By this method, however, any additional accuracy which might have accrued through taking six observations is lost completely, the result depending entirely on the first and last observations. Each of the intermediate observations is taken into account twice, once positively and once negatively, and is thus without effect on the result. If the six readings are denoted by A, B, C, D, E, and F, successive differences are A - B, B - C, etc., and their mean is

$$\frac{(A-B)+(B-C)+\ldots+(E-F)}{5}=\frac{A-F}{5}.$$

That the difference in the result may be considerable is shown in the sample set of observations given, the extension for 6 kgm. by this method being 0·576, as against 0·533 by the method shown in the table.

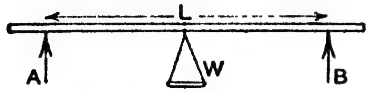
In the method given in the table each observation is taken into account once only, the result therefore depends on *all* the readings taken and is correspondingly more accurate.

The second point to which attention must be directed is concerned with the measurement of the length of the wire. The whole wire is stretched by the load on the scale-pan A, but the extension *measured* is only that suffered by the part of the wire between the point of support and the vernier. In calculating the *strain* this length is therefore the length to be used as the denominator.

YOUNG'S MODULUS FOR A MATERIAL IN THE FORM OF A BEAM

The apparatus required for this experiment consists of two knife-edges across which the 'beam' can be rested, a scale-pan or hook supported from the middle of the beam, and some convenient method of determining the extent to which the centre of the beam is depressed. If the rod is very thin so that the depression for (say) 1 kgm. is considerable, a metre scale placed vertically behind the rod can be used for measuring the depression of the centre, the division opposite to the top or bottom face of the rod being observed. If the rod were fairly stiff, very large forces would

be required to produce a bending which could be measured with sufficient accuracy in this way. It is preferable to use moderate forces and employ some more delicate method of measuring the depression. For this purpose a vertical scale may be attached to the centre of the rod and observed by a fixed low-power microscope. As the centre of the rod is depressed, the scale moves downwards relatively to the microscope. The reading of the scale seen coincident with the cross wire of the microscope eye-piece is taken for various loads, and thus the depression of the centre of the rod is measured. Or the scale may move over a fixed vernier and its depression may thus be measured with a fair degree of accuracy.



Beam on two supports A & B

FIG. 60.—Young's Modulus for a Beam.

EXPT. 19. **Determination of Young's Modulus for a Beam.**—Apply various loads on the scale-pan or hook and note the corresponding positions of the centre of the beam. Increase the load by equal amounts, taking six or eight observations both with increasing and with decreasing loads: the maximum load applied should be approaching the highest load which the beam can carry in safety, but should not exceed this amount.

Tabulate your observations as in finding Young's Modulus for a wire, and find the mean depression y for a load W by the method described therein for determining the elongation for 6 kgm.

Measure the length of the beam between the knife-edges; also the breadth and thickness of the beam.

Let these be L, b, and d respectively. Then it can be shown that *for a bar of rectangular section*, the relation between the depression of the centre y, the load W, and the dimensions of the beam is given by

$$y=\frac{WL^3}{4Ebd^3},$$

where E is Young's Modulus for the material of the beam.

Calculate Young's Modulus from the expression

$$E=\frac{WL^3}{4bd^3y}.$$

As an alternative method of working out the observations, the depressions y_1, y_2, y_3, etc., of the centre of the beam corresponding with loads W_1, W_2, W_3, etc., may be obtained, and the mean of the quotients $\frac{W_1}{y_1}$, $\frac{W_2}{y_2}$, $\frac{W_3}{y_3}$, etc., taken as the mean value for $\frac{W}{y}$.

This is substituted for $\frac{W}{y}$ in the expression

$$E = \frac{L^3}{4bd^3}\left(\frac{W}{y}\right),$$

and the value of E is calculated.

W must be expressed in **dynes**, and the other quantities on the right-hand side of the equation in **centimetres**.

As additional exercises on this type of experiment the following are suggested :—

EXPT. 50. **For a given load the depression of the centre of a beam varies directly as the cube of its length.**

This is shown by finding y for the same load when the supporting knife-edges are at different distances apart. If the depressions are y_1, y_2, y_3, etc., for distances apart equal to L_1, L_2, L_3, etc., the values of $\frac{y_1}{L_1^3}$, $\frac{y_2}{L_2^3}$, etc., should all be equal. The load must be acting at the point midway between the knife-edges in each case.

EXPT. 51. **The stiffness of a rectangular beam varies directly as its breadth and directly as the cube of its thickness (depth).**

Find y for the same load exerted on the same beam with the knife-edges kept at a constant distance, but with the beam resting first on the 'flat' side and then on the 'edge.' In the first case the flat side is the breadth (b) and the edge is the depth (d), these quantities being interchanged in the second case.

Show that bd^3y has the same value in the two cases.

YOUNG'S MODULUS FOR A CANTILEVER

A **Cantilever** is a loaded beam fixed horizontally at one end.

If a cantilever has a load W suspended at the extreme end, the depression y of the end is given by the equation

$$y = \frac{4WL^3}{Ebd}$$

if the beam is of rectangular section.

EXPT. 52. **Determination of Young's Modulus for a Cantilever.**—Clamp one end of a metre scale to the top of a table so that the metre scale extends horizontally about 90 cm. beyond the table edge. Place various loads at the extreme end of the scale and measure the depression of

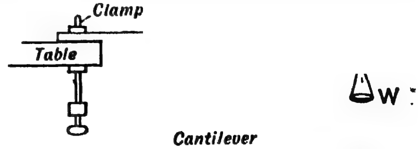


FIG. 61.—Young's Modulus for a Cantilever.

this end for each of the loads applied. One of the methods already described for a beam on two supports may be used for this measurement. Measure the length of the beam beyond the table, and also its breadth and thickness. Deduce the value of Young's Modulus, E, for the material (usually box-wood) by means of the equation above.

§ 3. MODULUS OF RIGIDITY

MODULUS OF RIGIDITY FOR A MATERIAL IN THE FORM OF A CYLINDRICAL WIRE

The Modulus of Rigidity has been defined earlier in the chapter, taking the case of a block fixed at the base and subjected to a tangential shearing force F distributed over its upper face. It is impossible, however, to determine the Modulus of Rigidity of any material (except india-rubber) in this way, as

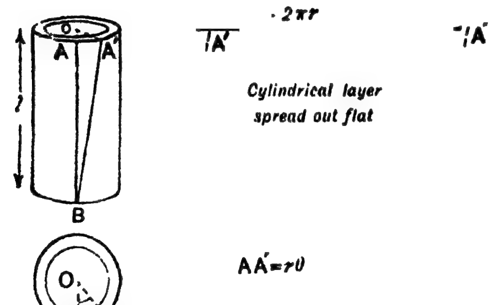


FIG. 62.—Twisting a Wire.

the deformation *d* produced by any practicable force F would be too small to measure.

Twisting a Wire.—When a couple is applied to one end of a wire, the other end being kept fixed, the wire is twisted through an angle which is proportional to the twisting couple applied.

The wire may be considered as consisting of a number of thin concentric cylindrical layers. Each of these is in a state of shear when the wire is twisted. Thus a layer of particles

originally along AB is displaced into the dotted line A′B, when the upper end is twisted through the angle θ (Fig. 62).

If the cylindrical layer could be spread out flat, it would form a rectangular sheet when the wire was not twisted, but its shape when the twist was produced would be as shown dotted (A′BBA′).

The relation between the angle of twist, the dimensions of the wire, and the twisting couple may be expressed by saying that to twist a wire l cm. long and of a cm. radius through an angle θ **radians** requires a couple C, given by

$$C = \frac{\pi n a^4 \theta}{2l}$$

where n is the **Modulus of Rigidity** of the wire.

In general, we measure the twist in **degrees**. Let the twist in the length l be ϕ°.

Then $$\theta \text{ radians} = \phi^\circ \times \frac{\pi}{180}.$$

Thus $$C = \frac{\pi n a^4}{2l}\left(\frac{\pi}{180}\phi^\circ\right),$$

or $$C = \frac{\pi^2 n a^4}{360 l}\phi^\circ.$$

APPARATUS FOR DETERMINING THE MODULUS OF RIGIDITY OF A WIRE BY TWISTING IT

The wire may be fixed either vertically (Figs. 63 and 64) or horizontally (Fig. 65). One end A is clamped rigidly to the supporting framework (not completely shown), and the other, passing through a suitable bearing to keep it steady, is clamped to the middle of a pulley B. A pointer C on the wire moves over a scale of degrees mounted near to the pulley end of the wire, and by its means the twist ϕ° produced *in that part of the wire between the pointer and the fixed end* by couples exerted on the pulley can be measured readily; sometimes the scale of degrees is mounted on the pulley, and a fixed pointer is used (Fig. 64).

The couple applied to twist the wire is exerted by weights suspended from cords which wrap round the pulley, as shown in the diagrams. It is preferable to use two equal forces, acting along parallel lines in opposite directions, so that there is no 'side pull'

on the wire, though in the case where the wire is fitted horizontally the usual method is to use a single force only, as shown. The effect

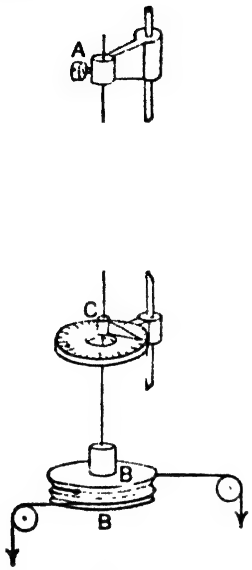


FIG. 63.—Modulus of Rigidity—Apparatus with Movable Pointer.

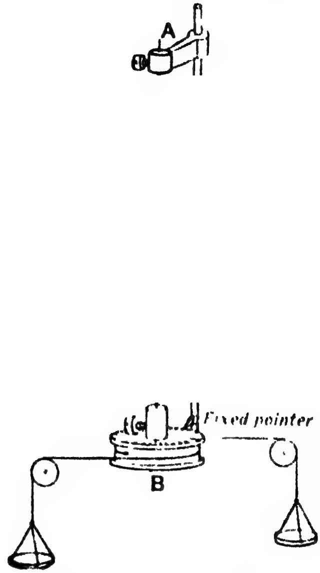


FIG. 64.—Modulus of Rigidity—Apparatus with Fixed Pointer.

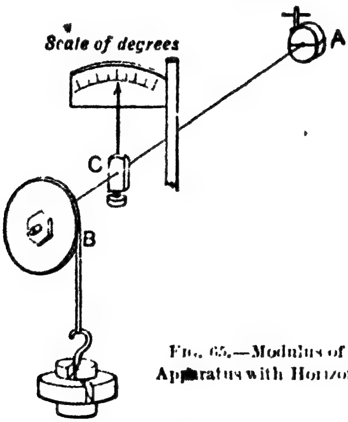


FIG. 65.—Modulus of Rigidity—Apparatus with Horizontal Wire.

of 'side pull' is to introduce friction between the bearing and the wire, which impedes somewhat the free turning of the wire under the twisting couple.

The twisting couple exerted on the wire by equal masses M hanging from cords passing round the pulley of diameter D is given by

$$C = MgD.$$

If a single cord is used,

$$C = \frac{MgD}{2}.$$

We have seen already that

$$C = \frac{\pi^2 a^4 n}{360 l}\phi^\circ,$$

therefore, considering the case **where the pulley is fitted with two cords carrying equal masses** M, we have

$$MgD = \frac{\pi^2 a^4 n}{360 l}\phi^\circ.$$

Whence
$$n = \frac{360 lgD}{\pi^2 a^4}\left(\frac{M}{\phi^\circ}\right).$$

If a single cord is fixed to the pulley and carries the load M,

$$C = \frac{MgD}{2} = \frac{\pi^2 a^4 n}{360 l}\phi^\circ,$$

and
$$n = \frac{180 lgD}{\pi^2 a^4}\left(\frac{M}{\phi^\circ}\right).$$

EXPT. 53. **Determination of the Modulus of Rigidity of a Wire.**—Note the **zero reading** of the pointer C, *i.e.* the reading of the pointer on the scale of degrees when the wire is subjected to no twisting couple. Attach various loads to the cords and note the corresponding twists produced. The load should be increased in equal steps up to the largest used. If the pulley is twisted by two cords, the loads attached to the cords should be equal. Take the readings of the angle of twist as the couple is being increased, and again for the same values of the couple when the loads are being removed.

If the observations taken with the couple decreasing do not correspond with those taken when the couple was being increased, the rod has either been twisted beyond the elastic limit, or else the clamps do not grip the rod sufficiently firmly and the rod has turned a little in the clamps.

These results should be discarded and a new series taken, using a smaller maximum load, the maximum being reduced to such a value that the two sets of readings are identical (within limits of experimental error).

Measure the diameter of the pulley B, the radius of the wire twisted, and the distance AC between the fixed end of the wire and the pointer. Tabulate the observations as shown below:—

Load on (each) cord M.	Twist in degrees ϕ. M increasing.	Twist in degrees ϕ. M diminishing	Mean twist ϕ°.	$\frac{M}{\phi^\circ}$

Mean $\frac{M}{\phi^\circ}$ . . . .

Length of wire from A to C $= l$ . . . cm.
Radius of wire (mean of 4) $= a =$ . . . cm.
Diameter of pulley B $=$ D $=$ . . . . cm.

If preferred, the mean value of ϕ° corresponding with an increase in the suspended masses equal to M can be found in the manner detailed in the determination of Young's Modulus for a wire.

Substitute the mean value of M ϕ° in the appropriate equation given above for n and calculate the value of n.

Plot a graph showing the way in which the angle of twist ϕ° varies, as M is increased.

CALIBRATION OF A SPRING AND METHOD OF USING A CALIBRATED SPRING AS A BALANCE

Hooke's law, that tension is proportional to extension, holds generally, even where the strain imposed on a body is not so simple as the strains in the cases just considered.

A typical case is offered by a spiral spring subjected to a tension along the axis of the spiral, the movement of the index along the scale (indicating the elongation of the spring) being accurately proportional to the applied force.

The object of the present experiment is to calibrate a Spring

Balance ; that is, for any point on the scale to determine the force required to elongate the spring till the index is at that point.

EXPT. 51. **Calibration of a Spring Balance.**—The apparatus employed usually consists of a frame of wood or metal to one end of which is attached the end of a spiral spring. At the other end of the spring is an index which moves freely over, and just in contact with, a scale screwed to the frame. A small scale-pan is hung at the end of a cord attached to the end of the spring.

Fix the framework so that the spring and scale are vertical, with the index just touching the scale, and note the zero reading, *i.e.* the reading when no load is applied to the spring.

Then take a series of readings of the position of the index for gradually increasing loads and tabulate the results.

Be careful not to exceed the elastic limit; the spring must never be stretched to such an extent as to bring the index beyond the scale, for the length of the scale is usually arranged to give almost the maximum motion allowable.

The results of the observations must now be plotted on squared paper, taking the load as abscissa and the scale reading as ordinate. The graph should be drawn on as large a scale as possible.

If the strain be exactly proportional to the load, the points should lie on a straight line. Draw a straight line passing between the observed points.

The graph may now be used to determine an unknown load. Find the extension which the load produces when attached to the spring, and read off from the graph the load corresponding to the observed scale reading.

§ 4. THE ENERGY OF A STRAINED BODY

If a body is deformed by a force, the force producing the deformation has acted through a certain distance. A certain amount of work has thus been done on the body by the force; **this work is stored up in the body as Strain Energy.** The C.G.S. unit of work is the **Erg,** which is defined as **the work done when a force of 1 dyne moves its point of application through 1 cm.**

When a wire is stretched l cm. by a force F dynes applied gently, it might appear at first sight that the force F acts through the whole distance l, and that therefore an amount of work equal

to Fl ergs should be stored in the wire due to its strained condition. Actually, however, the full force F does not act on the wire until the full elongation l has been produced; it is applied *gently* to the wire, *i.e.* at first the major part of the force is supported by the experimenter, and only a small fraction is allowed to act on the wire. As the wire stretches, the experimenter takes less and less of the force, allowing a continually increasing proportion to be supported by the wire until eventually the wire supports the whole force and is elongated the full amount l.

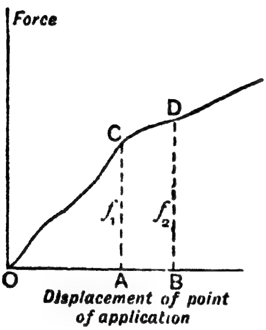


FIG. 66.—Work done in straining a Body.

While the force F has been acting, it has certainly done an amount of work equal to Fl ergs, but part of this has been taken by the experimenter in allowing the force to come into action gently, only a portion of the total work Fl having been done on the wire. Actually, *half* the total energy Fl is absorbed by each.

Consider the work done by a variable force f whose magnitude changes with the displacement of its point of application in the manner shown in the curve (Fig. 66). This curve is drawn irregular of set purpose so that the result obtained from its consideration may be taken as true generally.

When the point of application is at A, the force has a certain magnitude $f_1 = AC$; in moving its point of application to B, the force increases to a value $f_2 = BD$, its average value during the displacement being equal to some quantity $\bar{f}$ (called 'f bar')—

$$\bar{f} = \frac{f_1 + f_2}{2}.$$

The work done in this displacement is evidently equal to $\bar{f} \times AB$, and is represented by ABCD, the area under the curve between the ordinates considered.

The work done in any other displacement would similarly be equal to the corresponding area under the curve, and hence

the total work done by the force up to any displacement is equal to the area under the curve from the origin to the ordinate at the point considered. This rule is true for any force-displacement diagram, however the force may vary.

In dealing with strains, the force producing an elongation is proportional to the elongation produced, hence the force-displacement curve is a straight line. The area under the curve up to the ordinate corresponding with any given displacement is triangular; its area is $\frac{1}{2}Fl$ (Fig. 67).

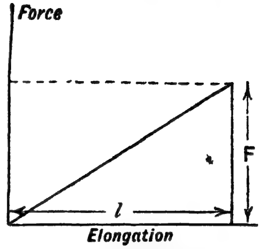


FIG. 67.—Strain Energy in a Wire.

Hence **the Strain Energy in a wire when a force F causes an elongation l cm. is equal to $\frac{1}{2}Fl$.**

If E represents the strain energy in the wire,

$$E = \tfrac{1}{2}Fl = \tfrac{1}{2} \text{ stretching force} \times \text{elongation}.$$

It is impossible to prove by simple experiment that $E = \frac{1}{2}Fl$ in the case of an ordinary straight wire, but with a spiral spring the truth of this statement can be demonstrated without difficulty.

The Energy stored in a spiral spring is equal to half the product of the force it exerts and its elongation

Steady Application of a Force.—Allow a mass M_1 to hang from a wire spring, letting its weight come into action gradually upon the wire: a steady elongation is produced equal to l_1 (say). The force now exerted by the spring is some force F_1, which is equal to M_1g, since the mass remains motionless at the end of the spring.

We wish to show that the energy now stored in the spring as Strain Energy is given by $E_1 = \frac{1}{2}F_1l_1$.

Sudden Application of a Force.—Suppose we allow a mass M_2 to rest lightly at the end of a spring which is entirely unstrained, the mass M_2 being supported by a small platform. If the platform is now removed quite suddenly, the whole of the weight of the mass M_2 comes into action on the spring. As the spring stretches, the Potential Energy lost by the falling mass is partly converted into Kinetic Energy of the mass and partly is stored in

the spring as Strain Energy. After moving some distance the mass begins to slacken in its fall, and eventually comes to rest *momentarily* after falling some distance l_2.

It now has no Kinetic Energy, and therefore **at the instant when the mass first comes to rest, the whole of the Potential Energy it has lost in falling is stored in the spring as Strain Energy.**

The Potential Energy lost by the mass in falling this distance is M_2gl_2, and we know, therefore, that when the spring is stretched a distance l_2, the amount of strain energy stored in the spring is M_2gl_2 ergs.

If we so adjust this mass M_2 that its sudden fall produces a maximum elongation equal to that produced by a mass M_1 applied steadily, we can test the truth of the equation $E = \frac{1}{2}Fl$ quite easily, for, writing l for l_1 or l_2, these being equal, the energy in the spring (E_1) is M_2gl, and the tension in the spring is $F_1 = M_1g$. If therefore $M_2 = \frac{1}{2}M_1$, we have verified experimentally that the energy of a strained body elongated an amount l by a force F is equal to $\frac{1}{2}Fl$.

Expt. 55. **Determination of the Energy of a Spiral Spring.**—Remove the scale-pan[1] from a spiral spring. Apply a load sufficient to stretch the spring nearly to the end of the scale when applied gently. Note the steady elongation and the mass used, M_1.

Adjust another load, M_2, so that when it is allowed to drop suddenly from a point where the spring is just *not* supporting it, the *first sudden* elongation of the spring may be equal to the steady elongation produced by the load M_1.

Repeat these observations for several different elongations.

Arrange your observations in tabular form thus:—

Elongation, l cm.	Load required to produce it.		$\frac{M_2}{M_1}$
	(a) Applied steadily, M_1 gm. wt.	(b) Applied suddenly, M_2 gm. wt.	
10·3	107	52	0·486
8·4	87	45	0·517
6·5	67	33	0·493
4·5	47	25	0·532
2·6	27	15	0·555

[1] If the scale-pan is not removed, its mass must be included in both M_1 and M_2.

It will be found that M_2/M_1 is approximately equal to 0·5, the accuracy being much less with the smaller masses and elongations than with the larger, owing to the *relatively* larger value of the errors of observation. The possible errors in reading being practically the same throughout, they will have a greater *proportional* value in the cases where the total quantities to be measured are smaller.

Thus since M_2/M_1 is found to be equal to 0·5 (within limits of experimental error), this experiment verifies the statement that $E = \frac{1}{2}Fl$.

CHAPTER VIII

DYNAMICS

§ 1. The Laws of Motion

Up to the present we have been concerned mainly with matter at rest, or, when we have allowed motion to take place, we have studied the results of the motion rather than the motion itself. In the division of the subject known as Dynamics, we are concerned with the motion itself, as well as with the forces producing the motion and the mass moved.

Newton's First Law of Motion is equivalent to a definition of **Force** as that which tends to change the state of rest or uniform motion of a material body.

Practically the whole of Dynamics may be said to be an application, more or less direct, of Newton's Second Law of Motion, or a study of one or other of the quantities mentioned in that law.

NEWTON'S SECOND LAW OF MOTION

The change in the quantity of motion possessed by a body when under the action of a force is proportional to the magnitude of the force and to the time during which it acts: it takes place in the direction of action of the force.

Quantity of Motion, or Momentum. — **The quantity of motion possessed by a body is now called the Momentum of**

the body: it is defined as the Mass of the body multiplied by its Velocity.

The Momentum of a body possesses direction as well as magnitude, *i.e.* **Momentum is a Vector quantity.**

The **second law** may be rewritten as **Rate of Change of Momentum is proportional to Force.**

We define our Force unit to be such that **Unit Force produces unit rate of change of momentum,** or

Force = Rate of Change of Momentum.

Now if a force acts on a body of constant mass the M
change is due solely to the resulting change of Veloc

Force = Mass × Rate of Change of Velocit

or finally

Force = Mass × Acceleration.

THE PRINCIPLE OF THE CONSERVATION OF MOMENTUM

If two bodies A and B come under the action of each other so that the motion of B is changed due to the action of A, and *vice versa*, these two bodies are said to have been in Collision: they need not necessarily have come into physical contact with each other.

The Principle of the Conservation of Momentum states that: **In any collision, there is, on the whole, neither gain nor loss of momentum.** This principle can be proved by purely theoretical considerations involving the use of the Third Law of Motion that Action and Reaction are equal and opposite. We shall deal here with its experimental verification, limiting ourselves to experiments dealing with actual physical contact, and to bodies moving in one straight line.

In considering the total momentum of the moving bodies, their directions of motion as well as the magnitudes of their momenta must be taken into account. Thus if two bodies are moving with velocities v_1 and v_2 along the same straight line, one moving to the right and the other to the left, one of the

bodies has a positive, the other a negative momentum, the total momentum being the *algebraic sum* of the momenta possessed by the two bodies. It is immaterial which direction is considered positive so long as this convention, once made, is adhered to during the whole of one experiment.

Consider a collision between two masses m_1 and m_2 moving along the same straight line with velocities v_1 and v_2 respectively. Their velocities after collision may be indicated by v_1' and v_2'. The principle of the conservation of momentum states that Total Momentum before = Total Momentum after impact, or

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2';$$

the velocities being reckoned positive in one direction and negative in the other.

THE BALLISTIC BALANCE

An apparatus which is convenient for the experimental demonstration of the principle of the conservation of momentum is that known by the name of the Ballistic Balance (Fig. 68).

Two scale-pans, usually of wood, are suspended by sets of cords

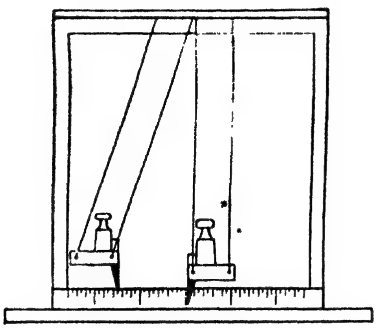


FIG. 68.—Hicks's Ballistic Balance.

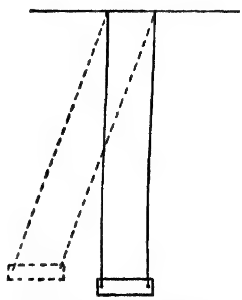


FIG. 69.—Motion of Scale-pan.

in such a way that they move over the arc of a circle of large radius. The suspending cords are so arranged that the scale-pans have no motion of rotation as they swing, their upper surfaces remaining *horizontal* in all positions (Fig. 69).

In one form, the scale-pans are supplied with pointers which

move over a scale stretching horizontally across the base of the apparatus, the pointers being quite clear of, but fairly close to, the scale.

EXPT. 56. **The Ballistic Balance.**—By placing known masses on the two pans, the total mass in motion can be altered in various ways. The masses added should always be placed close to the ledge which is fitted at the 'front' of each scale-pan, otherwise they will slide about when the pans collide, and this will diminish the accuracy of the result. In computing the masses of the moving systems, m_1 and m_2, the masses of the scale-pans must be taken into account.

If one scale-pan is drawn aside through a known distance and then released it will return to its equilibrium position with a velocity that is proportional to the initial horizontal displacement. The proof of this statement will be given later.

When the first scale-pan strikes the second (supposed to be at rest at the start) the velocity of each pan will be altered.

It is necessary to determine the velocity of each pan after the blow by observing the horizontal distance through which it travels; the horizontal distances may indeed be taken as the actual velocities in some arbitrary units not specified. As it is impossible to watch both pointers at the same time, it is necessary to repeat each experiment a number of times, noticing in one set of experiments the maximum horizontal displacement (after collision) of the first pointer, and in the other set the maximum displacement (after collision) of the other: the initial displacement and the masses must of course be made the same each time.

Tabulate the results in the following manner:—

Before Impact.			After Impact.							
Mass, m_1.	Velocity (displacement), v_1.	Total Momentum, m_1v_1.	Mass, m_1.	Velocity, v_1'.	Momentum, m_1v_1'.	Mass, m_2.	Velocity, v_2'.	Momentum, m_2v_2'.	Total Momentum, $m_1v_1'+m_2v_2'$.	% Error.

Since the mass m_2 was at rest at first, v_2 is zero. The third column m_1v_1 thus represents the total initial momentum, and the tenth column $m_1v_1'+m_2v_2'$ is the total momentum after collision.

Express the difference between these columns as a percentage of either, and enter this in the last column as the percentage error of each set of observations.

In the type of apparatus where a clip is supplied which locks the two scale-pans together after collision, the two move on with a common velocity, *i.e.* $v_1' = v_2'$. In this form the pointers can be dispensed with, and the scale-pans can be made to move a small rider along a bar of wood to indicate the first maximum horizontal displacement of the *two* masses after collision.

In this case the table of observations is somewhat simplified, as is also the taking of the observations and the experiment generally.

Before Impact.			After Impact.			Percentage Error.
Mass, m_1.	Velocity, v_1.	Total Momentum, m_1v_1.	Combined Mass, m_1+m_2.	Common Velocity, v_1'.	Combined Momentum, $(m_1+m_2)v_1'$.	

The last column in this case is the difference between the third and sixth columns expressed as a percentage of either.

Proof that the velocity in the equilibrium position is proportional to the horizontal displacement. Suppose the mass m to be displaced from its equilibrium position A to a point B along the arc AB: the point of support is the point O (Fig. 70), and the radius of the arc is OB = R. In moving from B back to A, A loses an amount of potential energy $= mgh$.

It possesses at A a velocity v in the direction shown, its kinetic energy at A being the result of the potential energy lost from B to A,

i.e. $$\tfrac{1}{2}mv^2 = mgh,$$

or $$v^2 \text{ is proportional to } h.$$

Now $$OB^2 = OC^2 + BC^2,$$

i.e. $$R^2 = (R - h)^2 + BC^2,$$

whence $$2Rh = h^2 + BC^2.$$

Now BC is large compared with h, and to a close degree of accuracy h^2 can be neglected compared with BC^2: h^2 is rarely equal to 1% of BC^2.

Hence *very closely* we may say that

$$BC^2 = 2Rh,$$

or $$BC^2 \text{ is proportional to } h.$$

BC is the initial horizontal displacement of the mass m.

Now BC^2 and v^2 are both proportional to h, hence BC must be proportional to v, *i.e.* the velocity as the body passes through its equilibrium position is proportional to the original horizontal displacement.

It may be shown, by reversing the proof, that the horizontal distance to which a body will swing after leaving its equilibrium position is proportional to the velocity it possessed when in its equilibrium position, *i.e.* the velocities *after* collision are proportional to the maximum horizontal displacements reached after colliding.

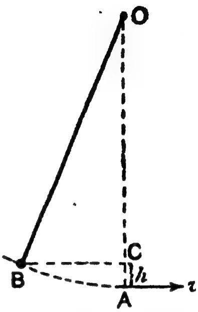


FIG. 70.—Velocity in Ballistic Balance.

The importance of ensuring that there is no motion of rotation of the scale-pans will be seen from the above proof. If there were motion of rotation, the potential energy at B would not appear entirely as *linear* kinetic energy at A but would partly exist as kinetic energy of rotation. Hence the statement $\frac{1}{2}mv^2 = mgh$ would be untrue and the proof would be invalid.

The way in which motion of rotation is prevented will be seen at once from Fig. 69.

§ 2. Methods employed for the Experimental Verification of the Second Law of Motion

If a body move with a constant acceleration a, the *distance* through which it moves in a time t is given by the equation

$$s = v_0 t + \tfrac{1}{2}at^2,$$

where v_0 is the initial velocity of the body.

If the body is initially at rest, v_0 is zero and we have

$$s = \tfrac{1}{2}at^2, \text{ when the initial velocity is zero.}$$

The *velocity* at the end of any time t is given by

$$v = v_0 + at,$$

which reduces to

$$v = at, \text{ when the initial velocity is zero.}$$

These equations are *absolute*; they are derived from the definitions of the various quantities involved, *and cannot be verified by experiment.* They may be used to determine whether a body is moving with uniform acceleration or not. Thus,

if the distance s moved over by a body in time t starting from rest obeys the law

$$\frac{s}{t^2} = \text{a constant},$$

that body is moving with uniform acceleration, and the value of the acceleration is *twice* the value of the constant obtained, for

$$a = \frac{2s}{t^2}.$$

WEIGHT AND MASS

As an example of the use of these equations, the case of a body falling freely under its own weight may be taken. Any body if allowed to drop quite freely towards the earth will describe in a time t from the start a distance s which is proportional to t^2. Thus in the first second it will fall 5 metres approximately, while in the first two seconds it falls 20 metres. Thus $s/t^2 = 5$ approximately for any body falling freely under its own weight, starting from rest, *i.e.* **the acceleration due to gravity is the same for all bodies** and is equal approximately to 10 metres per sec. per sec.—more accurately this acceleration is 9·81 metres per sec. per sec. in the British Isles.

Now from the second law of motion we define the unit of force in the C.G.S. system to be such that

Force in dynes = Mass in gm. × Acceleration produced (in cm. per sec. per sec.).

If we denote the acceleration due to gravity by the symbol g, in cm. per sec. per sec. when dealing with C.G.S. quantities, we have

$$\left.\begin{array}{l}\text{Force in dynes acting on}\\ \text{a body when falling freely}\end{array}\right\} = \text{Mass in gm.} \times g.$$

Now the force in dynes acting on a falling body is its **Weight.** Hence

$$\left.\begin{array}{l}\text{The Weight}\\ \text{of a body in}\\ \text{dynes}\end{array}\right\} = \left\{\begin{array}{l}\text{The Mass of}\\ \text{the body in}\\ \text{gm.}\end{array}\right\} \times \left\{\begin{array}{l}\text{The acceleration due to}\\ \text{gravity in cm. per sec.}\\ \text{per sec.}\end{array}\right.$$

If W indicate the weight in dynes, of a body having a mass m gm., we have

$$W = mg,$$

g being 981 cm. per sec. per sec. in the British Isles.

The simplest way of obtaining a uniform force is by suspending known masses from light cords, passing the cords over pulleys in order to direct the action of the force in any desired direction. The force acting along the cord in *dynes* is equal to the mass of the suspended body in gm. multiplied by g, the acceleration due to gravity in cm. per sec. per sec.

§ 3. Experiments to illustrate the Second Law of Motion

FLETCHER'S TROLLEY APPARATUS

In this apparatus (Fig. 71), a trolley is mounted on very light wheels so as to move in an almost frictionless manner along a horizontal table. To it is attached a cord which, passing over a pulley

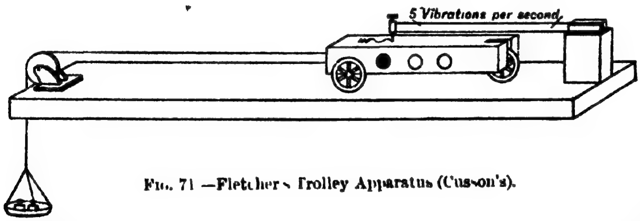


Fig. 71.—Fletcher's Trolley Apparatus (Cusson's).

mounted at the edge of the table, carries a small suspended mass. By hanging different masses from the cord, the trolley can be subjected to various forces, and its motion under these forces can be studied; the mass moved can also be varied by placing known masses in the holes in the side of the trolley.

The method of recording the distance moved through, and the time taken for this motion, is interesting. A long spring is mounted in a firm clamp and carries a light brush at the free end. Fixed on the trolley is a sheet of paper which is touched lightly by the brush. When the trolley is set in motion, the same mechanism which releases it sets the long spring in vibration, and a wavy trace is drawn on the paper, the brush having been moistened with ink.

The time taken to make a complete vibration is constant for the spring, and therefore the number of complete vibrations made between two given points may be taken as a measure of the time taken to move from one point to the other.

By taking the distances travelled from the start, during the time taken to make different numbers of vibrations, it is possible to find from the wavy trace if the relation s/t^2 is constant.

The Mass moved is the sum of the masses of the trolley, cord, and hanging mass, together with a small quantity which may be called the *equivalent mass of the pulley*, and another small quantity, the equivalent mass of the wheels. The trolley is usually sufficiently massive for these other masses to be neglected.

The Force acting is evidently the **weight** of the hanging mass, *plus* the weight of that part of the cord which is hanging over the pulley. In order to reduce the error due to the weight of the cord, an extremely fine, strong line (fishing-line) should be used, so that its weight is negligible compared with that of the hanging mass. Its effect may be allowed for, if necessary, by adding to the weight of the suspended mass a quantity equal to the weight of the *average* length of the cord hanging beyond the pulley.

The Acceleration produced.— The quotient s/t^2 will be found constant, therefore the trolley is moving with constant acceleration, the value of this acceleration being $2s/t^2$.

If only *relative* values are required, the units in which the time is measured may be taken as the time for one vibration of the spring.

For *absolute* results, the period of vibration of the spring must be known in order that the accelerations may be calculated in cm. per sec. per sec.

The period of vibration is usually stamped on the spring, having been determined by the maker of the apparatus, and this marked period may be used for this purpose. The spring seldom makes a sufficient number of vibrations for the period to be checked by simple means, and the calibration made by the maker has to be accepted: this constitutes a serious drawback to this method of timing when *absolute* results are required.

Experiments with Fletcher's Trolley Apparatus

EXPT. 57. **Acceleration is proportional to Force.**—Fix a sheet of paper on the trolley and attach small masses 10, 20, 30, 40 gm., etc., to the cord, obtaining traces for the motion of the trolley under the action of each force. It is interesting

to get all the traces on the same sheet of paper, starting from the same point in each case.

The mass moved is approximately the same in each case, being altered only by the change in the hanging mass.

The forces acting are proportional to the hanging masses in the various cases.

Show (*a*) that the distances travelled in equal times are proportional to the suspended masses; (*b*) that for each case $2s/t^2$ is a constant, and that the values of the constants in the different cases are proportional to the suspended masses used.

Friction Correction.—It is necessary to eliminate or neutralise the friction forces if accuracy is to be obtained. To do this a small mass is suspended from the cord, and adjusted until the trolley just continues to move when once started. The weight of this small mass is then just sufficient to overcome the friction of the apparatus *for the particular load on the trolley.* A piece of copper wire is a very convenient form of 'friction rider'; it should be twisted to the cord and cut to the required length with wire-cutters.

EXPT. 58. **The Acceleration under a given Force is Inversely proportional to the Mass moved.**—Using the same hanging mass each time, place different masses on the trolley, obtaining separate traces for each mass used. Calculate the value for the acceleration of each mass when under the action of this constant force, and show that

$$\text{Mass moved} \times \text{Acceleration}$$

is constant; *i.e.* For a given force, acceleration is inversely proportional to the mass moved.

In this experiment the equivalent masses of the pulley and wheels may be included if known, though, in general, they may be neglected.

Let M = mass of trolley and load placed on it,
m = mass of hanging weight,
x = equivalent mass of pulley,
and y = equivalent mass of wheels.

Total mass moved is taken as $(M + m + x + y)$.

NOTE.—It is possible to calculate a value for the acceleration due to gravity from the observations already made, provided the acceleration $2s/t^2$ has been calculated in cm. per sec. per sec. Thus in any case taken, $mg = (M + m + x + y) \cdot (2s/t^2)$, since the *force* is the weight of the hanging mass; hence g can be calculated.

The method is not, however, a good one for this purpose on account of the fact that the quantities x and y are not accurately known, and also on account of the difficulty in measuring t already discussed.

Another method of using the apparatus is to place the trolley on a plane inclined at an angle θ to the horizontal. In this case the force tending to produce motion along the plane is $mg - (M + y)g \sin \theta$.

ATWOOD'S MACHINE

This is an apparatus more widely known than Fletcher's Trolley Apparatus, and one of great historic interest. It was designed by Atwood (1746–1807), a famous English mathematician, for the purpose of illustrating the laws of motion, and for the determination of the acceleration due to gravity. In it, the weight of a small rider is compelled to move two much larger masses which, suspended at opposite ends of a cord moving over a pulley, counterpoise each other exactly. The weight of the small rider causes only a very small acceleration in the moving masses, since the total mass set in motion is large. Hence this acceleration can be measured with much greater accuracy than the acceleration the rider would have if falling freely alone.

Atwood's Machine: Pillar Type.—The two equal masses A and B are suspended from a cord. The cord passes over a pulley W supported on fine bearings at the top of a pillar from 2 to 2·5 metres in length. Attached beneath A and B there may be a compensating cord of the same type as the suspending cord; this keeps the masses of string on the two sides of the machine accurately balanced whatever the positions of the masses A and B: it is, however, inconvenient in practice, and is rarely used. The mass A carries a small rider which can be slipped on over the string, so that the only unbalanced force in the whole system is the weight of this small rider.

To perform the experiment, B is fixed lightly in the clip, so that the *top* of A is level with a known mark on the scale. The ring C is adjusted to some convenient distance below this zero position of A, so that A will traverse a known distance under the action of the

weight of the rider. A stop-watch is started and simultaneously the clip is opened, thereby releasing B with no *initial* velocity. When the rider on A is heard to strike the ring C, the watch is stopped, and thus the time t required to traverse a known distance s is determined.

Another method of making the experiment is to use a metronome, and to adjust the distance so that the time of fall occupies an exact number of beats of the metronome.

In some forms of machine a pneumatic release is fitted to the clip; in others the clip is replaced by a small electromagnet, and the mass B is held magnetically, iron masses (A and B) being used. All types of release are equally good provided B is released without any vertical motion, though simplicity in use and construction is generally associated with certainty of action, and therefore is to be recommended.

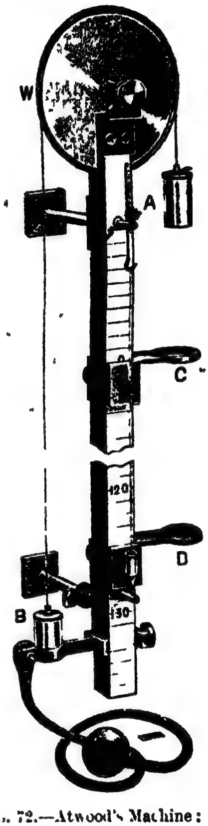


FIG. 72.—Atwood's Machine: Pillar Type.

Experiments with Atwood's Machine

EXPT. 59. **A Body moving under the Action of a Uniform Force moves with Uniform Acceleration.**—Place a small piece of copper wire on the mass A, adjusting its size so that the masses just continue in motion *without other rider* when gently started. Then the weight of this piece of copper wire just overcomes the friction of the machine—it is called the 'friction rider' and is kept on A all the time.

Adjust the ring C to different positions, so that the masses move through distances of 50, 100, 150, and 200 cm. under the action of the various riders.

Find the times taken for the masses to move through these various distances, when under the action of riders of different weights (say 2 gm., 4 gm., 6 gm., etc.). *At least three values of* t *should be taken for each distance, and each rider, used.*

Find the quotients $2s/t^2$ for each set of observations, and show that *for a given rider* $2s/t^2$ is constant.

That is, when acted on by a constant force, a mass moves with uniform acceleration.

Arrange the observations as in the following table :—

Mass of Rider used.	Distance s.	Time t.	$\frac{2s}{t^2}$.	
2 gm. rider . .	50 100 150 200			Mean acceleration with 2 gm. rider a_1 = . . .
4 gm. rider .	50 100 150 200			Mean acceleration with 4 gm. rider a_2 = ...
6 gm. rider . (if used)	50 100 150 200			Mean acceleration with 6 gm. rider a_3 = ...

It will be found that the sets of figures in the last column will approximate to a constant value for each of the riders, the value of the constant increasing as the mass of the rider is increased.

EXPT. 60. **Acceleration is proportional to the Force acting.**—This can be shown without further experiment from the results in the foregoing table. The total mass moved by any of the riders is practically the same, differing only by the small differences between the masses of the riders themselves. Thus if the acceleration produced in a mass is proportional to the magnitude of the force acting on the mass, the accelerations a_1, a_2, a_3 should be in the same proportions as the masses of the riders used, *i.e.* proportional to 2, 4, 6, etc., in the case considered.

EXPT. 61. **Acceleration for a Given Force is inversely proportional to Mass.**—By using pairs of masses A and B of different sizes, it is possible to show that a given force produces acceleration inversely proportional to the mass on which it acts.

This is done by finding from measurements of s and t the acceleration produced in different pairs of masses, when they

move under the action of the same rider. The products (Total mass moved × acceleration) should be constant.

It is necessary in this case to know the value of the 'equivalent' mass of the pulley.

The total mass moved in any one case is given by $(2M + m + x)$ gm. (see below).

EXPT. 62. **Acceleration due to Gravity.**—(i.) **Assuming the value of the equivalent mass of the pulley and cord.**

Let the mass of each of the masses A and B	$= M$ gm.
Let the mass of the rider	$= m$ gm.
Let the equivalent mass of the pulley (and cord)	$= x$ gm.
Let the acceleration produced	$= a$ cm. per sec. per sec.

Then Force acting = Wt. of rider $= mg$ dynes,
Mass moved $= (2M + m + x)$ gm.

$$\text{Force} = \text{Mass} \times \text{Acceleration}.$$

Therefore $$mg = (2M + m + x)a,$$

from which g can be determined.

Calculate g from the sets of observations for each of the riders used in Experiment 59.

(ii.) **Calculation of a value of the acceleration due to gravity, eliminating the equivalent mass of the pulley.**

If the same rider is used for different pairs of masses A and B, the value of g can be calculated without assuming the equivalent mass of the pulley to be known.

Thus if the acceleration produced with a pair of masses each equal to M' is a', and the acceleration produced by the *same* rider on a pair of masses each equal to M'' is a'', then

$$\left.\begin{aligned} mg &= (2M' + m + x)a' \\ \text{and } mg &= (2M'' + m + x)a'' \end{aligned}\right\} x \text{ being unknown,}$$

whence $$mg\left\{\frac{1}{a'} - \frac{1}{a''}\right\} = 2(M' - M'').$$

Calculate g, using the values of a' and a'' corresponding with the different masses used in Experiment 61.

The instrument was designed originally for the determination of g, the accurate pendulum methods of Kater having not

then been invented. Its main use nowadays is to demonstrate the laws of motion, determinations of g by its means being of a relatively low order of accuracy though possessing historic interest.

Atwood's Machine: Ribbon Pattern.—In this apparatus the equal masses are suspended by a paper ribbon which passes over the flat rim of the pulley. A compensating ribbon is attached below the masses as in Fig. 73. A steel spring is fixed at one end and at the other carries a fine brush, charged with ink, which marks the paper at the top of the pulley. A simple release sets free the spring and the moving masses simultaneously. Each wave traced on the ribbon represents a known period of time.

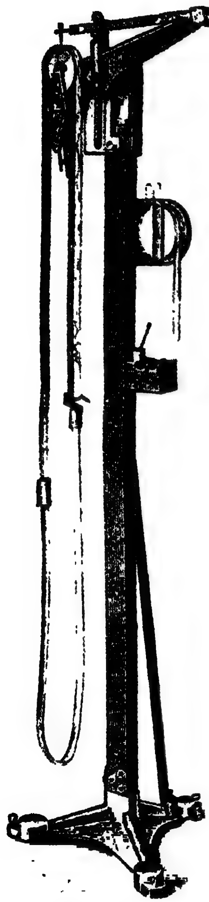


FIG. 73.—Atwood's Machine: Cusson's Ribbon Pattern.

The same type of experiment can be performed with this form as with the pillar form of machine, taking the distances and times as recorded on the ribbon. The value of the acceleration in each case can be obtained from the wavy trace, as already explained in Fletcher's Trolley Apparatus.

Carry out experiments exactly as with the pillar type of machine, using the ribbon and spring to measure the acceleration in the various cases instead of adjusting the distance s to different values and noting the corresponding times.

A way of using Atwood's machine which is sometimes employed is to find the velocity of the mass A after the rider has been removed by the ring and the system is supposed to be moving with constant velocity. This method, however, is neither so convenient nor so accurate as that detailed.

§ 4. Rotation of a Rigid Body

MOMENTS OF INERTIA

The 'effect' of a mass in rotation about an axis depends not only on the mass moving but also on the way in which the mass is distributed about the axis of rotation. Thus, consider the Kinetic Energy of a body as shown in Fig. 74, rotating with angular velocity ω radians per second, about an axis through O perpendicular to the plane of the paper.

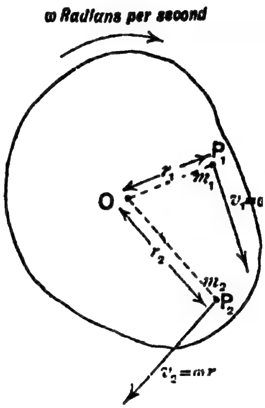


Fig. 74.—Moment of Inertia.

Velocity of particle m_1 at $P_1 = r_1\omega$,

Velocity of particle m_2 at $P_2 = r_2\omega$,

etc.

Kinetic Energy of particle m_1, at $P_1 = \frac{1}{2}m_1(r_1\omega)^2$,

Kinetic Energy of particle m_2, at $P_2 = \frac{1}{2}m_2(r_2\omega)^2$,

etc.

Total Kinetic Energy of the Body due to Rotation about O

$$\frac{1}{2}\omega^2\{m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \dots\};$$

or *indicating the sum of the quantities in the bracket by the symbol* I, we have

$$\text{Kinetic Energy of Rotation} = \tfrac{1}{2}I\omega^2.$$

The sum represented by I is a property of the body which has a perfectly definite value for that body with reference to the given axis O, its magnitude depending on the distribution of the mass about that axis. It is called the **Moment of Inertia of the body about the given axis**, and is defined by

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \dots$$

or
$$I = \Sigma mr^2,$$

where Σ denotes the sum of a number of terms of the same type, taken for all the particles in the body.

The Radius of Gyration.—If the total mass M of the body

were concentrated into a single particle, and if this particle were constrained to move in a circular path of radius k by means of a light rod with its centre at O, the Moment of Inertia of the particle about the given axis would be Mk^2. By properly choosing k the Moment of Inertia of this particle may be made the same as that of the given body. For this value of k, $Mk^2 = I$; this length k is called the **Radius of Gyration** of the body about that particular axis.

The Moment of Inertia would have the same value if the matter were arranged in the form of a ring of radius k.

THEOREM OF PARALLEL AXES

The Moment of Inertia of a body about any axis is equal to its Moment of Inertia about a parallel axis through the Centre of Gravity *plus* the product obtained by multiplying the mass of the body by the square of the distance between the axes.

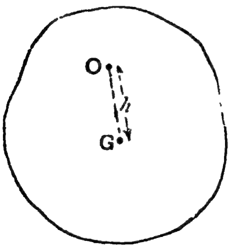


FIG. 75.—Parallel Axes through O and G.

Thus I_o, the Moment of Inertia about an axis through O,

$$= I_g + Mh^2.$$

But $\quad I_g = Mk_g^2,\ I_o = Mk_o^2,$

so $\quad Mk_o^2 = Mk_g^2 + Mh^2,$

or $\quad k_o^2 = k_g^2 + h^2.$

Consequently, if we know the Moment of Inertia, or the Radius of Gyration, for an axis through the Centre of Gravity, we can calculate the corresponding quantity for any parallel axis.

For a list of Moments of Inertia in some important cases, see the Appendix, p. 595.

LINEAR MOTION AND ANGULAR MOTION

The following parallels between quantities concerned with linear motion and quantities concerned with angular motion should be noticed :—

Linear Motion.		Angular Motion.	
Quantity.	Symbols.	Quantity.	Symbols.
Displacement or distance	s	Angle . .	θ
Velocity .	$v=\dot{s}=\frac{ds}{dt}$	Angular Velocity	$\omega=\dot{\theta}=\frac{d\theta}{dt}$
Acceleration .	a or $a=\dot{v}=\frac{dv}{dt}$ $=\ddot{s}=\frac{d^2s}{dt^2}$	Angular Acceleration	$\alpha=\dot{\omega}=\frac{d\omega}{dt}$ $=\ddot{\theta}=\frac{d^2\theta}{dt^2}$
Mass .	m	Moment of Inertia	I
Force . .	$f=m.a$	Couple . .	$c=\mathrm{I}\alpha$
Momentum .	mv	Angular Momentum	$\mathrm{I}\omega$
Kinetic Energy of Translation	$\frac{1}{2}mv^2$	Kinetic Energy of Rotation	$\frac{1}{2}\mathrm{I}\omega^2$
Work . .	force × distance moved $\mathrm{W}=fs$	Work . .	couple × angle turned through $\mathrm{W}=c\theta$

This table is of great value in dealing with angular motion. If a general expression be obtained connecting certain quantities in the case of linear motion, an exactly similar expression can be written at once for the corresponding quantities for angular motion. For examples of this, see Simple Harmonic Motion, Chapter IX.

§ 5. Measurement of Moment of Inertia

The idea of Moment of Inertia has been obtained above from a consideration of the Kinetic Energy of a rotating body. It is by measurement of the Kinetic Energy of a rotating body that we usually measure its Moment of Inertia. The experimental determination of the Moment of Inertia of a rotating body is usually carried out by giving to the body a definite or

measurable quantity of energy, and measuring the resulting angular velocity.

MOMENT OF INERTIA OF A FLYWHEEL

When the body is in the form of a wheel with a long axle, the following method is one of the most suitable for determining its Moment of Inertia about the axis of rotation. At some point in the axle, or in a cylindrical rim on the wheel itself, either a small hole or a small peg will be required.

A brass pin is made to fit into the hole and is tied firmly to a good length of cord. If, instead of a hole, a peg be found, a

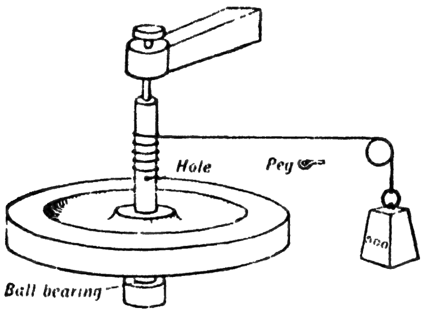


FIG. 76.—Flywheel with Axle vertical.

simple loop is made in the end of a cord and this is slipped over the peg. The cord having been attached in one of these ways, the wheel is turned so as to wind the cord round the rim a few times. The cord is passed over a pulley if the axis of the wheel is vertical, or allowed to hang straight down if the axis is horizontal; to the free end is attached a mass of suitable size.

If now the mass be allowed to fall, it will lose Potential Energy; the Potential Energy lost will be converted partly into Kinetic Energy of translation due to the motion acquired by the falling mass itself, and partly into Kinetic Energy of rotation of the fly-wheel. If we neglect any frictional losses for the time being, we may state from the principle of the conservation of energy that

$$\left\{\begin{matrix}\text{Potential Energy lost}\\ \text{by the falling mass}\end{matrix}\right\} = \left\{\begin{matrix}\text{Kinetic Energy}\\ \text{gained by mass}\end{matrix}\right\} + \left\{\begin{matrix}\text{Kinetic Energy}\\ \text{gained by wheel}\end{matrix}\right\}.$$

Now if the mass suspended be m gm., and if it fall through a vertical distance h cm. before the string is released from the wheel,

the Potential Energy lost is mgh ergs. Just as the end of the string is pulled off from the rim, the mass has acquired a velocity equal to v cm. per sec. (say), and the wheel an angular velocity equal to ω radians per second. The Kinetic Energy of the falling mass at this instant is thus $\frac{1}{2}mv^2$, and the Kinetic Energy of rotation of the wheel is $\frac{1}{2}I\omega^2$.

Thus, *neglecting friction*, we have

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2.$$

In this equation m and g are both known.

Determination of h.—The most convenient way of getting an accurate value of h is to *arrange the length of the cord so that the*

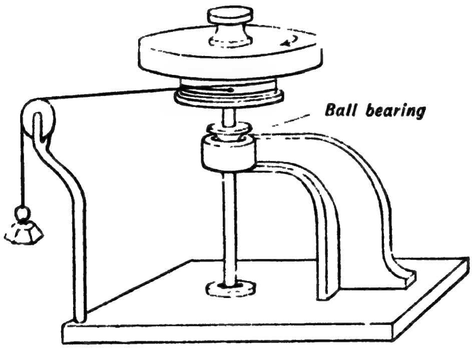


FIG. 77.—Flywheel with Axle vertical.

end separates from the wheel just as the bottom of the falling mass touches the ground. If the mass be started with its base level with the table, the height through which it falls while attached to the wheel is equal to the height of the table above the floor.

Determination of v and ω.—There are two ways in which v and ω may be determined; both these are described below, but the *second* method is preferable as giving much greater accuracy than the first. It also affords a useful means of correcting for friction losses (see later).

METHOD I.—The length of time taken for the falling mass to reach the ground is measured by means of a stop-watch. Let this be t_1 seconds.

During this time the mass falls a distance h cm. with a uniformly increasing velocity. Since the *initial* velocity is zero the final velocity will be double the average velocity.

$$\text{The Average Velocity } \bar{v} = \frac{h}{t_1}.$$

The final velocity, the velocity the mass has when it reaches the ground, is *double* this value,

i.e. $$v = \frac{2h}{t_1}.$$

The time t_1 is usually very short, and cannot be measured with great accuracy.

The quantities v and ω are connected by the relation $v = \omega r$, where r is the radius of the cylindrical rim round which the string is wound. If r be measured and v be determined as above, we can find ω, since

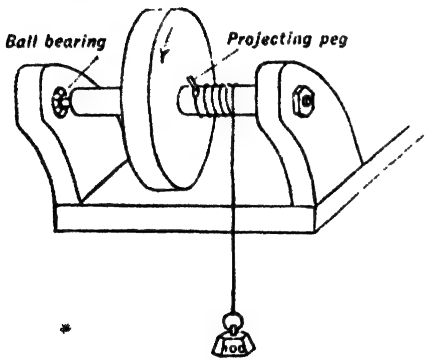


FIG. 78.—Flywheel with Axle horizontal.

METHOD II.—After the string has become detached from the wheel the wheel continues to revolve for a considerable time. Its angular velocity decreases, however, on account of friction, and eventually the wheel comes to rest again.

If the friction be constant, the wheel will be retarded quite uniformly, and the average angular velocity taken over the whole time required to come to rest will be equal to *one-half* the initial angular velocity ω.

If the wheel make n_2 revolutions after the string has become detached, and take t_2 seconds to come to rest, the average angular velocity, while coming to rest, is given by

$$\bar{\omega} = \frac{2\pi n_2}{t_2} \text{ radians per second.}$$

Therefore ω, the angular velocity at the moment when the string became detached, is given by

$$\omega = 2\bar{\omega} = \frac{4\pi n_2}{t_2}.$$

The value of t_2 is much greater than that of t_1 in Method I., and can therefore be determined with much greater accuracy; thus the resulting values of v and ω obtained by this method will be more accurate than those obtained by Method I.

Having found ω, v is calculated by means of the relation

$$v=\omega r.$$

Determine v in cm. per second and ω in radians per second.

Expt. 63. Moment of Inertia of a Flywheel.—Set the wheel in rotation by allowing different masses to hang from the string, adjusting and measuring the heights of fall as already described; this gives m and h.

Measure the radius of the cylindrical rim round which the cord is wound, adding half the thickness of the cord to this if the cord is an appreciable thickness compared with the radius of the rim; this gives r.

Count the number of revolutions made by the wheel after the string has become detached, n_2.

Find the time taken to come to rest, t_2.

Repeat the observations three times each, taking the *means* of the observed values of n_2 and t_2, if these vary for the same m and h.

Calculate mean values of ω and v corresponding with each value of m and h, and substitute in the equation

$$mgh=\tfrac{1}{2}mv^2+\tfrac{1}{2}I\omega^2.$$

Calculate each of the quantities mgh, $\frac{1}{2}mv^2$, and $\frac{1}{2}\omega^2$ *separately* before solving for I.

Express the Moment of Inertia in *gm. cm.*^{2}

Correction for Friction.—If the friction of the supporting bearings is considerable, it must be allowed for. Suppose a certain amount of work f is done against friction every time the wheel revolves *once*. While the mass was falling, a certain number of revolutions n_1 were made, and therefore an amount of work n_1f was done against friction.

The equation $mgh=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$ is therefore no longer quite true: it must be modified to

$$mgh=\tfrac{1}{2}mv^2+\tfrac{1}{2}I\omega^2+n_1f,$$

since the work n_1f was done while the falling mass was losing its Potential Energy.

Now, after the string was detached from the wheel, the wheel possessed a certain amount of Kinetic Energy $=\frac{1}{2}I\omega^2$. This energy was gradually lost in overcoming friction, the whole amount being absorbed in a certain number of revolutions n_2. Hence

$$\tfrac{1}{2}I\omega^2=n_2f.$$

We thus have a value for f expressed in terms of known quantities,

$$f=\frac{\frac{1}{2}I\omega^2}{n_2},$$

and therefore $$n_1 f = \frac{n_1}{n_2}\tfrac{1}{2}I\omega^2$$

The equation may now be rewritten as

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + \frac{n_1}{n_2}\tfrac{1}{2}I\omega^2,$$

or $$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2\left(1 + \frac{n_1}{n_2}\right),$$

the friction correction being represented by the term $\frac{n_1}{n_2}$ in the bracket.

Determine n_1, the number of turns made by the wheel while the mass is falling, and recalculate I, introducing this small correction.

SOLID OF REVOLUTION ROLLING DOWN AN INCLINED PLANE

When a body is allowed to roll down an inclined plane, the Potential Energy it loses is converted into Kinetic Energy. When the body reaches the foot of the plane it possesses two kinds of motion—

(*a*) Motion of translation,
and (*b*) Motion of rotation.

Hence the Kinetic Energy of the body is made up of two parts—

(*a*) Kinetic Energy of linear motion $= \tfrac{1}{2}mv^2$,
and (*b*) Kinetic Energy of rotation $= \tfrac{1}{2}I\omega^2$.

Where m = Mass of body,
v = Linear Velocity,
I = Moment of Inertia about the axis through the Centre of Gravity,
ω = Angular Velocity.

If the top of the plane where the body starts be h cm. above the place where the body is stopped, the body loses Potential Energy $= mgh$.

Hence $$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2.$$

Actually the motion is one of rotation about the axis of

contact with the plane, as it is assumed that no slipping takes place here. This can, however, be shown to be equivalent to a linear motion of the C.G., together with an angular motion about the axis through the C.G., therefore the above statement is correct.

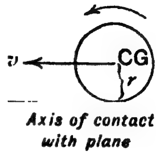


*FIG. 79. Proof that $v=\omega r$.

If r is the perpendicular distance between the *axis of contact* and the Centre of Gravity, the linear velocity of the Centre of Gravity v is given by the equation

$$v=\omega r,$$

as is readily seen by reference to Fig. 79.

Now v may be determined by observing the time taken for the rolling body to traverse a length s on the plane; let this be t secs.

Final Velocity = twice Average Velocity.

Thus ω also is known, for $\omega=\frac{v}{r}$; *i.e.* $\omega=\frac{2s}{tr}$.

h and m may be determined directly, so that everything in the equation is known except I.

Substituting the known values for the various quantities in the equation

$$mgh=\tfrac{1}{2}mv^2+\tfrac{1}{2}I\omega^2,$$

the value of I may be obtained.

EXPT. 64. **Wheel and Axle on an Inclined Plane.**—A large disk fitted with a steel axle is allowed to roll down a set of

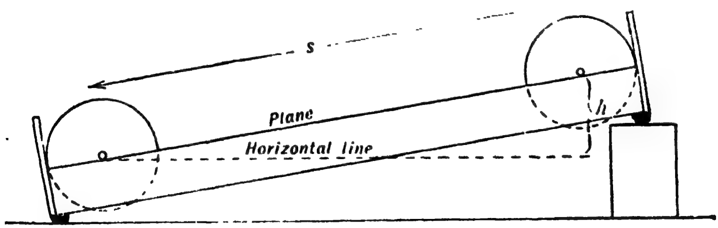


FIG. 80.—Wheel and Axle on Inclined Plane.

rails on an inclined plane, and the time taken for the disk to roll down the plane is noted; let this be t sec.

The length of the plane traversed by the axle is also determined; call this s cm. The total height fallen through by the axle is measured by means of a simple cathetometer;

let this be h cm. Then the Potential Energy lost $= mgh$. Weigh the disk, and calculate the value of mgh.

The linear velocity of the disk as it reaches the foot of the plane is $2s/t = v$.

Calculate the value of v in cm. per sec.

Calculate the value of $\frac{1}{2}mv^2$, the Kinetic Energy of translation at the foot of the plane.

In this case, the distance from the fixed axis to the Centre of Gravity is equal to the radius of the axle.

Measure the radius of the axle, using a micrometer screw gauge; let this be r cm.

The angular velocity at the foot of the plane $= \omega = \frac{v}{r} = \frac{2s}{rt}$.

Calculate the value of ω in radians per second.

Substitute these values in the equation

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\mathrm{I}\omega^2,$$

and solve for I.

Repeat the experiment, using various values of h (5, 10, 15, and 20 cm.).

Verify your result by calculation, assuming the Moment of Inertia of the disk is $\frac{1}{2}ma^2$, where a is the radius of the *disk*.

It has been mentioned in the foregoing that although the actual motion is one of rotation about the axis of contact with the plane, this motion is equivalent to a linear motion of the Centre of Gravity together with an angular motion about an axis through the Centre of Gravity.

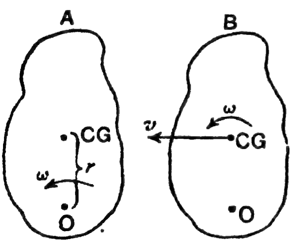


FIG. 81.—Motion about Instantaneous Axis of Rotation.

The proof of this is as follows: Consider the body A to be subject to an angular velocity ω about the fixed axis O, and let B be an exactly similar body with a linear velocity v of the C.G., and an angular velocity ω about the C.G.

Let the distance from the Centre of Gravity to O be r, and let the linear velocity of the C.G. in case B, be at right angles to this line and equal to ωr.

Consider the motion of the C.G. in the two cases.

Case A. Linear velocity of C.G. $= \omega r$, from right to left, due to angular motion about O.

Case B. Linear velocity of C.G. $= v = \omega r$, from right to left, by hypothesis.

Motion due to rotation is zero.

Next consider the motion of the point O in the two cases.

Case A. Motion is zero.

Case B. Motion is $v = \omega r$, from right to left, due to linear motion and ωr, from left to right due to rotation, $\therefore$ O is at rest.

Thus in either case two points of this rigid body have the same motion. Therefore the motion of *all* points is the same, *i.e.* a motion of rotation about an axis distant r from the C.G. may be resolved into an equal motion of rotation about a parallel axis through the C.G., together with a linear motion of the C.G. $= \omega r$.

DISK SUPPORTED BY CORDS PASSING ROUND AN AXLE THROUGH THE DISK

The disk is mounted on a steel spindle suspended by strings in such a way that the axle is horizontal. The disk is raised by turning the axle so as to roll the string evenly on the two sides of the axle as shown in Fig. 82. When released the disk falls, acquiring a motion of rotation, due to the unwinding of the cord, as well as a motion vertically downwards.

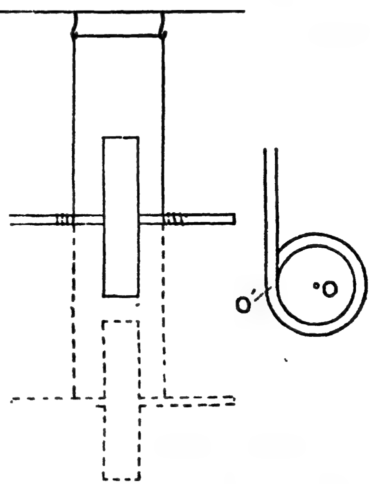


FIG. 82.—Disk suspended by Strings.

If it falls a distance h, its mass being m, then as before

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2,$$

v being the linear, and ω the angular velocity of the disk, when it has fallen through this distance h.

The relation between v and ω is $v = \omega r$, where r is the radius of the axle plus half the thickness of the string. This is easily seen from Fig. 82. The point O′ is at rest, and the centre of the axle O will therefore have a velocity $v = \omega OO'$.

Determination of v and ω.—The linear velocity of the disk as it reaches the bottom of the string is double the average linear velocity during the fall, for it starts with zero velocity and is accelerated uniformly throughout.

The average velocity of fall is obtained by finding the time t

taken for the disk to reach its lowest position. The distance moved through is h, therefore the average velocity

$$\bar{v} = \frac{h}{t},$$

or

$$v = 2\bar{v} = \frac{2h}{t}.$$

From this ω is obtained, since $\omega = \frac{v}{r}$.

EXPT. 65. **Determination of the Moment of Inertia of a Disk suspended by Strings.**—See that the axle is horizontal when the disk is in its lowest position. Rotate the disk about its axis so that the strings are wound evenly round the spindle, and the disk is raised to its highest point. Release the disk and start a stop-watch simultaneously, and take the time of descent from the highest to the lowest point. This observation must be repeated several times, and the mean value found. Measure the distance h through which the disk falls, and calculate the final velocity v in cm. per sec. from the formula

$$v = \frac{2h}{t}.$$

Measure the diameter of the spindle and the diameter of the cord with a micrometer screw, and determine r, which is the sum of the radii of the spindle and the cord. Determine the angular velocity ω from the formula $v = \omega r$, expressing the result in radians per second.

The mass of the disk (and axle) is determined by actual weighing, and thus the data necessary for the calculation of I are all known. Calculate the value of I from the energy equation.

An approximate value of I can be calculated from the mass and dimensions of the disk,

$$I = \frac{ma^2}{2},$$

a being the radius of the disk.

This calculated value is only approximate; the formula is only true if the mass of the disk is distributed uniformly throughout its volume, and of course, this is not the case, the axle having an appreciable mass which is obviously not distributed uniformly throughout the disk.

CHAPTER IX

PERIODIC MOTION

§ 1. Linear Simple Harmonic Motion

In all branches of Physics cases occur in which the motion of a point or particle is of the nature of an oscillation or vibration. The motion of a point is said to be **periodic** when the same series of movements is repeated at regular intervals of time. **The Period of the motion is the time required for the complete series of movements.** The simplest type of periodic motion is that which is known as **Simple Harmonic Motion (S.H.M.).**

Linear Simple Harmonic Motion may be defined geometrically as the **projection of uniform circular motion on a diameter of the circle.**

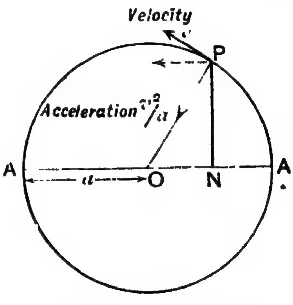


Fig. 83.—Simple Harmonic Motion.

Imagine a point P moving round a circle with uniform speed. Take any diameter AA′ of the circle, and draw PN the perpendicular from P to this diameter. Then the point N, the foot of the perpendicular, executes S.H.M. across the diameter AA′. The **Displacement** of the point N is ON, **the distance from its mean position O.** The **Amplitude** of the motion is the **maximum displacement from the mean position O.** It is equal to a or OA, the radius of the circle of reference. The **Phase** of the motion is the time, or **the fraction of a period, that has elapsed since**

P passed some fixed point, such as A. It may be expressed also as the angle POA. **The Period of the motion is the time taken for the point N to travel backwards and forwards along the diameter AA′**; it is the same as the time taken by the point P to travel round the circle of reference.

Let the velocity of P at any point be v, and let the angular velocity of OP be ω radians per second, then the Period t is given by

$$t = \frac{2\pi a}{v} = \frac{2\pi}{\omega}.$$

The point N has a velocity along AA′ which is always equal to the component of the velocity of P parallel to AA′. Hence any change in the velocity of P which affects its velocity parallel to AA′ also affects the velocity of N. Thus **the acceleration of the point N along AA′ is equal to the component of the acceleration of P parallel to AA′.** But the acceleration of P can be shown to be equal to $\frac{v^2}{a}$ in the direction PO. Therefore the acceleration of N

$$= \frac{v^2}{a} \cos \text{PON, towards O,}$$

$$\frac{v^2}{a} \times \frac{\text{ON}}{\text{OP}}$$

$$= \left(\frac{v}{a}\right)^2 \times \text{Displacement of N}$$

$$= \omega^2 \times \text{Displacement of N.}$$

Hence we see that **in Linear Simple Harmonic Motion a point moves along a straight line with an acceleration that is always directed towards a fixed point in that line, and is directly proportional to the distance from that fixed point.**

We may regard this statement as an alternative definition of S.H.M.

Thus if we know that a point moves with an acceleration of this type, we know that the point executes a S.H.M. with this fixed point as its mean position. The period of this S.H.M. can

be expressed in terms of the acceleration at any known distance from the fixed point, even if no other property of the motion is given.

The moving point would correspond with the point N in the preceding discussion. For any given value of the amplitude of the motion of N, we may imagine a point P moving round in a circle as therein described.

The angular velocity of this point P will require to be some value ω such that the acceleration of $N = \omega^2 \times$ displacement of N.

Now the acceleration of the point is given by some equation of the form

$$\text{Acceleration} = Ax.$$

The particular value of ω to fit this equation is obviously $\omega = \sqrt{A}$, and hence the period of the S.H.M.

is $$\frac{2\pi}{\sqrt{A}}.$$

In Dynamics it is frequently found that the force acting on a particle is directed towards a fixed point for all positions of the particle, and is directly proportional to the distance of the particle from that point. It is easy to see that if the particle move in a straight line passing through the fixed point its motion must be S.H.M.

For let the force be μx, where μ is a constant and x is the displacement. The constant μ is the value of the force when the displacement is unity. If the mass of the particle be m, the acceleration a, in accordance with Newton's Second Law, is given by

$$ma = \mu x,$$

or $$a = \frac{\mu}{m}x;$$

i.e. the acceleration is directly proportional to the displacement, and the motion is S.H.M.

We see that μ/m replaces A, ω^2, or $(v/a)^2$ in the preceding discussion.

The Period, t, can be written down at once

for $$t = \frac{2\pi}{\sqrt{A}}, \frac{2\pi a}{r}, \text{ or } \frac{2\pi}{\omega},$$

i.e. $$t = 2\pi\sqrt{\frac{m}{\mu}}.$$

Notice that **the period t is independent of the amplitude a.**
The equation for the period

$$t = 2\pi\sqrt{\frac{m}{\mu}}$$

is of far-reaching application. If we know the mass of a body, and the force acting on it in terms of the displacement of the body from its mean position, the period can be determined at once from this relation.

μ is often spoken of as the **force for unit displacement**, *i.e.* μ is the value of the force that would act on the body if it were displaced one centimetre from its mean position.

§ 2. Angular Simple Harmonic Motion

The analogy between certain quantities connected with linear motion and other quantities connected with angular motion has already been pointed out (see the Table on p. 145). This may be made use of in dealing with linear and angular S.H.M.s. Thus we may at once deduce the following statement:—

When a body rotates about an axis under a couple which is proportional to the angular displacement from a certain position, the body executes an Angular S.H.M.

Again by simple analogy, if the couple acting on a body is related to its angular displacement by the equation

$$\text{Couple} = c\theta,$$

the period of the Angular Simple Harmonic Motion executed is

$$t = 2\pi\sqrt{\frac{I}{c}},$$

I being the **Moment of Inertia** of the body about the axis of rotation.

The coefficient c is frequently spoken of as the **couple for unit twist**, *i.e.* c is the value of the couple that would act on the body if it were displaced *one radian* from its mean position.

§ 3. Examples of Periodic Motion

The types of periodic motion met with in practice are rarely pure Simple Harmonic Motions. They may, however, be considered as Simple Harmonic Motions in many cases, provided the extent of motion allowed does not exceed certain small limits. One of the most important cases of periodic motion is that of a pendulum. Not only is the pendulum widely used for time-keeping purposes, but very important results in Physics can be obtained from determinations of pendulum periods.

PERIOD OF A SIMPLE PENDULUM

A simple pendulum consists of a heavy particle of matter suspended from a perfectly rigid point of support by a flexible, weightless, inextensible string. When displaced to one side of its mean position O, the 'bob' swings back towards O along the arc, under the action of the forces on the bob (Fig. 84). The only force which has any component along the arc is the weight of the bob, and the component of this tending to set the body moving back towards O is $mg \sin \theta$.

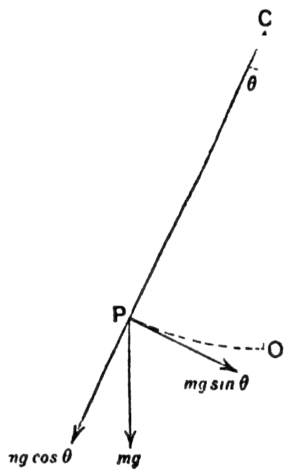


FIG. 84.—Forces on Bob of Simple Pendulum.

Thus the tangential force on the bob is given by

$$f = mg \sin \theta.$$

If the angle of displacement θ *is very small* we may write $\theta = \sin \theta$, and then

$$f = mg\,\theta.$$

Now if the displacement of the bob along the arc is x

and the length of the pendulum is l,

$$\theta = \frac{x}{l},$$

and $$f = \frac{mg}{l}x.$$

This is an equation of the form $f = \mu x$, and hence the motion of the bob is a Simple Harmonic Motion if the displacement x is never large. Its period is given by

$$t = 2\pi\sqrt{\frac{m}{\mu}} \text{ where } \mu = \frac{mg}{l}.$$

Hence $$t = 2\pi\sqrt{\frac{l}{g}}.$$

PERIOD OF A COMPOUND PENDULUM

A body the mass of which is distributed throughout its volume can be used as a pendulum by allowing it to oscillate about some axis. Suppose a body to be suspended from the axis O (Fig. 85).

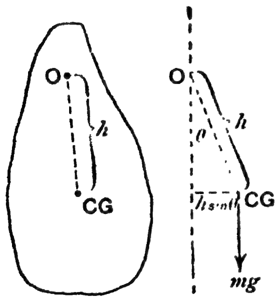


FIG. 85.—Compound Pendulum.

If displaced sideways, its weight acts downwards through its Centre of Gravity and exerts a restoring moment on the body, the restoring moment about the axis O being $mgh \sin \theta$, when the body is displaced through an angle θ.

If θ is small we may write $\theta = \sin \theta$, and then the

$$\text{Restoring Couple} = mgh\ \theta.$$

This is of the form, couple $= c\theta$, where $c = mgh$. The body thus executes an angular S.H.M. about the axis through O, its period being given by

$$t = 2\pi\sqrt{\frac{I_0}{mgh}}.$$

I_0 is the Moment of Inertia about the axis through O, and

can be expressed as $I_o = m(k^2 + h^2)$, where k is the radius of gyration about the Centre of Gravity.

Hence $$t = 2\pi\sqrt{\frac{m(k^2 + h^2)}{mhg}},$$

i.e. $$t = 2\pi\sqrt{\frac{k^2 + h^2}{hg}}.$$

PERIOD OF AN OSCILLATING MAGNET

When a magnet of pole strength m is suspended in a field of strength H, each pole is subject to a force mH.

When the magnet is displaced through an angle θ from its mean position, these forces exert a couple on the magnet

$$= mH \times 2l \sin\theta,$$

the length of the magnet being $2l$.

$2lm$ is called the Magnetic Moment of the magnet, and is denoted by M. Hence

$$\text{Couple} = MH\sin\theta.$$

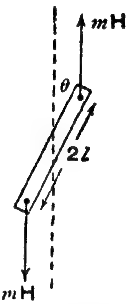


Fig. 86.—Couple on Magnet.

This reduces to MHθ, *if the swings are small*, and we find at once for the period of swing

$$t = 2\pi\sqrt{\frac{I}{MH}},$$

I being the Moment of Inertia of the magnet about its axis of rotation.

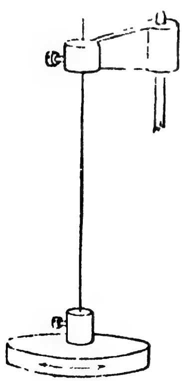


Fig. 87.—Torsion Pendulum.

PERIOD OF A TORSION PENDULUM

In all the foregoing examples of periodic motion, the motion has only been a Simple Harmonic Motion if the angle of swing has been small. **In the case of a Torsion Pendulum the motion is accurately Simple Harmonic, even for large angular displacements.**

If the upper end of a wire is fixed, and the lower end twisted

through an angle θ radians, the restoring couple exerted by the wire is equal to $\frac{\pi n r^4 \theta}{2l}$, where r = radius of wire (*see* Determination of Modulus of Rigidity, p. 119). Hence a body of Moment of Inertia I, supported from a wire in this way, would execute a Simple Harmonic Motion, the period of which is given by

$$t = 2\pi\sqrt{\frac{I}{\pi n r^4 / 2l}},$$

or

$$t = 2\pi\sqrt{\frac{2lI}{\pi n r^4}}.$$

With a body of known Moment of Inertia, this result could be used for measurement of the modulus of rigidity n by an oscillation method.

PERIOD OF MASS SUSPENDED FROM SPIRAL SPRING

The periodic motion of a mass suspended from a spiral spring furnishes another example of a pure Simple Harmonic Motion. Suppose a mass M, suspended from a spring, extends the spring l cm. Then the force exerted by the spring is just equal to the weight, Mg, of the suspended mass, and the force for unit extension is Mg/l. If the spring is stretched a little further, say 1 cm., the extension being now equal to $l + 1$ cm., the spring will pull upwards on the mass with a force equal to

$$\frac{Mg}{l} \times (l+1) \text{ dynes.}$$

The forces acting on the mass are now (a) the *upward* force, $Mg\left(1 + \frac{1}{l}\right)$, exerted by the spring, and (b) its weight, Mg, acting *downwards*, the resultant being Mg/l *upwards*. Thus, when displaced 1 cm. from its steady position, the mass is pulled upwards by a force = Mg/l dynes. This is the force for unit displacement.

The period of vibration will therefore be

$$2\pi\sqrt{\frac{M}{Mg/l}} = 2\pi\sqrt{\frac{l}{g}},$$

where l is the steady elongation produced by the suspended mass.

§ 4. Experiments on Periodic Motions

DETERMINATION OF THE ACCELERATION DUE TO GRAVITY BY MEANS OF A SIMPLE PENDULUM

The direct determination of the acceleration due to gravity is a matter of considerable difficulty, if even approximate accuracy is required, and therefore other, less direct, methods are usually employed. One of the easiest of these is by using a simple pendulum, observations being taken of the times of vibration for different lengths, and the values thus observed substituted in the formula (p. 160),

$$t = 2\pi\sqrt{\frac{l}{g}},$$

which connects the time of a complete vibration, t, with the length of the pendulum, l, g being the acceleration due to gravity.

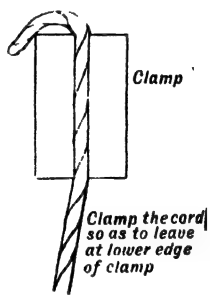


FIG. 88.—Simple Pendulum.

EXPT. 66. **Determination of *g* by the Simple Pendulum.**—The theoretical simple pendulum consists of a heavy particle of infinitely small dimensions suspended from an inextensible massless fibre held in a perfectly rigid clip at the upper end.

In practice, we usually employ a small heavy ball or 'bob' of metal suspended from a fine strong cord such as is used for fishing. The upper end of the cord is clamped as rigidly as possible. The length of the pendulum is the length from the point *where the cord leaves the clamp*, to the *centre* of the bob. *Note.*—The string leaves the clamp at the lower edge.

The time of a complete vibration backwards and forwards is the time between the 'bob' passing through the centre

of its swing *in the same direction* on two consecutive occasions. The centre of the swing should be marked in some way, *e.g.* by means of a vertical chalk line on an object close behind the cord. In order to determine the period accurately, a large number of swings, say 50, should be made by the pendulum, and the total time taken for these should be observed. Suppose 50 swings take T secs., the period is T/50 secs.

In timing the swings with a stop-watch it is advisable to start counting by calling the first passage of the cord past the mark 3, and then to count *backwards*, thus

$$3,\ 2,\ 1,\ \overset{\downarrow}{0},\ 1,\ 2,\ 3\ \ldots\ \overset{\downarrow}{50}.$$

The stop-watch is started at passage 0 and stopped at 50. This avoids the error made by calling the first passage 1 and after counting on to 50, saying that 50 swings have been made, while in reality only 49 periods have been observed. The method also enables the observer to learn the *rhythm* of the swings before actually beginning to time them.

Take various lengths of string, from 30 cm. to about 100 or 120 cm., *not less than* **six** different lengths being used, and observe the period for each length. **The angle of swing must be small.**

Enter the observations thus:

Length l in cm.	Time T for 50 swings.	Period t in secs.	t^2.	$\frac{l}{t^2}$	
					Mean Value of $\frac{l}{t^2}$ . . .

The mean value of l/t^2 (given in the last column) is now used to calculate a value of g, the acceleration due to gravity. This, the important experimental result, should be entered *prominently* at the conclusion of the account of the experiment as

$$\text{Acceleration due to Gravity, } g = 4\pi^2\left(\frac{l}{t^2}\right) = \quad \text{cm. per sec. per sec.}$$

Plot a graph using values of l as abscissæ, and the corresponding

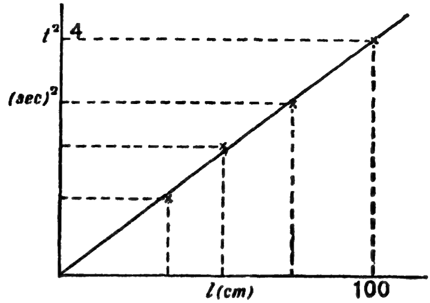


FIG. 89.—Graph for Simple Pendulum.

values of t^2 as ordinates. This should be a straight line passing through the origin, showing that l is proportional to t^2.

DETERMINATION OF g BY SPHERE ON CONCAVE MIRROR

An interesting example of periodic motion, involving the Moment of Inertia of the moving body, is afforded by the present experiment. In the case of a simple pendulum the motion of the 'bob' is along the arc of a circle of radius equal to the length of the string, the motion being purely translational, *i.e.* there is no rotation of the bob about its Centre of Gravity, but only to-and-fro motion.

A sphere rolling over a concave mirror has a motion similar in all respects to the motion of the bob of a simple pendulum, with the exception that the sphere *rolls*, *i.e.* rotates as it moves forward. Thus, in dealing with a sphere rolling over a concave mirror, we must take the motion of rotation into account, as well as that of translation. In either case, the Kinetic Energy of the moving body at any point is equal to the Potential Energy it has lost in falling from the end of its swing to the point in question.

In the case of the simple pendulum, where the motion is one of pure translation, the Kinetic Energy is $\frac{1}{2}mv^2$ and the Potential Energy lost $= mgh$;

$$\tfrac{1}{2}mv^2 = mgh,$$

or

$$v^2 = 2gh.$$

In the case of the sphere rolling on the mirror the Kinetic Energy $= \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega^2$, and the Potential Energy lost $= mgh$;

$$\therefore \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}I\omega^2 = mgh;$$

Now the Moment of Inertia (I) of a sphere about its centre is $\frac{2}{5}ma^2$, and the equation becomes

$$mgh = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}\left(\tfrac{2}{5}ma^2\right)\omega^2.$$

Again, the point of contact with the surface is at rest, therefore the linear velocity of the centre (v_1) is equal to the angular velocity (ω) times the radius of the ball (a),

i.e. $$v_1 = \omega a \text{ (see p. 151)},$$
or $$v_1^2 = \omega^2 a^2.$$

The equation with regard to the motion of the sphere on the concave mirror thus reduces to the form

$$\begin{aligned} mgh &= \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}\cdot\tfrac{2}{5}m(a^2\omega^2), \\ &= \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}\cdot\tfrac{2}{5}mv_1^2, \\ &= \left(\tfrac{7}{10}\right)mv_1^2, \end{aligned}$$
or $$v_1^2 = \left(\tfrac{5}{7}\right)2gh.$$

Thus, if a simple pendulum bob and a sphere on a concave mirror trace out similar paths, the velocity of the sphere is always less than the velocity of the pendulum bob in the same position.

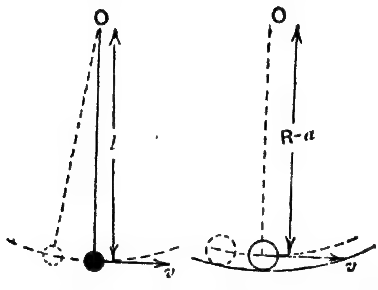


FIG. 90.—Simple Pendulum and Sphere on Concave Mirror.

If v = the velocity of the pendulum bob, and v_1 = the velocity of the sphere at the same point, then $v^2 = 2gh$, and

$$v_1^2 = \tfrac{5}{7}(2gh) = \tfrac{5}{7}v^2,$$

so that at all points in its path the velocity of the rolling sphere is only $\sqrt{\frac{5}{7}}$ of the velocity of the pendulum bob.

Therefore, to complete any given motion, the sphere will take $\sqrt{\frac{7}{5}}$ the time taken by the more rapidly moving pendulum bob.

Now the period of a simple pendulum is $t = 2\pi\sqrt{\dfrac{l}{g}}$.

Therefore in going over a similar path, the period of the rolling sphere (t_1) will be $t \times \sqrt{\frac{7}{5}}$, *i.e.* $t_1 = 2\pi\sqrt{\frac{7}{5}}\sqrt{\frac{l}{g}}$.

The path traced out by the sphere is a curve of radius $(R - a)$ where $R =$ the radius of curvature of the mirror and $a =$ the radius of the sphere. The length of the simple pendulum, the bob of which would trace out a similar path, is $(R - a)$.

The period of the similar simple pendulum is thus

$$t = 2\pi\sqrt{\frac{(R-a)}{g}}.$$

The period of the sphere on the mirror is $\sqrt{\frac{7}{5}}$ of this, *i.e.*

$$t_1 = 2\pi\sqrt{\frac{\frac{7}{5}(R-a)}{g}}.$$

Expt. 67. **Determination of *g* by Sphere on Concave Mirror.**—Measure the radius of curvature of the mirror by means of a spherometer, and the radius of the ball by callipers. Wipe away all dust from the surfaces of the sphere and the mirror. Take the time for 10 or 20 complete oscillations of the ball on the mirror by means of a stop-clock, and deduce the time of vibration. Repeat these observations for determining the period three times.

Calculate g from the formula

$$t_1 = 2\pi\sqrt{\frac{\frac{7}{5}(R-a)}{g}},$$

or

$$g = \frac{4\pi^2\frac{7}{5}(R-a)}{t_1^2}$$

COMPOUND PENDULUM

When a rigid body is supported so that it can turn about a horizontal axis, it will oscillate about an equilibrium position (see the discussion on p. 160).

Let $I_G = Mk^2$ denote the Moment of Inertia of the body about a horizontal axis through its centre of gravity, G (Fig. 91).

The body is supposed to be oscillating about an axis at right angles to the plane of the paper through O. The point O is called the 'centre of suspension.' The Moment of Inertia

about the axis through O is given by $I_o = I_G + Mh_1^2 = M(k^2 + h_1^2)$, where $h_1 = OG$.

The time of vibration of the compound pendulum has been shown on p. 161 to be

$$t = 2\pi\sqrt{\frac{I_o}{mgh_1}}, \text{ or } 2\pi\sqrt{\quad h_1^2}$$

The length of the simple pendulum that vibrates in the same period is

$$l$$

This is called the length of the **simple equivalent pendulum.**

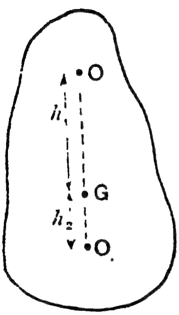


FIG. 21.—Compound Pendulum.

If the whole mass of the body could be concentrated at this distance from the axis of suspension, at a point in the line joining OG produced, the period of oscillation and the equilibrium position would be unaltered. O_1 would be such a point, if the distance OO_1 is equal to l. **O_1 is called the Centre of Oscillation corresponding with the Axis of Suspension at O.**

The distance h_2 from G to O_1 is given by the relation $h_1 + h_2 = l$.

The centre of suspension and the centre of oscillation are interchangeable, for since $h_1 + h_2 = l$,

$$h_1 + h_2 = \frac{k^2 + h_1^2}{}$$

i.e. $$h_1 h_2 = k^2. \quad . \quad . \quad (1)$$

Adding h_2^2 to each side, we have

$$h_2^2 + h_1 h_2 = k^2 + h_2^2,$$

or $$h_2 + h_1 = \frac{k^2 + h_2^2}{h_2}. \quad . \quad (2)$$

Now $\frac{k^2 + h_2^2}{h_2}$ is the length of the simple pendulum, which has the same period as that of this body when suspended about O_1.

This is equal to $h_1 + h_2$ by equation (2), *i.e.* **the period will be the same whether the body is suspended about O or O_1.** This is equivalent to saying that the centre of suspension and the centre of oscillation are interchangeable.

Locus of points for which t is constant.—There are other axes about which the body will oscillate in the same period as about O and O_1.

If we describe two circles with centre G and radii h_1 and h_2, any parallel axis of oscillation taken on either of these circles would give the same value for t.

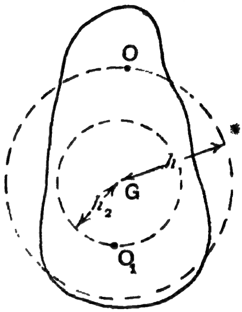


Fig. 92.—Locus of Points for which t is constant.

Variation of t with h—Minimum Period.—When the axis of suspension passes through the Centre of Gravity, the periodic time becomes infinitely great. If the axis is at an infinite distance the periodic time is again infinite. Consequently there must be some intermediate position for which the periodic time is a minimum.

Now $$t = 2\pi\sqrt{\frac{k^2 + h^2}{hg}}.$$

This will be a minimum when $\frac{k^2 + h^2}{h}$ is a minimum.

But $$\frac{k^2 + h^2}{h} = \frac{(k - h)^2 + 2kh}{h},$$

$$= \frac{(k - h)^2}{h} + 2k.$$

This is clearly a minimum when $h = k$.

The length of the simple equivalent pendulum in this case $= 2k$, and the points O and O_1 are at distances from G each equal to k.

The minimum period is $t_0 = 2\pi\sqrt{\frac{2k}{g}}$

EXPT. 68. **The Compound Pendulum.**—To illustrate these results, a bar about 1 metre long, in which holes have been bored at equal distances (about 2 cm.) along its length, may be suspended from an axis which must be made horizontal (Fig. 93).

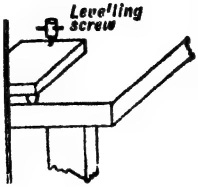


FIG. 93.—Bar Pendulum.

i. Take the periodic time for every third hole, starting from one end of the bar, noting the time for 50 complete swings.

ii. Plot a curve showing the periodic time at different distances from the Centre of Gravity. It will consist of two symmetrical branches corresponding to the two halves of the bar, showing a minimum period for the points A and B.

iii. Find the holes on the bar corresponding to these minimum periods and determine *very accurately* the period for the two holes on either side of that

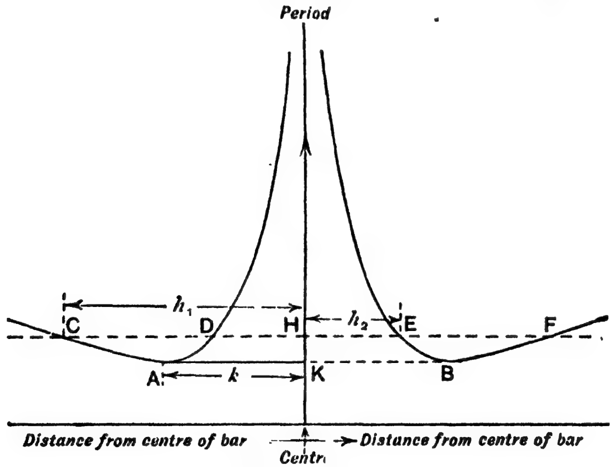


FIG. 94.—Graph for Compound Pendulum.

giving t a minimum, and also for the hole corresponding with the minimum itself. At least 100 swings should be taken for each of these five holes, so as to get very exact points on the curve in this part. Find k from the positions of A and B:

$$k = AK = BK.$$

iv. Find points such as C, D, E, and F in Fig. 94, for which the periodic time is the same.

Then if CH is taken as h_1, HE is h_2.

Also $$\mathrm{HF} = h_1 \text{ and } \mathrm{HD} = h_2.$$

Calculate the radius of gyration from the formula

$$k^2 = h_1 h_2.$$

Find the length of the simple equivalent pendulum

$$l = h_1 + h_2,$$

and calculate g from the formula

$$t = 2\pi\sqrt{\frac{l}{g}}$$

v. Find the minimum periodic time t_0 corresponding to the points A, B in the graph and calculate k from the formula

$$t_0 = 2\pi\sqrt{\frac{2k}{g}},$$

assuming $g = 981$ cm./sec.^{2}.

vi. Find the mass M of the body and calculate its Moment of Inertia

$$\mathrm{I} = \mathrm{M}k^2.$$

The values of k obtained in iii., iv., and v. should all agree, and should be approximately equal to the length of the bar divided by $\sqrt{12}$, if the breadth of the bar is negligible compared with its length (Appendix, p. 595).

EXPT. 69. **Determination of Modulus of Rigidity by Oscillation.**—Suspend a bar or a disk, or some other body of *known* Moment of Inertia, from a wire which is fixed firmly at its upper end.

Determine the period of oscillation of the body when moving as a torsion pendulum.

Measure the length of the wire and its radius. Calculate the Moment of Inertia of the suspended body from its mass and dimensions.

Deduce the value of the Modulus of Rigidity from the equation

$$t = 2\pi\sqrt{\frac{2\mathrm{I}l}{\pi n r^4}},$$

or $$n = \frac{8\pi \mathrm{I}l}{t^2 r^4}.$$

Compare the Moments of Inertia of two bodies by using them as torsion pendulums from the *same* wire

$$t_1 = 2\pi\sqrt{\frac{2l}{\pi n r^4} I_1}, \qquad t_2 = 2\pi\sqrt{\frac{2l}{\pi n r^4} I_2}$$

The same wire is used, $\therefore \frac{I_1}{I_2} = \frac{t_1^2}{t_2^2}$

EXPT. 70. **Determination of g by observing the Period of Oscillation of a Mass suspended from a Spiral Spring.**—Suspend a mass from a spiral spring. Note the elongation l it produces when applied gently. Observe the period of oscillation of the mass when vibrating vertically.

Calculate g from the expression (proved earlier, p. 162)

$$t = 2\pi\sqrt{\frac{l}{g}}.$$

CHAPTER X

GASES: THE BAROMETER AND BOYLE'S LAW

§ 1. Properties of Gases

A **gas** differs from the other forms of matter in that it **always fills any space in which it is placed.** However small the quantity of gas contained in an enclosed volume, the gas distributes itself so as to fill the volume. This property is the most striking property possessed by substances in gaseous form, and may be described as their **expansibility.**

In the present chapter we shall study only the gaseous phenomena associated with constant temperature, leaving the investigation of the effects of change of temperature to the section on Heat.

BOYLE'S LAW

When the volume of a fixed mass of gas is changed, the *pressure* exerted by the gas alters in a definite way, and, **provided the temperature is maintained constant, the pressure varies inversely as the volume.** This relation may be stated conveniently in the form

Pressure × Volume = Constant.

This law was first enunciated by Robert Boyle in 1662, and is known generally as Boyle's Law. It was stated, however, by the French physicist Mariotte fourteen years later, and is known on the Continent as Mariotte's Law.[1]

[1] See Tait's *Properties of Matter*, Appendix IV.

It will be seen that, in order to investigate the phenomena presented by a gas at constant temperature, we must be able to measure its **volume** and its **pressure.**

The measurement of volume presents little difficulty, but that of pressure requires some explanation.

§ 2. Measurement of Atmospheric Pressure

Before dealing with the measurement of the pressure of gases in confined volumes, the first point to realise is that the air exerts a considerable pressure.

THE BAROMETER

To show this, a long tube of glass about 1 metre in length is closed at one end; it is then filled with mercury, and inverted so that the unclosed end is beneath the level of a bath of mercury in a trough. At once the mercury falls down a little way from the top of the tube, although no air or any other substance is admitted. The mercury does not run out of the tube entirely; a considerable column, about 75 cm. in height, remains in the tube, this column being supported by the pressure of the atmosphere. **A Tube of this construction is called a Barometer Tube.**

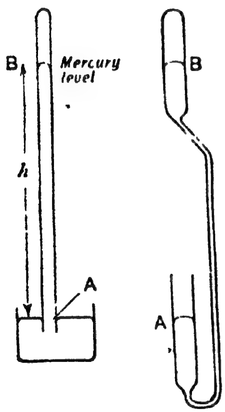


FIG. 95.—Simple Forms of Barometer.

Consider the pressure at the point A in the tube (Fig. 95) at the same level as the *free surface* of the mercury outside. Above this point is a column of mercury h cm. high and of density ρ, exerting a pressure at the point A equal to $h\rho g$ dynes per sq. cm. Above the mercury inside the tube, the space B is vacuous, except for a minute trace of mercury vapour, the effect of which is negligible. The total pressure exerted at the point A is therefore that due to the column of liquid alone,

i.e. Pressure inside tube at A $= h\rho g$ dynes per sq. cm.

Outside the tube, at the surface of the liquid, the only pressure acting is the pressure of the external atmosphere.

Now, **at all points on the same level in a liquid, there is the same pressure,** hence **the pressure at A inside the tube is exactly equal to the pressure at the surface outside,** since A is at the same level as the external surface.

The pressure at A is $h\rho g$; the pressure at the surface of the liquid outside is the atmospheric pressure.

Therefore the Atmospheric Pressure $= h\rho g$ dynes per sq. cm.

The Density of Mercury is approximately constant over the small range of temperature through which the atmosphere varies, and it is customary, therefore, to speak of the **pressure in cm. of mercury.** Strictly speaking, the expression of the pressure in this way should be in terms of the length of a column of mercury **at 0° C.**, but for ordinary purposes the variation of density is so small that the correction for temperature need not be applied. When required, it can be calculated without much difficulty (p. 180).

Further, the value of g is not uniform over the whole of the earth's surface, and a correction for latitude and height above sea-level should be introduced, to bring the height of the mercury column to what it would be at sea-level in latitude 45°. This correction is never required except in work of the highest degree of accuracy.

Since ρ and g may be considered approximately constant, **the Atmospheric Pressure can be described as equivalent to a certain height *h* of mercury, this height being called the Barometric Height.** It is the height of the column of mercury supported by the atmosphere in a barometer tube constructed as described.

The Meteorological Office now expresses atmospheric pressure in units that are multiples of the absolute C.G.S. unit, and some modern barometers are graduated so that the pressure can be read directly in such units.

The unit of pressure used in practice is called the **bar**, and is equal to **one million dynes per sq. cm.** Two smaller units are also used, the **centibar** and the **millibar**, being respectively **one-hundredth** and **one-thousandth** of a **bar**.

The **bar** is equivalent to a pressure of 75·01 cm. of mercury at 0° C. in latitude 45°.

A standard atmosphere (76 cm. of mercury) is rather greater than one bar, being equal to 1013·2 millibars.

COMMON TYPES OF BAROMETER

U-Tube Form.—For rough work a simple U-tube form of barometer is sufficiently good (Fig. 95). The free surface in A corresponds with the surface in the trough in the type already described. The tube is bent near the top, so that the two mercury surfaces of which the difference in level is to be measured are in the same vertical line.

These portions of the tube should be of fairly wide diameter, and equal to each other in bore, so as to avoid any error in the height observed due to surface tension effects. The heights are read on a scale usually engraved on the tubes themselves, and the Barometric Height is the difference between the levels B and A.

The accuracy obtainable with this form is not very great: the error in each reading may be as much as half a millimetre. As *two* readings have to be taken, the possible error is thus equal to one millimetre.

Fortin's Barometer.—This barometer is usually to be found in physical laboratories where accurate observation of atmospheric pressure is required. The apparatus does not differ from that of the simple type of barometer tube first described (p. 174), except in the manner of taking the readings of the two levels of mercury.

EXPT. 71. **Reading Fortin's Barometer.**—To determine the height of the mercury column two adjustments are necessary.

(i.) **Adjustment of the Reservoir.**—At the bottom end of the tube the mercury is carried in a leather bag, which can be altered in shape by a screw A (Fig. 96) bearing against its base. Fixed to the framework of the barometer is a small ivory point P which is the zero of the barometer scale.

The mercury surface is adjusted up to this ivory point by turning the screw A, the adjustment being made until the point and its image (reflected in the mercury) are just seen to touch *without any depression being visible in the mercury surface* at the point (Fig. 98). An extremely fine adjustment is thus rendered possible, if the mercury surface is suitably illuminated.

(ii.) **Adjustment at the Upper Surface.**—At the upper surface the adjustment is not quite so simple. Round the glass tube is fitted a brass tube S, which is moved up and down by a milled head B (Fig. 96) at the side of the apparatus. This tube has the bottom edge cut so that the back **D** and front C of the bottom edge are exactly on the same level (Fig. 100). If the eye is placed *below* the level of these two, the back edge can be seen as well as the front. As the eye is raised, the back edge gradually shows less and less, until *when the eye is exactly level with the bottom of this movable tube, the back edge is just covered by the front edge.*

The eye should be placed in

Fig. 96.—Fortin's Barometer.

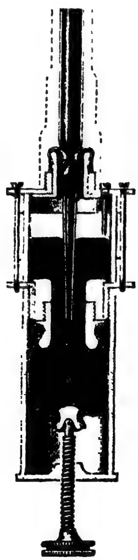


Fig. 97.—Reservoir of Fortin's Barometer.

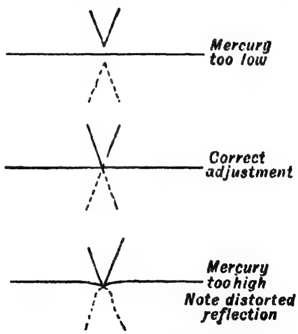


Fig. 98.—Adjustment of Reservoir.

such a position that the back edge is just *covered by* the front edge in this way.

The brass tube should then be moved until the front edge is just on a level with the top B (Fig. 100) of the curve or 'meniscus' formed by the mercury surface, keeping the eye exactly level with the edge of the tube as it moves.

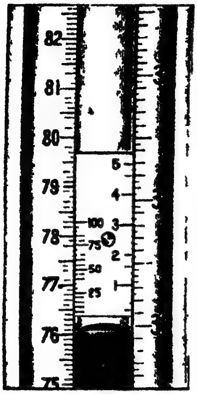


FIG. 99.—Barometer Scales and Verniers

NOTE.—If the eye is placed too high, the back edge will be covered by the front edge and the adjustment will be inaccurate. It is therefore important to *raise the eye until the back edge only just disappears behind the front*: this is the *only* test of accurate level.

When the movable tube has been adjusted accurately, the slightest movement of the eye downwards should bring the back edge of the tube into view. When the eye is raised to the correct level again, the middle of the front edge should just be tangential to the surface of the mercury, a little light being seen through between the edge and the mercury surface at the sides.

The small brass tube carries a vernier scale the zero of which is level with the lower edge: this edge usually projects downwards

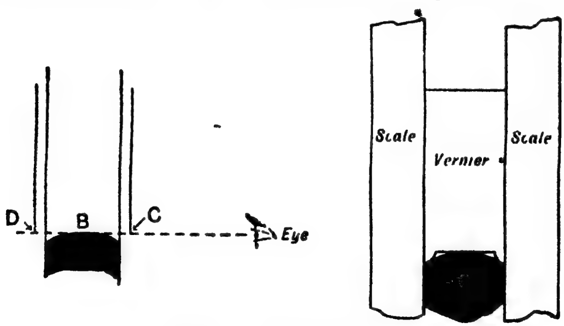


FIG. 100.—Adjustment of Vernier.

just at the sides to avoid wear on the corner. The frame of the instrument carries a scale along which the vernier slides, and the reading of the vernier on this scale will give the position of the surface of the mercury. The scale is graduated only over a few centimetres near the top, but its zero is the ivory point in

the lower reservoir. Hence the reading of the scale shown by the vernier gives the Barometric Height.

Read and record the Barometric Height as given by the scale and vernier.

Among other forms of barometer in common use is the recording U-tube type, which records the motion of the level in the lower tube by a float which is geared to a pointer. Another form is like Fortin's in construction, but has no adjustment of the reservoir, the divisions on the scale at the top being made not quite true inches or cm., in order to compensate for the alteration in level in the reservoir. Neither type is of any value for scientific work.

The Aneroid Barometer.—A very convenient type of barometer, and one which, in its modern form, is capable of considerable accuracy, is the Aneroid. It consists of a metal vessel which is evacuated completely and hermetically sealed. Any variation in barometric pressure will deform this vessel somewhat, the deformation produced being proportional to the change of pressure (see Hooke's Law, p. 109). By levers and watch-gearing, this slight deformation is magnified so as to move a pointer over a scale, and by the motion of this pointer the variation in the Barometric Height can be obtained.

The apparatus is of course not *absolute*; it has to be calibrated by comparing its indications with a mercury barometer of the Fortin type, but when once calibrated, it can be relied on to give consistent readings for an almost unlimited time, and with well-made watch-gearing is quite free from 'back-lash.' As it can be constructed in a compact form, it is extremely useful where portability is desirable, though if subjected to considerable variations of temperature its readings will not be quite accurate, as the elasticity of the metal vessel is affected by temperature.

EXPT. 72. **Measurement of the Height of a Building using an Aneroid Barometer.**—Take an Aneroid Barometer with a finely divided scale and observe the *difference* between its readings when at the bottom and at the top of a building. Let the observed difference be *x cm. of mercury*.

This difference corresponds with a difference of level in air equal to the height of the building h (say). For small differences in level the air may be treated as a fluid of approximately uniform density. The difference in pressure between the two points would then be $h\rho_1 g$, ρ_1 being the density of the air.

This difference in pressure has been measured, and is found to be that exerted by a column of mercury x cm. long.

Thus $h\rho_1 g = x\rho g$, where ρ is the density of mercury.

Neglecting slight variation of ρ_1 due to temperature, we may with sufficient accuracy take $\rho = 13{\cdot}6$ and $\rho_1 = 0{\cdot}00129$, both in gm. per c.c., and we obtain

$$h = x \cdot \frac{13{\cdot}6}{0{\cdot}00129}.$$

Verify the result obtained by actual measurement.

Aneroid Barometers for mountaineering are often graduated directly in *feet* or *metres*.

CORRECTION OF A MERCURY BAROMETER FOR TEMPERATURE

The reading of a barometer requires correcting for temperature, in order to express the pressure of the atmosphere in cm. of mercury at 0° C., or in dynes per sq. cm.

Let the Barometer Reading be H cm. This reading is really not obtained in cm. but in *scale divisions*, the scale divisions being cm. only at some temperature t_0° C. (usually 15° C.). If the temperature of the room is t° C., each division is of length $\{1 + b(t - t_0)\}$ cm., where b is the coefficient of *linear* expansion of the scale (usually of brass).

The *actual height* of the mercury column is thus

$$H_1 \text{ cm.} = H\{1 + b(t - t_0)\}.$$

We have therefore a column of mercury of height H_1 cm. at a temperature t° C.

It is required to find what height of mercury H_0 at 0° C. would exert the same pressure as this column H_1 exerts at t° C.

The pressure of H_0 cm. of mercury at 0° C. is

$$H_0\rho_0 g \text{ dynes per sq. cm.},$$

ρ_0 being the density of mercury at 0° C.

The pressure of the column H_1 cm. high at t° C. is

$$H_1\rho_1 g \text{ dynes per sq. cm.},$$

ρ_1 being the density of mercury at t° C.

We have to find H_0 such that

$$H_0\rho_0 g = H_1\rho_1 g.$$

Now $\rho_0 = \rho_1(1 + at)$, where a is the coefficient of cubical expansion of mercury.

Hence $$H_0 = \frac{H_1\rho_1}{\rho_0} = \frac{H_1}{1+at},$$

i.e. substituting for H_1 in terms of the reading H we have

$$H_0 = \frac{H\{1+b(t-t_0)\}}{1+at}.$$

When this value H_0, the equivalent height of mercury at 0° C., has been calculated, the pressure in dynes per sq. cm. can be obtained from the equation

$$P = H_0\rho_0 g.$$

ρ_0 is 13·596 gm. per sq. cm., g is 981·18 dynes per gm. or cm. per sec. per sec. (in London).

Hence $P = H_0 \times 13{\cdot}596 \times 981{\cdot}18$ dynes per sq. cm.

A numerical example may be of assistance in explaining this method of correcting the Barometric Height for temperature.

A barometer with a brass scale reads 75·933 cm. at 18° C. The scale is graduated to be accurate at 15° C. What is the height of the barometer reduced to 0° C.? Also what is the pressure of the atmosphere in dynes per sq. cm.?

The coefficient of linear expansion of brass = 0·0000189 per 1° C.

The coefficient of cubical expansion of mercury = 0·000180 per 1° C.

$$H_0 = \frac{75{\cdot}933(1+0{\cdot}0000189(18°-15°))}{(1+0{\cdot}00018 \times 18)}$$

$$= \frac{75{\cdot}933(1+0{\cdot}0000567)}{1+0{\cdot}00324}.$$

This can be written as

$$H_0 = 75{\cdot}933(1+0{\cdot}0000567)(1-0{\cdot}00324),$$

and then as $$H_0 = 75{\cdot}933(1+0{\cdot}0000567-0{\cdot}00324)$$

to a very close degree of accuracy, and we obtain

$$H_0 = 75{\cdot}933(0{\cdot}99682),$$

whence $$H_0 = 75{\cdot}690 \text{ cm.}$$

The pressure in dynes per sq. cm. in the example given is

$$P = H_0\rho_0 g \text{ dynes per sq. cm.}$$
$$= 75{\cdot}690 \times 13{\cdot}596 \times 981{\cdot}18$$
$$= 1{,}009{,}700 \text{ dynes per sq. cm.}$$
or 1009·7 millibars.

EXPT. 73. **Determination of the Atmospheric Pressure in Absolute Units.**—Read the height of the barometer as in Expt. 71. Read the temperature of the barometer by means

of the thermometer attached. Apply the correction for temperature as in the foregoing example, so as to find the pressure of the atmosphere in cm. of mercury at 0° C., and deduce the pressure in absolute units.

Correction Table for Thermometer Reading.—It is convenient to calculate in this way the correction that has to be applied at each temperature from 0° to 25° C., and to have this arranged by the side of the thermometer for reference. If the corrections are calculated for a reading assumed to be 76·0 cm., they will be sufficiently accurate to apply to all ordinary barometric readings without modification.

Formula for Correction at any Temperature.—The correction is expressed sometimes also in the form: **deduct B cm. from the reading obtained, and from the remainder subtract C cm. for every degree above 0° C.** A formula of this type can be worked out without much difficulty as an exercise on the equation

$$H_0 = \frac{H(1 + b(t - t_0))}{1 + at}.$$

This gives $$H_0 = H(1 - bt_0 - (a - b)t).$$

B (above) is Hbt_0, an approximately constant quantity, and is worked out for $H = 76$ cm.

C (above) is $H(a - b)$, and is also approximately constant; it is worked out on the supposition that $H = 76$ cm.

§ 3. Pressure of a Gas in a Closed Volume

The measurement of the pressure of a gas in a closed volume is usually achieved by means of a U-tube containing mercury. One side of this communicates with the chamber within which the pressure is to be measured, and the other is open to the atmosphere.

The *difference in level between the mercury surfaces in the U-tube indicates the difference between the pressure inside the space and the atmospheric pressure outside.*

Thus, if the pressure inside the space C is P (cm. of mercury),

and the atmospheric pressure (or Barometric Height) is H, the relation between P and H is given by

$$P = H + (B - A).$$

If B is below A, $(B - A)$ is a negative quantity, so that the pressure P is less than H.

If desired, the above expression can be rewritten as $P = H - (A - B)$ to suit this case: the two expressions are algebraically identical, and both perfectly general.

Sometimes, where it is desired to avoid reading the Barometric Height, the surface B is exposed to an evacuated chamber, when

$$P = B - A,$$

but this method is rarely used.

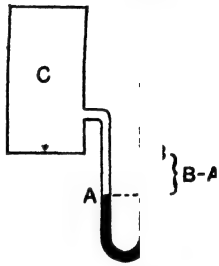


FIG. 101.—Measurement of Pressure.

When the tube B is open to the atmosphere it is necessary to read the barometer as well as the difference in level between B and A before the pressure in the chamber C can be measured—this point must be noted specially. If during the course of an experiment the Barometric Height varies, the quantity H used in the later readings will be different from that used earlier.

Strictly speaking, the barometer should be read *immediately after every observation* of corresponding values of B and A, though this is not required except in the most accurate work. The barometer should, however, be read both *before and after* any experiment on gases, and the difference distributed among the observations, according to the order in which they were made.

§ 4. VERIFICATION OF BOYLE'S LAW

To verify Boyle's Law (p. 173) a quantity of gas is enclosed in a glass tube, and is separated from the external atmosphere by a column of mercury. This mercury is contained in a flexible rubber tube, joining the tube containing the gas to a glass tube in which the level of the mercury exposed to the air can be seen; or, as an alternative arrangement, the two glass tubes may be sealed together, and both connected with a movable reservoir as shown in Fig. 102.

In the best forms of instrument the closed tube containing the air is graduated in c.c., a burette calibrated to the tap being convenient for this purpose. If this form of burette is not available, however, a square-ended glass tube of uniform bore may be used, the volume of air it contains being proportional to the length of tube between the level of the mercury and the square end.

The apparatus of the burette type is more convenient to adjust and to read than this simpler form. If great precautions are to be taken, it may be fitted with a drying tube, so that the air in the tube can be quite dry before closing the tap. It is important that the tap should fit very accurately, otherwise leaks will occur at high pressures and the quantity of gas experimented on will be changed, thus vitiating all the readings.

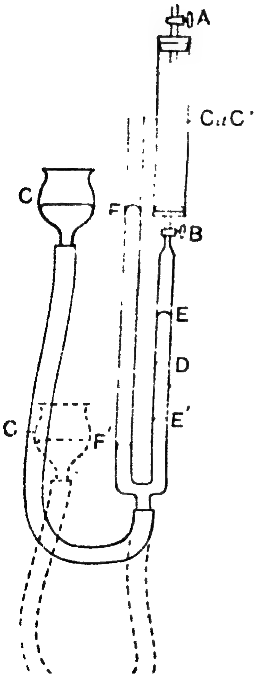


FIG. 102.—Boyle's Law (Apparatus I.).

EXPT. 74. **Verification of Boyle's Law (Apparatus I.).**—The method of using this type of apparatus is as follows:—

The taps A and B are both opened, and mercury is forced up the burette to the tap B by raising the reservoir C. Air is then allowed to enter through these taps by lowering C again, till the burette is filled to D with air. About 30 c.c. of air should be admitted between B and D.

The tap A is closed, and the air all forced up into the drying tube again by raising C till the mercury reaches B.

The gas is left between A and B for a few minutes to dry it completely, and C is then lowered till the level of the mercury in the burette is at D again: by this means BD is filled with air which is practically dry. B is then closed so as to confine a definite quantity of air between B and D, and the apparatus is ready for the experiment.

By raising and lowering C, various pressures can be exerted on the gas in the burette, and the volume of the gas will be altered until its pressure is equal to the pressure exerted on it.

The level of the mercury in the other tube (F) will be the same as the level in the reservoir C, since both surfaces are exposed to the atmosphere.

If for any position of the reservoir the level in the side tube is F and the level in the burette is E, the pressure on the gas inside the burette is given by

$$P = H + (F - E).$$

where H is the Barometric Height.

The levels F and E are read on a vertical scale placed immediately behind the tubes.

Pressures both above and below atmospheric can be used in this form of apparatus if the burette and the side tube are both long nough, the levels F' and E' being two corresponding values, when the pressure is below atmospheric. Such a case is obtained by lowering C to some position such as that shown dotted (Fig. 102).

The volumes are the spaces between B and E, B and E', etc.

Adjust the level of the reservoir to several different heights, so that half the observations are made with pressures below and half with pressures above atmospheric.

Calculate the total pressure in the burette in each case (the barometer must be read *before* this can be done), and note also the *volume* of the gas in the burette under each pressure. Show that the product Pressure × Volume is the same for each adjustment made.

Arrange your observations thus—

Barometric Pressure H = . . . cm.

Reading in Side Tube F.	Reading in Burette E.	F - E.	Total Pressure H + (F - E) = P.	Volume of Gas V.	PV.
		(Half these quantities will be negative.)			

Be careful not to add F - E in cm. to H measured in mm. Both H and (F - E) must be in cm.

If the tube enclosing the gas is not provided with a tap at the top it may be filled with gas and the experiment then carried out

as already described. If, however, it is required to obtain observations at pressures below atmospheric, the tube will have to be heated considerably before closing with mercury.

The tube must be allowed to get quite cold again before the experiment is commenced.

Adjustment of the quantity of air in the tube in this way is not very easy, and frequently leads to fracture of the tube: *it should never be attempted by the student.*

Where the apparatus with the burette and tap is not available, it is often preferable to use two forms of apparatus for verification of Boyle's Law instead of adjusting the quantity of air as described above. One form of apparatus may be used for pressures above, and the other for pressures below atmospheric. It is advantageous to use apparatus of these two forms even where the first type described is also used. A greater *total* variation of pressure is possible when two such forms of apparatus are used than with the single form described, and thus the law is verified over a wider range. The student is also made familiar with different forms of apparatus which may be used for the measurement of gas pressure.

Fig. 108.—Boyle's Law (Apparatus II.)

Expt. 75. **Verification of Boyle's Law, Apparatus II. (for Pressures above Atmospheric Pressure).**—The present apparatus is used for verifying Boyle's Law for cases in which the pressure is *greater than* the pressure of the atmosphere.

The air to be experimented on is contained in the glass tube A. The lower part of this tube is in connection with a reservoir of mercury C and a pressure tube B. A certain mass of air is enclosed in A at a pressure equal to the atmospheric pressure plus the pressure due to difference of levels in A and B. The atmospheric pressure must be found by reading the barometer; let it be H cm. of mercury. The volume of the gas may be taken to be proportional to AE measured on the scale attached to the apparatus. The position of the top of the tube A containing the gas is noted on the scale.

If we raise the reservoir the pressure on the air in A is increased and the volume diminished.

The pressure is equal to the pressure of the atmosphere plus the pressure due to the column of mercury FE, that is,

$$P = H + (F - E).$$

The new volume is equal to V, and is proportional to AE.

In the same way determine the values of P and V corresponding to other positions of the mercury reservoir.

Calculate the values of the products $P \times V$. These should be constant if Boyle's Law is obeyed.

Enter the results in tabular form as for Apparatus I. (p. 185).

Plot a curve showing the relation between the pressure (as ordinate) and the volume (as abscissa). This should be a rectangular hyperbola.

EXPT. 76. **Verification of Boyle's Law, Apparatus III. (for Pressures below Atmospheric Pressure).**—This third type of apparatus used for the verification of Boyle's Law enables us to work over a wide range from atmospheric pressure *downwards*.

A very convenient form consists of a uniform glass tube, which can be raised or lowered inside an iron tube filled with mercury. The iron tube widens at the top into a bowl-shaped vessel, this widening enabling the inner tube to be raised or lowered considerably, without causing any *large* change in the level of the external surface of mercury.

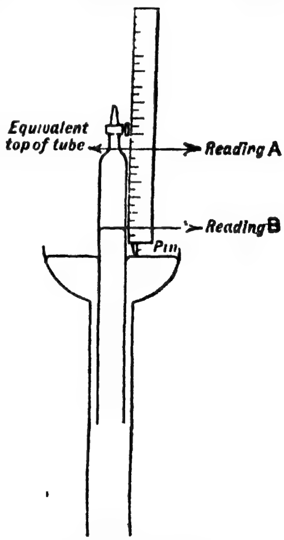


FIG. 104.—Boyle's Law (Apparatus III.).

The pressure of the gas in the inner (glass) tube is less than atmospheric pressure, by the height to which the mercury in the inner tube stands above the level of the mercury outside.

This is measured by adjusting a steel pin, fixed to a vertical metre scale (Fig. 104), until its point just touches the surface of the mercury in the bowl.

The height of the column inside the tube is given by the reading B (Fig. 104) *plus* the length of the pin (x cm.). This

is the amount by which the pressure of the gas inside is *less* than the atmospheric pressure.

The volume of the gas is proportional to the length of tube it fills, if the bore is uniform.

A mark on the neck of the tube just beneath the tap represents the 'equivalent top' of the tube, *i.e.* where the top of the tube would be *if it were of uniform section all the way up, and of the same volume as the actual volume*.

The distance between this mark and the level of the mercury in the tube is proportional to the *volume* of the gas enclosed.

Depress the glass tube, with the tap *open*, until the top of the tube stands about 15 cm. above the level of the mercury in the trough.

Close the tap carefully, pressing it inwards gently whilst turning it, and *do not touch it again during the whole of the experiment*, otherwise more air will be admitted into the tube, and the mass of gas used will thus be altered.

The pressure of the air enclosed in the tube is now equal to the atmospheric pressure.

Adjust the metre scale so that the point of the steel pin just touches the surface of the mercury in the bowl, and find the reading on the scale, level with the 'equivalent top' of the tube.

Raise the tube until the mercury level inside the tube is above the zero of the metre scale. Adjust the pin until it just touches the mercury surface in the bowl, and take the reading (A) on the scale level with the 'equivalent top' of the tube; also read the level B of the surface of the mercury in the tube.

Raise the tube a few centimetres at a time and repeat readings A and B, *taking care to adjust the pin until it touches the mercury in the bowl before taking each set of readings.*

Continue this until, if the tube were raised any higher, there would be no mercury left in the bowl.

At least *six* sets of observations should be taken, distributed uniformly over the range of pressures used.

Depress the tube again to the first position in which readings were taken. If the first readings are not repeated, some air must have leaked in through faulty closing of the tap, and the experiment must be repeated after seeing that the tap is properly closed and air-tight.

Read the barometer and express the pressure of the atmosphere in *centimetres* of mercury.

Tabulate the results of the observations :—

Length of pin, $x = \ldots$ cm.

Reading A.	Reading B.	Pressure P = Atm. − (B + x).	Volume V = A − B.	PV.

The values in the last column should be constant.

Plot a graph taking the values of the pressure as ordinates and the values of the volume as abscissae. The curve should be a rectangular hyperbola.

In this way it is possible to verify the law of Boyle that the volume of a fixed mass of gas varies inversely as the pressure when the temperature is kept constant.

CHAPTER XI

SURFACE TENSION

§ 1. Definition of Surface Tension

The surface of a liquid acts everywhere as though it were in tension; the analogy of a stretched rubber membrane is frequently used to illustrate this, but there is one important difference to be noted. If a rubber membrane is stretched, the tension exerted across any line in the membrane is increased as the extension is increased, while no such increase of tension occurs in the case of a liquid surface.

The tension in dynes exerted across unit length of any line imagined in the surface of a liquid is called the Surface Tension of that liquid.

The surface tension depends not only on the liquid itself but also on the medium on the other side of the surface. Thus the surface tension of a mercury surface exposed to air is entirely different from the surface tension in a surface between mercury and water. The effect of the second medium is extremely marked if the mercury is put in a weak solution of potassium dichromate. The mercury then loses its 'mercurial' character and exhibits a sluggishness entirely different from its mobility when in contact with air.

Whenever we speak of '*the* surface tension' of a liquid, therefore, it must be understood that we refer to the surface tension in a surface bounded by the liquid and by *air*.

§ 2. Effects of Surface Tension

CAPILLARITY

When a fine tube is filled with liquid and its lower end is placed beneath the surface of some of that liquid in a large vessel, at first the liquid will flow down and out of the tube, but eventually a column of liquid of measurable height will be left in the tube, projecting above the level of the liquid in the large vessel.

This column of liquid is supported by the tube as a result of surface tension of the liquid, and the surface tension can be determined from the height of the liquid column and the dimensions of the tube.

Let the radius of the tube be r cm. and the surface tension of the liquid be T dynes per cm. At the line where the liquid surface and the tube meet there is exerted at right angles to their line of contact a force T dynes on each centimetre of that line.

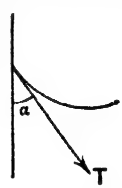


Fig. 105.—Force due to Surface Tension.

This force is exerted by the surface of the liquid, and acts in the liquid surface at right angles to the line of contact. Thus, if the tangent to the liquid surface at this line is at an angle a with the side of the tube (Fig. 105), we have a force acting at an angle a to the vertical, the magnitude of this force being T dynes per cm.

On the whole line, of length $2\pi r$, the total force would be $2\pi r$T. But this is acting at all points at an angle a to the vertical; therefore the vertical component only will have any effect, opposite sections of the line exerting forces whose horizontal components neutralise each other. There will thus be a total force equal to $2\pi r\text{T} \cos a$ exerted by the liquid on the tube across the line of contact, this force being exerted *downwards* by the liquid on the tube.

There is therefore an *upward* force of this magnitude exerted by the tube on the liquid since action and reaction are equal

and opposite, *i.e.* the tube exerts forces on the liquid, across the line of contact, such that the **total upward force exerted on the liquid by the tube** is

$$2\pi r\mathrm{T} \cos \alpha \text{ dynes.}$$

This force supports the column of liquid raised above the level outside, hence, if we can find the weight of the liquid column, its weight must be equal to the above force.

Weight of Column raised.—The column is cylindrical up to the base of the meniscus. Above this its volume is approximately the difference between a hemisphere of radius r and the circumscribing cylinder.

If the bottom of the meniscus is at a height h above the free surface outside, we have

$$\text{Volume of column raised} = \pi r^2 h + \tfrac{1}{3}\pi r^3$$
$$= \pi r^2[h + \tfrac{1}{3}r].$$

Let $h + \frac{1}{3}r$ be indicated by h'.

Then if ρ is the density of the liquid, the *Mass* of the column raised is $\pi r^2 h'\rho$, and its *Weight* is $\pi r^2 h'\rho g$ dynes.

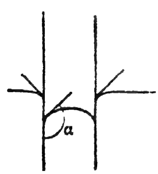


FIG. 106.—Effects of Surface Tension of Mercury.

$$\text{Thus} \qquad 2\pi r\mathrm{T}\cos\alpha = \pi r^2 h'\rho g,$$

$$\text{whence} \qquad \mathrm{T} = \frac{h'r\rho g}{2\cos\alpha}.$$

For all liquids which *wet* the surface, $\alpha = 0$, and therefore $\cos\alpha = 1$, so that, in this case,

$$\mathrm{T} = \frac{h'r\rho g}{2}.$$

The chief exception is mercury, for which α is greater than 90°, so that $\cos\alpha$ is negative; thus h is negative in the case of mercury, due to the negative value of $\cos\alpha$.

EXPT. 77. **Determination of the Surface Tension of Water by the rise in a Capillary Tube.**—Clean a capillary tube carefully with caustic soda and then with nitric acid,[1] washing out the nitric acid with considerable quantities of

[1] Caustic soda is used first to remove any grease in the tube; it is used *before* the acid because it cannot be washed out so easily with water as the acid can.

water. Place the tube in a *thin* glass beaker with vertical sides containing water, depressing it so as to fill the tube with water and then raising it till a column of water is supported in the tube. Tap water should be used rather than distilled water, as the surface of the latter is often contaminated with a film of grease.

Measuring the Height of the Column.—This may be measured directly with dividers, setting the dividers so that when one point is just at the surface of the liquid in the beaker the other is at the level of the meniscus in the tube.

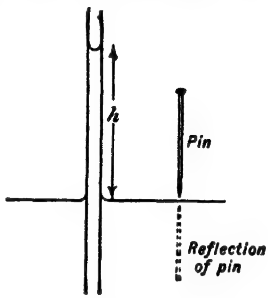


FIG. 107.—Rise of Liquid in Capillary Tube.

Frequently, however, a cathetometer microscope is used. The microscope is focussed first on the meniscus, then on the point of a pin which is just *not* touching the water, the microscope being set so that when the images of the pin and its reflection are viewed through it, the cross-hair is exactly between them. The vertical distance through which the microscope has to be raised between these two positions, is measured on the cathetometer stand of the microscope, and h is thus obtained quite accurately.

Measuring the Bore of the Tube.—The bore of the tube is measured by drying the tube, drawing a thread of mercury into it, and measuring the length of the thread while in the tube. The thread is then run out into a weighed watch-glass and its mass determined. From this mass the radius of the tube may be calculated, assuming a knowledge of the density of mercury.

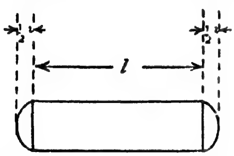


FIG. 108.—Mercury Thread.

Mass of mercury thread $= \pi r^2 l' \rho'$,

ρ' being the density of the mercury and l' the length of the thread.

In measuring the length of the thread, it will be noted that the ends of the mercury are not flat, but curved. If the length of the cylindrical part of the mercury thread is l, and the total length of the *two* curved ends together is x, the volume of the mercury thread may be taken as

$$\pi r^2 l + \frac{2}{3}\pi r^2 x,$$

and opposite, *i.e.* the tube exerts forces on the liquid, across the line of contact, such that the **total upward force exerted on the liquid by the tube** is

$$2\pi r \mathrm{T} \cos \alpha \text{ dynes.}$$

This force supports the column of liquid raised above the level outside, hence, if we can find the weight of the liquid column, its weight must be equal to the above force.

Weight of Column raised.—The column is cylindrical up to the base of the meniscus. Above this its volume is approximately the difference between a hemisphere of radius r and the circumscribing cylinder.

If the bottom of the meniscus is at a height h above the free surface outside, we have

$$\text{Volume of column raised} = \pi r^2 h + \tfrac{1}{3}\pi r^3$$
$$= \pi r^2[h + \tfrac{1}{3}r].$$

Let $h + \frac{1}{3}r$ be indicated by h'.

Then if ρ is the density of the liquid, the *Mass* of the column raised is $\pi r^2 h' \rho$, and its *Weight* is $\pi r^2 h' \rho g$ dynes.

$$\text{Thus} \qquad 2\pi r \mathrm{T} \cos \alpha = \pi r^2 h' \rho g,$$

$$\text{whence} \qquad \mathrm{T} = \frac{h' r \rho g}{2 \cos \alpha}.$$

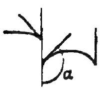


FIG. 106.—Effects of Surface Tension of Mercury.

For all liquids which *wet* the surface, $\alpha = 0$, and therefore $\cos \alpha = 1$, so that, in this case,

$$\mathrm{T} = \frac{h' r \rho g}{2}.$$

The chief exception is mercury, for which α is greater than 90°, so that $\cos \alpha$ is negative; thus h is negative in the case of mercury, due to the negative value of $\cos \alpha$.

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Measuring the Height of the Column.—This may be measured directly with dividers, setting the dividers so that when one point is just at the surface of the liquid in the beaker the other is at the level of the meniscus in the tube.

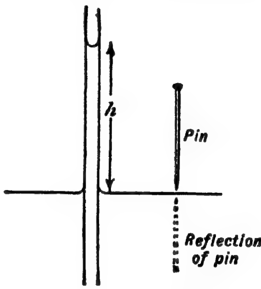


FIG. 107.—Rise of Liquid in Capillary Tube.

Frequently, however, a cathetometer microscope is used. The microscope is focussed first on the meniscus, then on the point of a pin which is just *not* touching the water, the microscope being set so that when the images of the pin and its reflection are viewed through it, the cross-hair is exactly between them. The vertical distance through which the microscope has to be raised between these two positions, is measured on the cathetometer stand of the microscope, and h is thus obtained quite accurately.

Measuring the Bore of the Tube.—The bore of the tube is measured by drying the tube, drawing a thread of mercury into it, and measuring the length of the thread while in the tube. The thread is then run out into a weighed watch-glass and its mass determined. From this mass the radius of the tube may be calculated, assuming a knowledge of the density of mercury.

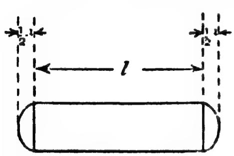


FIG. 108.—Mercury Thread.

$$\text{Mass of mercury thread} = \pi r^2 l' \rho',$$

ρ' being the density of the mercury and l' the length of the thread.

In measuring the length of the thread, it will be noted that the ends of the mercury are not flat, but curved. If the length of the cylindrical part of the mercury thread is l, and the total length of the *two* curved ends together is x, the volume of the mercury thread may be taken as

$$\pi r^2 l + \frac{2}{3}\pi r^2 x,$$

the value of the second term being obtained on the assumption that the ends are semi-ellipsoidal in shape.

Thus, the mass of the mercury is

$$\pi r^2\left(l + \frac{2}{3}r\right)\rho'.$$

The lengths l and l' are to be measured as accurately as possible, and the value of x found by taking the difference between them. The value of r must then be calculated.

Another method is to cut off the tube at the place where the meniscus stood, and to mount this in a stand so as to view it in section with a microscope. The size of the image of the hole as viewed in the microscope is measured on a micrometer scale in the focal plane of the eye-piece. This micrometer is then calibrated by viewing a standard scale, and finding the number of divisions of the micrometer eye-piece which correspond with one millimetre of the standard scale, the scale being viewed with the microscope in the same adjustment as when viewing the tube section. (See p. 27.)

The method with the mercury thread is much more accurate than this second method. The tube can be tested to see whether the bore is uniform or not by measuring the thread in different positions. This might be done before carrying out the rest of the experiment, any tube which exhibits marked inequalities in the bore being discarded.

The experiment should be performed for three tubes of different bore, and h should be shown to be inversely proportional to r.

If any liquid other than water is used, its density must be determined before T can be calculated.

PRESSURE DUE TO CURVED SURFACES

Pressure inside a Soap Bubble.—Inside a soap bubble, the pressure is greater than atmospheric by a small quantity p. Considering the equilibrium of the upper hemisphere, this excess pressure acts on the upper hemisphere, and produces a resultant *upward* force on the hemisphere of magnitude $p\pi r^2$, tending to blow the upper and lower hemispheres apart.

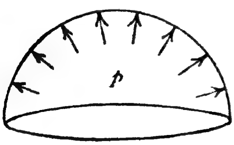


FIG. 109.—Pressure in Soap Bubble.

The two hemispheres are kept together by the surface tension

forces acting in the *two* surfaces of the film round the line of contact, and the bubble expands until the surface tension forces just neutralise the disruptive force $p\pi r^2$.

The line of contact between the hemispheres in *each* surface is of length $2\pi r$, and hence the *total* force due to surface tension keeping the hemispheres together is $2(2\pi r\mathrm{T})$, since the film has two surfaces.

Thus
$$4\pi r\mathrm{T} = p\pi r^2,$$
or
$$\mathrm{T} = \frac{pr}{4}.$$

EXPT. 78. **Surface Tension of Soap Solution by Pressure in Soap Bubble.**—Blow a small bubble on the end of an

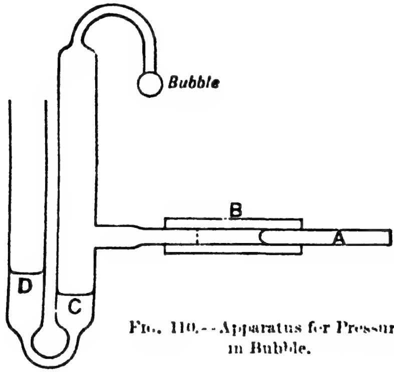


FIG. 110.—Apparatus for Pressure in Bubble.

apparatus as depicted in Fig. 110, by gently pushing a glass rod A into the rubber tube B attached to the side.

By means of a travelling microscope with vertical and horizontal scales, measure the dimension of the bubble across a horizontal diameter, using the vertical cross-hair of the microscope adjusted tangentially, first on one side, then on the other, of the image of the bubble.

Measure the difference in height h between the levels of the water in the tubes C and D, using the vertical scale of the cathetometer.

Then the pressure excess inside the bubble over the atmospheric pressure is
$$p = h\rho g \text{ dynes per sq. cm.}$$

ρ = density of water in bend of U-tube; r has been measured already.

Calculate the value of the surface tension in dynes per cm. from the equation

$$T = \frac{pr}{4}.$$

Make observations on two or three bubbles of different sizes.

PART I

ADDITIONAL EXERCISES ON PROPERTIES OF MATTER

1. Draw a sector of a circle (radius 15 cm.), containing an angle of 150°, measure its area with the planimeter, and verify the result by means of the balance.

2. Draw an ellipse of major axis 20 cm. and minor axis 10 cm. and measure its area with the planimeter.

3. Find the area and density of the given plate by weighing it in air and water and measuring the thickness.

4. Find the average area of cross section of the given wire by means of a metre scale and a hydrostatic balance.

5. Find the length and specific gravity of a given tangle of wire, using a hydrostatic balance and a micrometer screw.

6. Find the specific gravity of a solid by weighing it in a liquid whose specific gravity is given.

7. Weigh the given solid in air, in water, and in the given liquid. Deduce the specific gravity of the metal and of the liquid.

8. Make a solution of sugar and water which contains accurately 10 per cent by weight of sugar, and determine the specific gravity of the solution.

9. Prepare a solution of common salt in water containing 15 gm. of salt in 100 gm. of solution and find its density.

10. Find the density of a liquid which is denser than water and does not mix with it, by preparing a salt solution of equal density and determining the density of this solution.

11. Calibrate the given burette by means of a balance.

12. Find the internal volume of a definite length of the narrow bore tube provided, and calculate the mean internal diameter.

13. Measure the radius of the sphere provided by means of the spherometer, find its weight and deduce the density of the material.

14. A body is supported on an inclined plane by a force acting parallel to the plane. Plot a graph showing the relation between the magnitude of the force and the height of the plane.

15. Find the mass of the roller supplied, using an inclined plane.

16. Support a metre scale from various points along its length, and balance it by hanging weights on the shorter side. Deduce the weight of the metre scale.

17. Find the angle of static friction between the given surfaces.

18. Find the velocity ratio and the force ratio for the given machine and deduce the efficiency.

19. A bar is supported at its ends and loaded at its centre. Plot a curve showing how the depression of the centre varies with the load.

20. Plot a curve showing the relation between the angle of twist and the length of wire twisted, when a given couple is applied to the end of the wire.

21. A ball is projected horizontally with a given velocity by allowing it to roll down one quadrant of a circle. Plot a curve showing the relation between the horizontal range and the height of the point of projection.

22. Find the relation between the distance traversed and the time when a body starts from rest and moves with uniform acceleration, using Atwood's machine.

23. Find the combined mass of the two weights and the pulley of an Atwood's machine, given $g=981$ cm. per sec. per sec.

24. Find the relation between the distance traversed and the time when the given body rolls down an inclined plane, starting from rest.

25. Prove that the acceleration of a body rolling down an inclined plane of given length is proportional to the difference in height between the ends.

26. Plot a graph showing how the acceleration of a body rolling down an inclined plane varies with the height of the plane.

27. Calculate the kinetic energy of the given body when it is rotating with unit angular velocity about its axis.

28. Plot a curve showing the relation between the time of swing of a simple pendulum and the square root of the length. Deduce a value of g from the results.

29. Find the length of the simple pendulum supplied by determining the time of swing, assuming $g=981$ in C.G.S. units. Adjust the length so that the time of one complete swing is 2 sec.

30. Plot a curve showing the variation of period with length for a simple pendulum, and deduce the length of a 'quarter seconds' pendulum, *i.e.* a pendulum requiring *half* a second to swing both ways.

31. Find the period of swing of a pendulum 20 inches long, without actually using one of that length.

32. Plot a graph showing the relation between the length of the simple pendulum and (*a*) the time of swing, (*b*) the square of the time of swing. Deduce the value of g, the acceleration due to gravity.

33. Plot a graph showing how the time of oscillation of the given torsion pendulum varies with the distance from the axis of the added symmetrical load.

34. A thin lath is fixed horizontally at one end and loaded at the other. Plot a curve showing how the time of vibration varies with the load.

35. From observations on the oscillations of a loaded spring prove that the square of the time of oscillation divided by the load is approximately constant.

36. The two ends of a heavy bar are connected by a string whose middle point is attached to a fixed support. Find how the time of oscillation of the system under gravity depends on the length of the string, when the bar remains in the same vertical plane during the oscillation.

37. Verify Boyle's law by pouring mercury into the open limb of a U-tube the other limb of which is closed.

38. A U-tube contains mercury in the bend and air in the closed limb; use it to find the height of the barometer.

PART II

SOUND

CHAPTER I

INTRODUCTORY THEORY

§ 1. Velocity: Frequency and Wave-Length

Most of the experimental determinations in sound are either determinations of the velocity of sound in different media or else determinations of *pitch* and the associated quantities, frequency, and wave-length. Sound is propagated through any material medium as a wave-motion, the disturbance being produced by the sounding body and causing the sensation of sound when it reaches the ear.

The Velocity of Sound varies with the medium through which it travels. It can be shown that the velocity with which sound is propagated through a medium of elasticity E and density ρ, is given by the equation

$$V = \sqrt{\frac{E}{\rho}}.$$

E is the modulus of elasticity corresponding with the particular strain caused by the wave-motion.

EFFECT OF TEMPERATURE ON THE VELOCITY OF SOUND IN A GAS

The modulus of elasticity concerned when a sound wave travels through a gas, is equal to γP, where γ is the ratio of the specific heats of the gas (a constant) and P is its pressure. Hence the velocity, V, of sound in the gas is equal to $\sqrt{\gamma P/\rho}$, where ρ is the density.

Now $P/\rho = RT$ (see section on Heat, p. 341); therefore V, the velocity of sound in the gas, is equal to $\sqrt{\gamma RT}$, or is proportional to the square root of the absolute temperature.

If α is the gas coefficient ($\frac{1}{273}$),

$$T/T_0 = 1 + \alpha t,$$

where t is temperature in degrees centigrade.

$$\therefore \frac{\text{Velocity of Sound at } t^\circ \text{ C}}{\text{Velocity of Sound at } 0^\circ \text{ C}} = \sqrt{\frac{T}{T_0}} = \sqrt{1 + \alpha t},$$

or

$$V_t = V_0\sqrt{1 + \alpha t}.$$

When t is not large, this is written as

$$V_t = V_0(1 + \tfrac{1}{2}\alpha t);$$

and hence the velocity of sound at any temperature can be calculated if the velocity at 0° C. is known.

PITCH AND FREQUENCY

The musical pitch of a note depends on the **number of vibrations made by the sounding body in one second** or the **vibration frequency** of the note. The note called middle C on a piano is taken to correspond with a vibration frequency of 256. This is called the *scientific pitch* of middle C. The middle C of *concert pitch* has a frequency considerably above this, while other standards of pitch are some above and some below this scientific standard.

The reason for the choice of 256 as middle C in scientific work is in order that the number of vibrations corresponding with any C shall be a whole number, 256 being equal to 2^8 (see below).

Musical Interval depends on the **ratio of the vibration frequencies** of the two notes, the ratios for various intervals being:

octave	1 : 2	minor third	5 : 6
fifth	2 : 3	major tone	8 : 9
fourth	3 : 4	minor tone	9 : 10
major third	4 : 5	semitone	15 : 16

The Relation between Velocity, Frequency, and Wave-Length. —Let the velocity of sound in any medium be indicated by

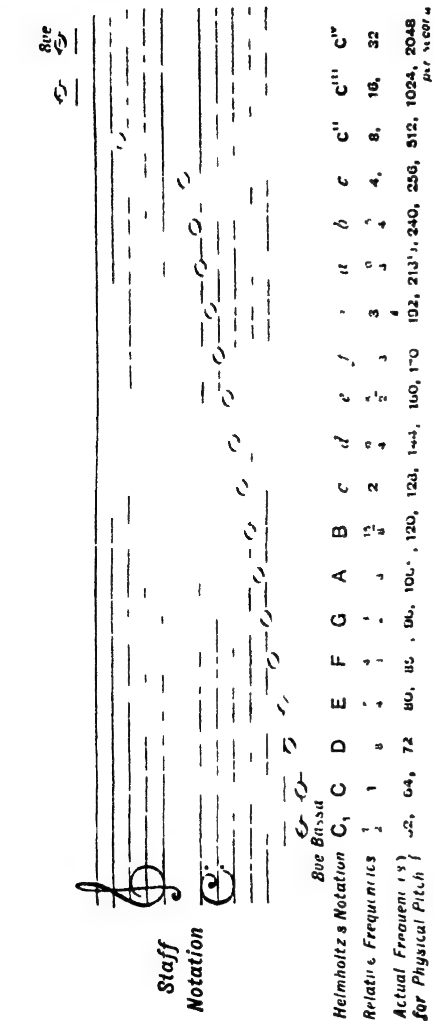


Fig. 111.—Musical Scale. Frequencies and

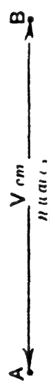


Fig. 112.—Velocity and Frequency.

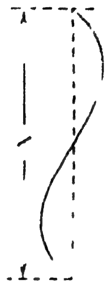


Fig. 113.—Wave-Length.

V cm. per sec. Choose two points A and B (Fig. 112), such that their distance apart is V cm.

At A an observer is stationed, and at B a body is set into vibration, emitting a note the vibration frequency of which is n. The time taken for the first wave to reach A will be one second, since the distance AB is equal to V; thus the observer at A will just be receiving the first wave as the nth wave is emitted from B.

Between A and B there will thus be n waves moving towards A. If each wave is of length λ (Fig. 113), then the length AB must be equal to $n\lambda$, and we obtain the important result that

$$V = n\lambda.$$

§ 2. Resonance

THE PRINCIPLE OF RESONANCE

If two neighbouring bodies have identical frequencies and one of them is set in vibration, the other will take up a rhythmic vibration due to the vibration of the first. This vibration, which is communicated to the second body, may attain considerable amplitude; so that if the first body is stopped the second may continue to vibrate for some time afterwards.

This principle is not limited to sound but is common to all forms of vibratory motion. It can be understood readily by considering a simple case, such as that of two accurately attuned tuning-forks. One of these is set in motion, and wave disturbances from it reach the other. The prongs of the second fork are alternately pressed away from and drawn towards the first fork, in consequence of the alternate compressions and rarefactions reaching the prongs through the air. These occur exactly in step with the natural motion of this second fork, which is just starting to spring back into its mean position after the first pressure wave, when the rarefaction commences, its backward motion being thereby assisted. The fork moves beyond its equilibrium position under its own momentum and this assisting force, and is just turning to move forward again when the next pressure wave arrives; under the pressure wave it is *urged* forward instead of being merely allowed to move forward under its own

elastic forces. Thus every vibration of the fork is assisted by a force exerted by the air near to it, and the succession of light impulses, insignificant when considered singly, has a cumulative effect which results in the fork taking up a vibration of considerable amplitude. All other cases of resonance can be explained in a similar way.

STATIONARY VIBRATIONS

When two wave-trains of equal intensity are passing through the same medium in opposite directions, stationary vibrations

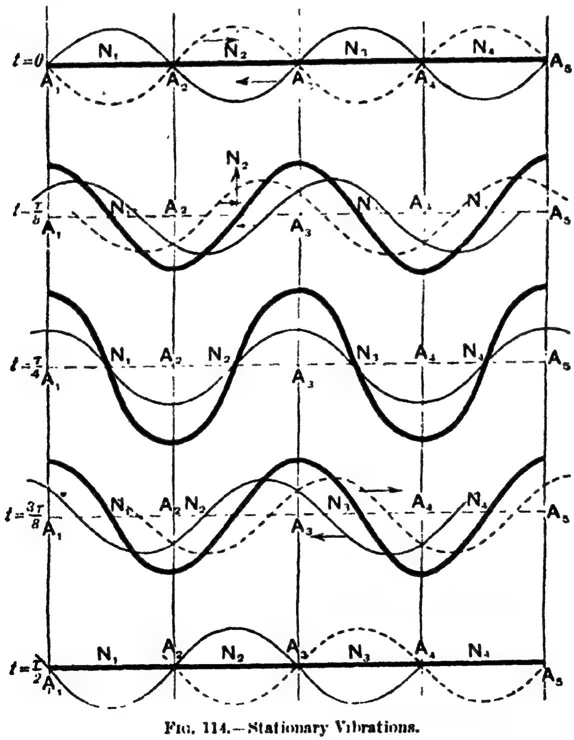


FIG. 114.—Stationary Vibrations.

are set up. If, in Fig. 114, the thin wavy line represent a wave motion moving to the left, and the dotted line a similar motion

moving to the right, their *resultant* action on the medium between A_1 and A_5 is represented by the *thick* line in each case considered. Certain points N_1, N_2, N_3 are never disturbed at all, while points A_1, A_2, A_3 are disturbed more than any other points in the whole line. N_1, N_2, N_3 are called **nodes,** and A_1, A_2, A_3 are called **antinodes.**

It will be seen that **the distance between two consecutive nodes or between two consecutive antinodes is equal to half a wave-length;** or, **from a node to the next antinode is a quarter of a wave-length.**

This result is made use of in connection with the experiments described below.

THE RESONANCE TUBE

If a tube is arranged so that its length can be varied by suitable means, it can be adjusted so that the column of air in it resounds to various notes. Resonance occurs for any note which corresponds with any possible mode of vibration of the air in the tube.

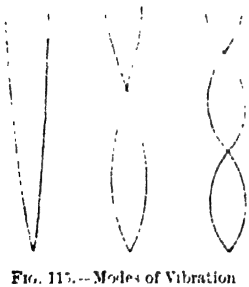


FIG. 115.—Modes of Vibration in Resonance Tube.

If the tube is closed at one end, the air in it can vibrate in any way which permits of free motion at the open end and zero motion at the closed end, *i.e.* **the open end is an antinode and the closed end is a node.** The different forms of vibration possible are indicated diagrammatically. It will be seen that the length of the pipe is respectively $\lambda_1/4$, $3\lambda_2/4$, $5\lambda_3/4$, etc., λ_1, λ_2, λ_3 being the wave-lengths of the possible vibrations.

A fork placed over the end of a tube causes the air in the tube to be set in stationary vibration, the air being under the action of the waves sent out by the fork, and of the waves reflected from the closed end.

Thus, if a fork giving a certain note be placed over the end of

the tube, and the tube be adjusted to resound to the note emitted by the fork, the shortest length of tube which will give resonance is a length l_1, such that $l_1 = \lambda/4$, λ being the wave-length in air of the note emitted by the fork.

The next length giving resonance will be l_2, such that $l_2 = 3\lambda/4$, and so on; so that we can determine the wave-length in air of the note emitted by the fork.

A slight correction has to be applied for the diameter of the pipe; the length l_1 is not exactly equal to $\lambda/4$, nor is l_2 exactly $3\lambda/4$. The correction for a cylindrical pipe is approximately $3/5$ the radius.

Thus
$$l'_1 = l_1 + \frac{3R}{5} = \frac{\lambda}{4},$$
and
$$l'_2 = l_2 + \frac{3}{5}R = \frac{3\lambda}{4}.$$

These corrected lengths l'_1 and l'_2 should be used in calculating λ.

The correction need not be known if l_1 and l_2 can both be found; for
$$l_2 - l_1 = \frac{\lambda}{2}$$
quite accurately, the correction being eliminated by taking the difference between l_2 and l_1.

If λ is found by this means for a fork of known frequency, the velocity of sound in the air in the pipe can be calculated, for
$$V = n\lambda.$$

λ, the wave-length in the air in the pipe, is known, and n is given; therefore V can be obtained. Or if V is given, n can be calculated.

If two forks are used, the ratio of their frequencies can be determined by obtaining the values of λ corresponding with the notes they emit.
$$V = n_1\lambda_1,$$
$$V = n_2\lambda_2,$$
$$\therefore \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}.$$

Expt. 79. **The Resonance Tube.**—The Resonance tube is conveniently constructed in either of the forms shown in Fig. 116. The first consists of a counterpoised brass tube which projects from a tall standard tube containing water.

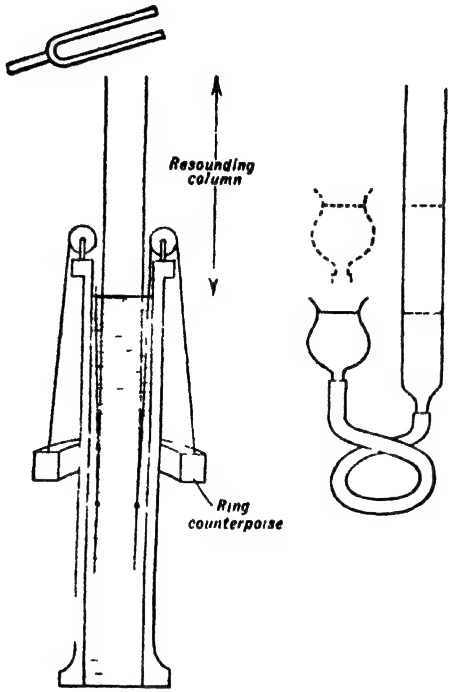


Fig. 116.—Resonance Tubes.

A scale of cm. is graduated on the movable brass tube, the zero of the scale being at the top of the tube. The level of the water can be read on the scale by a window in the standard tube, and hence the length of tube giving resonance can be determined conveniently.

The other form is so simple that it needs little explanation. The level of the water is adjusted by moving the reservoir, and the length of the tube containing air is measured with a metre scale.

Adjust the resonance tube to give resonance with several

different forks, obtaining both the first and second resonance length, if possible, with each fork.

(i.) Calculate the velocity of sound in the air in the pipe from the known frequency of one of the forks Observe the temperature of the room and reduce the value of V obtained at this temperature to give the velocity at 0° C., by means of the formula (p. 202)

$$V_t = V_0 \left(1 + \tfrac{1}{2}at\right).$$

Or (ii.) Being given the velocity of sound in air at 0° C., calculate the velocity of sound in air at the room temperature, and hence deduce the frequency of a given tuning-fork.

(iii.) Compare the frequencies of two forks from observations with the resonance tube, and check the values so calculated by means of the frequencies marked on the forks.

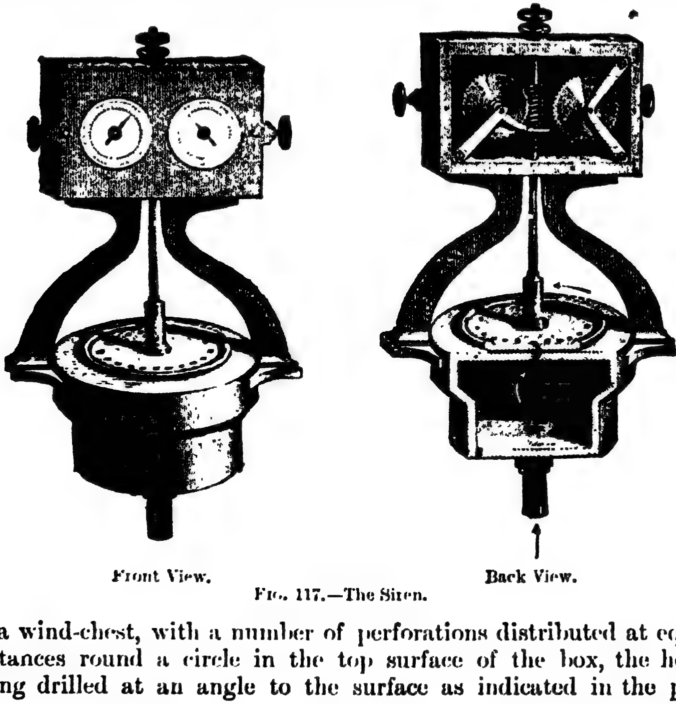
CHAPTER II

FREQUENCY

§ 1. Determinations of Frequency

THE SIREN

The **Siren** is an apparatus which, in the form most convenient for scientific use, was invented by Caignard de la Tour. It consists



Front View. Back View.

Fig. 117.—The Siren.

of a wind-chest, with a number of perforations distributed at equal distances round a circle in the top surface of the box, the holes being drilled at an angle to the surface as indicated in the part

section shown in Fig. 117. Above this box, and very close to it, is a circular plate drilled with a similar set of holes, inclined the opposite way to those in the top of the chest. This plate is mounted so that it can rotate on the wind-chest in such a manner that the one set of holes passes over the other as the plate rotates.

When the chest is filled with air under pressure, the air emerges through the holes, and, impinging on the holes in the disk, sets the latter in rotation about its axis.

The holes in the chest are thus covered and uncovered at regular intervals, puffs of air escaping periodically as the two sets of holes coincide momentarily, and thus periodic pressure waves are sent out into the air, producing the periodic disturbance called 'sound.'

A worm gearing and toothed wheels in the case at the top cause the revolutions to be recorded on the dials shown. By measuring the time required for a definite number of revolutions the rate of revolution of the disk can be obtained.

Suppose that the disk and the top of the chest have n holes in the ring, and N revolutions of the disk are made in t seconds, the total number of 'puffs' made is nN in t seconds, and the frequency is therefore $n\text{N}/t$.

By adjusting the speed of revolution till the note of the siren is in unison with the note of the vibrating body, we can obtain the vibration frequency of this body, as it is then equal to that of the note given by the siren, viz. $\text{N}n/t$.

EXPT. 80. **Determination of Pitch by means of a Siren.**—Adjust the siren to unison with a vibrating tuning-fork or to a sounding organ-pipe.[1] Keep the rate of revolution constant by adjusting the tap and the pressure in the bellows, and find the vibration frequency of the siren. This is given by $\text{N}n/t$, and is the same as the frequency of the note of the fork or organ-pipe used.

Other methods of determining frequency are employed in the following experiments:—

EXPT. 81. **Determination of the Pitch of a Tuning-fork by the Dropping Plate Method.**—Place a light style of stiff hair on the prong of a tuning-fork, fixing it in place with wax. Suspend a smoke-blackened sheet of glass in a heavy frame from a stand (Fig. 118), using a piece of cotton passing over two pins for this purpose. Mount the fork so that the hair gently touches the glass near its bottom end, bow the

[1] See notes on tuning, p. 224.

fork with a violin-bow, and release the glass by burning the cotton thread between the pins.

When the glass is examined, a wavy trace will be found on the blackened surface, the trace being made in the soot by

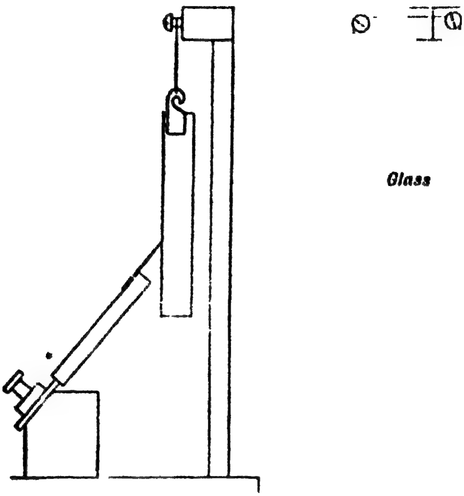


FIG. 118.—Dropping Plate Apparatus.

the style as the plate fell. From the trace thus formed, the vibration frequency of the fork can be found as follows:

(i.) If the beginning of the trace is quite clearly defined, measure the distance s from the beginning to the last wave shown on the glass, and count the number of complete waves in this distance; let this be N.

The glass falls under its own weight, and in t seconds it will fall a distance $s = \frac{1}{2}gt^2$, since it has no initial velocity. Thus the time t required to fall the measured distance s can be calculated,

$$t = \sqrt{\frac{2s}{g}}.$$

During this time the fork made N vibrations, therefore the vibration frequency of the fork is N/t.

(ii.) If the trace is not clear at the beginning, start at the first clearly defined crest and count off a certain number of waves n (say 20) marking the nth crest. Count beyond this a further set of n waves. Measure the length s_1 of the first set, and s_2 the length of the second set of n waves.

If the velocity of the glass when passing the first crest was v_0, and the time taken to make n waves is t, we have

$$s_1 = v_0 t + \tfrac{1}{2}gt^2.$$

When the second of the marked crests was being passed, the velocity of the glass was $v_1 = v_0 + gt$.

The distance s_2 is given by

$$s_2 = v_1 t + \tfrac{1}{2}gt^2$$

or

$$s_2 = v_0 t + gt^2 + \tfrac{1}{2}gt^2,$$

since s_2 is described in the same length of time as s_1.

Thus

$$s_2 - s_1 = gt^2$$

or

$$t = \sqrt{\frac{s_2 - s_1}{g}}.$$

In this time n vibrations are made; therefore the vibration frequency $= n/t$.

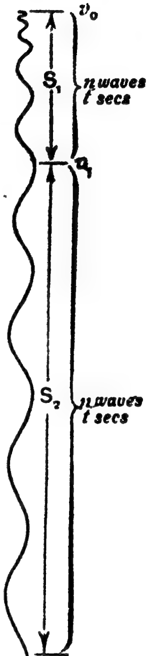


FIG. 119.—Trace of Tuning-fork.

EXPT. 82. **Kundt's Tube.**—A glass tube, about 1 metre long and 5 cm. internal diameter, is dried thoroughly over a Bunsen flame. The tube, closed at one end with a cork, is lightly dusted inside with *dry* cork powder or lycopodium dust. Rotate the tube about a horizontal axis till the dust is just on the point of slipping down the walls. A rod, fitted with a light disk of cork or ebonite at the end, is fixed with this end just projecting into the tube. The disk is made rather smaller than the tube, so that the end of the rod and the attached disk can vibrate freely inside the tube.

The rod is clamped exactly at its middle point, and is set into longitudinal vibration by stroking lengthways with a resined leather or cloth. By this means the air

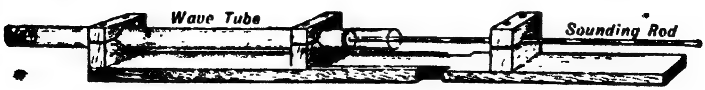


FIG. 120.—Kundt's Tube.

in the tube is set in vibration, waves being sent down the tube, which give rise to returning waves by reflection from the fixed cork at the other end of the tube.

The tube is pushed by *very small* amounts over the end of the rod, the rod being stroked so as to cause it to sound after each fresh adjustment. A position is reached where the air or gas in the tube resounds to the note of the rod. When this is the case the dust in the tube is caught up in the swirl of

FIG. 121.– Antinodes in Kundt's Tube.

the air, which is set in violent vibration. After the motion has died down, the dust settles in characteristic ridges at the *antinodes*.[1] Several antinodes are thus fairly clearly marked, and the distance between two at some considerable distance apart is measured.

From the number of heaps between these two, the number of intervening antinodes is found, and the wave-length of the note in the gas in the tube is obtained, the distance between two consecutive antinodes being half a wave-length.

The pitch of the rod is now found by means of a sonometer (see experiments with sonometer, p. 219), using a tuning fork of known frequency for comparison; and from this the velocity of sound in the gas can be determined using the relation

$$V = n\lambda.$$

If the velocity of sound in the gas is assumed to be known, the frequency of the note of the rod can be determined by this same equation.

Calculation of Young's Modulus for the Rod.– The rod vibrates with a node at its middle point and an antinode at each end; hence **its length is equal to half the wave-length of the note in the material of the rod.**

The velocity of sound in the rod is $\sqrt{E/\rho}$, where ρ is the density of the rod and E the modulus of elasticity for longitudinal strain, *i.e.* E is Young's Modulus.

If $V' =$ Velocity of sound in the rod
and $\lambda' =$ Wave-length of sound in the rod,

$$V' = n\lambda'.$$

[1] By continued sounding it is possible, though difficult, to get the dust to settle in little heaps at the *nodes*.

Here n is known, λ' is equal to twice the length of the rod, which can be measured; hence V' can be calculated.

The density of the rod is also known, and

$$V' = \sqrt{\frac{E}{\rho}},$$

hence we can find Young's Modulus for the material of the rod.

§ 2. Beats

When two pure notes of nearly the same pitch are sounded together, periodic variations in the intensity of the sound are heard. These alternations of sound and comparative silence are termed **Beats**. They can be plainly recognised when two tuning-forks of nearly the same frequency are set in vibration together. If the forks have frequencies N_1 and N_2 respectively, the number of beats per second is the difference between these frequencies, $n = N_1 - N_2$. Here N_1 is supposed greater than N_2.

This result can be explained by the principle of interference. The velocity of propagation is the same for the two notes, but the wave-lengths differ slightly. Where the waves agree in phase they will strengthen each other, but where they are opposed they will neutralise one another (Fig. 122). Let us take as starting-point an instant when waves from the two sources reach the ear in the same phase. At the end of one second, the higher note has made N_1 complete vibrations, the lower note only N_2, that is,

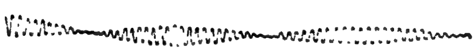


FIG. 122.—Variations of Amplitude in Beats.

the higher note has made $N_1 - N_2$ more vibrations than the lower. During the second, one system of waves has been falling behind the other, and the loss amounts to $N_1 - N_2$ wave-lengths. Hence there must have been $N_1 - N_2$ occasions in the course of the second when the two systems agreed in phase, and $N_1 - N_2$ occasions when the phases were opposed so that there was comparative silence. In other words, the number of beats in one second is $n = N_1 - N_2$.

If two notes are nearly in unison, the beats are very slow and it is difficult to distinguish them. On the other hand, if the number of beats is more than four per second it is difficult to count them. When the beats become so rapid that they cannot be separately perceived, a 'discord' or 'dissonance' is produced.

EXPT. 83. **Beats between Tuning-forks.**—Take two tuning-forks of nearly the same pitch mounted on resonance boxes. The frequency of one fork can be altered by means of a movable mass which can be clamped at any part of the prong (Fig. 123).

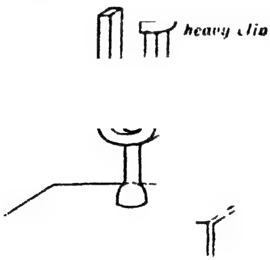


FIG. 123.—Loaded Tuning-fork.

Fix the mass at a definite distance from the end of the prong, and count the number of beats made with the other fork in a measured interval of time.

The number of beats per second should be determined by counting as many beats as possible, taking the time with a stop-clock or stop-watch.

Repeat the observations with the mass at other points on the prong, and plot a curve showing the relation between the distance of the mass from the free end of the prong and the number of beats per second.

CHAPTER III

TRANSVERSE VIBRATIONS OF A STRETCHED STRING

§ 1. Propagation of Transverse Waves along a Stretched String

The expression for the velocity of a transverse wave along a string under tension, can be shown to be

$$V=\sqrt{\frac{T}{m}},$$

where T is the force or tension exerted on the string, and m is the mass of unit length of the string.

If T is measured in *poundals* and m in pounds per foot of length, the velocity is given in feet per second; with T in *dynes* and m in gm. per cm. length, the velocity is obtained in centimetres per second.

Expt. 84. **Determination of the Velocity of a Wave along a String.**—Set up a cord several metres in length, fixed at one end and with the other passing over a pulley and carrying a scale-pan. Stretch the cord by various weights placed in the scale-pan. Pluck the cord at one end, and find the time taken for the disturbance to travel, say 10 or 15 times, from end to end of the string. This can be done easily, for the motion of the disturbance is plainly visible and the time taken can be noted with a stop-watch.

Calculate the tension of the string in dynes from the known mass M hanging from the end,

$$T=Mg \text{ dynes.}$$

M here includes the mass of the scale-pan.

Determine the mass of 1 cm. of the cord by weighing a known length of a similar cord.

From the observations of the motion of the disturbance, calculate the velocity of the wave along the string, and show that the observed velocity is equal to $\sqrt{T/m}$ cm. per sec.

Expt. 85. **Determination of an Unknown Mass from Observations on the Velocity of a Wave.**— Hang an unknown mass M on the string used in the last experiment, and observe the velocity of the wave disturbance as before. Calculate the mass of the suspended body from the equations

$$M \text{ (unknown)} = \frac{T}{g},$$

and

$$T = V^2 m.$$

Verify your result by actual observation of M, using a balance.

§ 2. Stationary Vibrations of a Stretched String

If a string is stretched between two points A and B

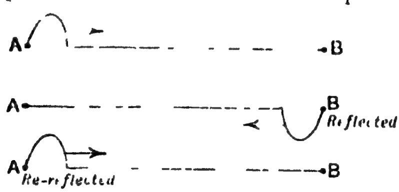


Fig. 124. Reflection of Disturbance from Ends of Stretched String.

(Fig. 124), and a disturbance is created at any point on the string, the disturbance travels to one end, and is there reflected *reversed* to the other end. Here it is again reflected, but now in the same form as the original disturbance, *i.e.* the string is in exactly the same condition as at first, after the disturbance has travelled *twice* along its length; or when the disturbance has travelled *once along and once back*, the vibration of the string has completed one *cycle*.

Now the velocity of propagation is $\sqrt{T/m}$, and the distance travelled by the wave in a complete cycle is $2l$. Hence the period of vibration is

$$t = \frac{2l}{\sqrt{T/m}},$$

or the **vibration frequency**

$$n = \frac{1}{t} = \frac{1}{2l}\sqrt{\frac{T}{m}}.$$

This expression enables us to calculate the frequency of a string if the various quantities l, T, and m are known.

THE SONOMETER OR MONOCHORD

A sonometer consists of a firm frame carrying two fixed bridges over which one or more strings or wires can be stretched. Usually one string is fixed permanently to the apparatus, and its pitch is varied by 'keying up,' wrench keys or other means being supplied whereby the tension can be adjusted as desired. Another string is also used, one end being fixed over one of the fixed bridges, while the other end, passing over the other bridge, carries a scale-pan. The tension in this second string is adjusted by suspending masses in the scale-pan hanging from it, and for this reason the sonometer is best supported in a vertical position. If a horizontal position be used, the string has to pass over a pulley so that the weights can hang downwards. There is usually considerable friction at the pulley, so that the tension on the string is not necessarily the same as the weight hanging from the end. A pair of movable bridges is also supplied, one for each string; by moving these bridges along the strings, the *sounding length* can be altered at will, and the pitch of either string changed as a result of the altered length.

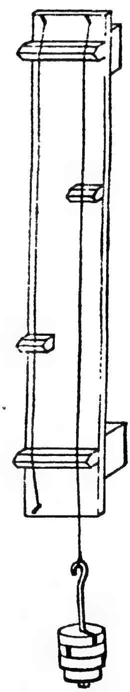


FIG. 125.—Vertical Sonometer.

EXPT. 86. **Determination of the Variation of Pitch with Length.**—The sonometer is set up and the tension of the keyed wire adjusted so that a musical tone is emitted on plucking the wire. Several forks of known frequency are supplied, and the length of the keyed wire is altered by shifting the movable bridge, so as to give unison with each fork in turn, the tension of the wire being kept constant throughout. See the notes on tuning on p. 324.

The lengths l_1, l_2, etc., corresponding with the frequencies n_1, n_2, etc., of the forks (and of the tuned wire), are determined.

It will be found that $n_1 l_1 = n_2 l_2 = n_3 l_3$, showing that **the frequency of the wire under constant tension is inversely proportional to its length.**

Use this result to find the pitch of an unmarked tuning-fork, tuning the wire first to unison with a known fork, and afterwards tuning to the unknown fork.

$$\frac{n_1}{n_2} = \frac{l_2}{l_1},$$

or $$n_1 \text{ (unknown)} = n_2 \frac{l_2}{l_1} \text{ (all known).}$$

To determine the way in which the frequency of a wire of constant length varies with tension, etc., is a matter of some difficulty, requiring the use of a large number of tuning forks of known pitch. In the following experiments an indirect method is used, the wire being tuned by altering the length as well as the tension, the effect of the alteration of length being allowed for by a simple calculation involving the result obtained in the experiment just described.

EXPT. 87. **Determination of the Variation of Pitch with Tension.**—Apply different tensions to the second wire of the sonometer and find the lengths of this wire which vibrate in unison with a certain length of the fixed wire, the fixed wire being kept under constant tension. Let the tensions be T_1, T_2, T_3, etc., and the attuned lengths be l_1, l_2, l_3, etc.

In order to find how the pitch of a *constant* length of the wire varies with the tension acting in the wire, we apply the result obtained in Experiment 86 in the following manner.

Let the pitch of the length l_1, when pulled with a tension T_1, be n_1. A length l_2 of the same wire had the same frequency n_1 when pulled with a force T_2. If we had used the same *length* of wire as at first (l_1), the pitch under tension T_2 would have been

$$n_2 = n_1 \frac{l_2}{l_1},$$

and we can therefore *calculate* the pitch n_2 of a length of wire l_1 when under a tension T_2.

Similarly the length l_1 would have had a vibration frequency $n_3 = n_1 \frac{l_3}{l_1}$, if it had been sounding under a tension T_3.

Calculate n_2, n_3, etc., and show that n is proportional to $\sqrt{T}$.

Arrange the results of the observations as in the Table :—

Frequency of note of fixed string $= n_1$.

Tension of Weighted Wire in Gm.-wt. T.	Length giving Note of Frequency n_1. l.	*Calculated* Frequency for a Length l_1. $n = n_1 \frac{l_2}{l_1}$, etc.	$\frac{\sqrt{T}}{n}$.
$T_1 =$	$l_1 =$	$n_1 =$	$\frac{\sqrt{T_1}}{n_1} =$
$T_2 =$	$l_2 =$	$n_2 = \frac{l_2}{l_1} n_1 =$	$\frac{\sqrt{T_2}}{n_2} =$
$T_3 =$	$l_3 =$	$n_3 = \frac{l_3}{l_1} n_1 =$	$\frac{\sqrt{T_3}}{n_3} =$

The last column of the table will be found to be constant, *i.e.* ***n*** **is proportional to** $\sqrt{T}$.

EXPT. 88. **Variation of Frequency with Mass per Unit Length.**—Stretch a wire on the sonometer with a given load, and find what length of it vibrates in unison with the fixed wire.

Remove the wire, and replace it by a second wire stretched

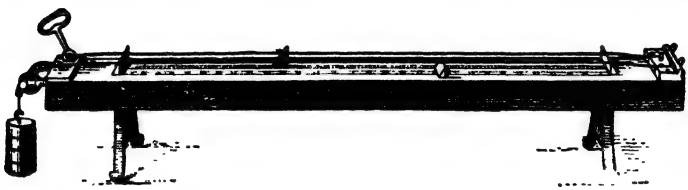


FIG. 126.—Horizontal Sonometer.

with the same load; again find the length giving unison with the fixed wire.

Repeat this for three or four different wires, using wires of different material, or different diameter, each time.

Weigh each wire or a portion of each wire, measuring the length of the part weighed: a considerable length should be used to obtain accuracy. Calculate the mass of 1 cm. of each of the wires used.

By the result obtained in Experiment 86 calculate what frequency each of the wires would have had, if each had been under the same tension but of length equal to the length of the first wire used.

This gives the value of n for each of the wires when equal lengths are vibrating under the same tension.

Show that $n\sqrt{m}$ is the same for each wire, *i.e.* that n is proportional to $1/\sqrt{m}$.

Arrange the results as below :—

Frequency of fixed wire $= n_1$.

Lengths of Wire vibrating in Unison with Fixed W. (l_1, l_2, l_3, etc.)	Mass of each Wire per cm. Length (m_1, m_2, etc.)	Calculated Frequency of a Length l_1 of each Wire under the given Tension (n_1, n_2, etc.)	$n\sqrt{m}$
l_1 =	m_1 =	n_1 (as above) =	$n_1\sqrt{m_1}$
l_2 =	m_2	$n_2 = \frac{n_1 l_2}{l_1} =$	$n_2\sqrt{m_2}$
l_3 =	m_3	$n_3 = \frac{n_1 l_3}{l_1} =$	$n_3\sqrt{m_3}$

The last column of the table will be found to be constant, *i.e.* ***n* varies as $1/\sqrt{m}$.**

EXPT. 89. **Absolute Determination of Pitch with a Sonometer.**—Stretch a wire with a known force T dynes. Find the length l cm. vibrating in unison with the fork whose frequency is to be determined.

Cut off a length of the wire, weigh it, and determine its mass per unit length, m gm. per cm.

Calculate the vibration frequency of the wire, which is the same as that of the fork with which it is in unison.

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}.$$

These results can be applied in various ways. The following exercises are suggested as corollaries on the vibration of strings :—

EXPT. 90. **Determination of the Density of the Material of a Wire, using the Sonometer.**—In this determination the wire must not be removed from the sonometer board. A fork of known pitch is supplied. Stretch the wire with a known force, and tune a length of it to unison with the given fork.

In the equation $$n = \frac{1}{2l}\sqrt{\frac{T}{m}}.$$

n is given, T is known, and l is measured. Hence m can be calculated.

Now m is the mass of a cylinder of metal 1 cm. long, and of diameter equal to that of the wire, *i.e.*

$$m = \pi r^2 \rho,$$

where r is the radius of the wire, and ρ its density. Hence ρ can be determined from the calculated value of m, if the radius of the wire is measured with a micrometer screw.

EXPT. 91. **Determination of the Weight of a Given Load by the Sonometer.**—Another useful exercise is to weigh a bag of weights by means of a sonometer. A fork of known pitch is supplied, and a length of wire l, when stretched with the bag of weights, is tuned to unison with the fork. The mass of 1 cm. of the wire is determined by weighing a considerable length of the wire. Thus in the equation

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}},$$

n, l, and m are all known or determinable, and hence T can be calculated.

Now $T = Mg$ dynes, where M is the mass of the bag in grams. Hence

$$M = \frac{T}{g},$$

and the mass of the bag can thus be found.

NOTE.—Avoid such 'formulae' as

$$n = \frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}},$$

$$\rho = \frac{T}{4l^2r^2n^2\pi}, \text{ etc.}$$

These are quite true, but to remember them is a useless burden on the memory, and the results required can always be found from the fundamental equation

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

by an application of first principles. Even the fundamental equation itself need not be remembered, if the student will deduce it as shown on p. 218.

§ 3. NOTES ON TUNING

In tuning a fork and a string, two strings, or any two notes to unison, there may be a difficulty if the student has no musical 'ear.' In this connection certain aids may be used to indicate when the tuning is correct. One of these is given by the phenomenon of *beats*. When tuning is close but not perfect, the two notes will 'beat,' and the rapid variations in intensity of the sound will be observable even if there is no musical appreciation of approximate unison.

The notes will be in unison when the beats are so slow that they cannot be detected. In using the sonometer, adjust the string by *very small alterations in length* so that the beats get slower and slower. When they can no longer be distinguished, the notes may be considered as identical, *i.e.* the frequencies of the sounding bodies are equal.

Another method, when a horizontal string is used, is to place a small rider of paper on the middle of the string to be tuned. If the other string or the fork be sounded, the fork being allowed to rest on the sonometer board, the paper rider will *flutter* if tuning is close; it will be thrown violently off the wire if the tuning is exact. Thus, by altering the length so as to increase the fluttering, the wire may be tuned to the other string or fork. The reason for this has already been dealt with under the 'Principle of Resonance' (p. 204).

In plucking a string to excite its note, care should be taken to avoid touching the string with the finger nail, as this may introduce overtones; the string should be pulled aside between the thumb and finger.

When listening to the note of a string, a small plate of wood fitted with a wooden handle may be held to the ear, the end of the handle being placed in contact with the base-board of the sonometer.

PART II

ADDITIONAL EXERCISES IN SOUND

1. Having given two tubes arranged so that one can slide inside th other, measure the frequency of a fork by finding the position of resonanc for the compound tube open at both ends.

2. Compare the frequencies of two tuning-forks by the dropping plat method.

3. Find the velocity of sound in the given glass rod.

4. Find the mass of 1 cm. of the given cord by determining the velocit of a transverse wave along it under a known tension.

5. Compare the densities of the materials of two wires by means of th sonometer.

6. Compare the frequencies of two forks by means of the sonometer.

7. Compare the weights of two bags by means of the sonometer.

8. Stretch a wire with different weights, and find what lengths of th wire are in unison with a fork of known frequency. Deduce the stretching force required to make a similar wire, 2 metres long, vibrate with a frequency of 50 vibrations per second.

9. Plot a curve showing the relation between the volume of the given resonator and the frequency of the resonant vibration. (The resonator in this experiment is an ordinary medicine bottle with a narrow neck. The volume may be altered by adding water.)

PART III

LIGHT

CHAPTER I

THE LAWS OF GEOMETRICAL OPTICS

§ 1. Parallax

Light travels in straight lines so long as its path lies in a medium the properties of which are the same at all points and in all directions round each point. From the **Rectilinear Propagation of Light** it follows that a ray of light, *i.e.* a beam of light of very small cross section, can be represented by a geometrical straight line.

The direction in which any object is seen depends on the direction of the ray entering the eye of the observer. The term **parallax**, originally used in connection with astronomical observations, means the apparent displacement of an object caused by an actual change in position of the observer; if a difference is made in the position of the point of observation, there is a corresponding difference in the apparent position of the object. Thus, if two bodies are viewed from a certain point, their relative positions will be altered if the observer move to another position. To illustrate this, set up two retort stands on a table and view them from such a position that the more distant rod appears to be directly behind the nearer. If the observer now move to the right, the further rod will appear to the right of the nearer, as viewed from this new position. Similarly, if the observer move to the left, the more distant rod will appear to move to the left of the nearer. **Thus, the object at a greater distance moves in the same direction as the observer, relatively to an object at a smaller distance.**

When the rods are placed nearer together, the relative motion of the rods for a given displacement of the observer is reduced. If the rods are placed one in continuation of the other, they appear together from any position in which the observer places himself. The same principle applies in dealing with images formed by mirrors or lenses. When two bodies are coincident, or in continuation of each other, there is *no parallax* between them, and this test is frequently used to determine if two bodies or two images are coincident. If parallax is observed, the rule given above determines at once which of the two is more distant. This method of experimenting is termed the **method of parallax.** Interesting examples of parallax may be met with in everyday life, as in the case of objects seen from the windows of a rapidly moving conveyance.

§ 2. Reflection at Plane Surfaces

THE LAWS OF REFLECTION

When a ray of light falls upon a polished surface it is reflected in accordance with the following laws :—

Law I. The incident ray, the reflected ray, and the normal to the surface lie in one plane.

Law II. The angle between the incident ray and the normal (the angle of incidence) is equal to the angle between the reflected ray and the normal (the angle of reflection).

Expt. 92. **Verification of the Laws of Reflection.**— These laws may be verified by means of a plane mirror and a number of pins in the following way :—

Attach a sheet of drawing-paper to a drawing-board, and on it place a strip of mirror with its plane vertical. The mirror can be supported by a wooden block with a vertical groove. The mirror should be of good plate glass but quite *thin*. If possible, glass silvered on the *front* surface should be used.

Draw a line on the paper to mark the position of the *reflecting* surface. Fix two pins into the board at points such as P and Q (Fig. 127). On looking into the mirror the images of these pins will be seen. Move the head till these images

appear in a straight line, and then fix two other pins, R and S, in line with the images. P and Q should be some distance apart, say 10 cm. or 15 cm. R and S should be at a similar distance apart. The line PQ is the trace of an incident ray, RS that of a reflected ray.

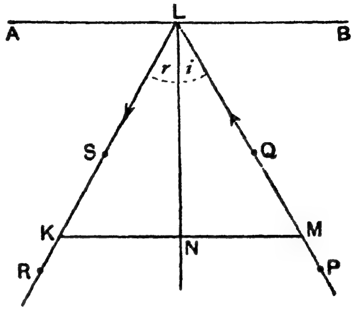


FIG. 127.—Laws of Reflection.

If the mirror is perpendicular to the drawing-board, the feet of the pins R and S should appear in an unbroken line with the feet of the pins P and Q. For in this case the normal to the mirror lies in the plane of the drawing-board, and according to the first law of reflection the incident ray, the reflected ray, and the normal to the mirror must lie in one plane.

Let the rays PQ and RS meet the mirror at the point L. Draw LN, the normal to the mirror at L. Measure the angles MLN and KLN with a protractor, and also measure equal distances LK, LM (say 10 cm.) along the two rays and join KM.

If KN and MN are equal, the triangles are equal in all respects, and the angles MLN and KLN are equal to each other. Measure the lengths of KN and MN and record the results.

In order to verify the second law of reflection, the determination must be repeated for *at least two* other directions of the incident ray. In each case the angle of incidence should be found approximately equal to the angle of reflection.

If the mirror is *thick*, PQ and RS will meet at a point behind the front surface about two-thirds of the way through the glass. The place where they intersect should be taken as the equivalent reflecting surface.

The image of any object formed by a plane mirror is at the same distance behind the mirror as the object is in front of it.

EXPT. 93. **The Image formed by a Plane Mirror.**—Fix a pin somewhere in front of a plane mirror and find by trial the position of the image. This can be done by viewing a second pin over the top of the mirror and moving it about till

there is no parallax between the second pin and the image of the first, *i.e.* there is no relative motion between the image and the second pin on moving the head from side to side (see p. 229).

Measure the perpendicular distance from the first pin to the equivalent reflecting surface, and also from the image to the equivalent reflecting surface. Make a note of the distances which should be approximately equal and enter the results in your notebook.

Two mirrors inclined at an angle give rise to a series of images.

EXPT. 94. **Inclined Mirrors.**— Draw two lines at an angle with each other, say at 90° and again at 60°, on a horizontal sheet of paper. Set up two mirrors on these lines. Place a pin somewhere in the angle between the mirrors and find the positions of *all* the images produced by reflection in the mirrors in each case. Verify the fact that all the images lie on a circle whose centre is at the point of intersection of the two mirrors, and that the number of images formed is $\frac{360}{\theta} - 1$, where θ is the angle in degrees between the mirrors.

One image will be found in the angle behind both mirrors; trace the path of the rays of light from the pin to the eye of the observer by means of which this image is seen.

Each image should be labelled I_1, I_2, $I_{1,2}$, according as it is formed by single reflection in mirror 1, single reflection in mirror 2, double reflection first in mirror 1, then in mirror 2, etc.

ROTATION OF A MIRROR

When a mirror is rotated about an axis perpendicular to the plane of incidence, the reflected ray is turned through an angle twice as great as that through which the mirror turns.

Let AB (Fig. 128) be the original position of the mirror, ML the incident, LK the reflected ray, and LN the normal to the surface. Let the accented letters denote corresponding quantities after the mirror has turned through a certain angle. The student should deduce from the laws of reflection already investigated that K′LK—the angle through which the reflected ray turns—is twice N′LN—the angle through which the mirror turns.

EXPT. 95. **Reflection from a Mirror which is rotated.**—The result that the reflected ray turns through twice the angle turned through by the mirror, may be verified experimentally by tracing the rays by means of pins. Two pins are used to define the incident ray ML, and two to define the reflected ray LK. After the position of the reflecting surface has been marked, the mirror is turned through a certain angle and the new position of the reflected ray is determined by pins as before. The angle through which the mirror is turned and the angle through which the reflected ray is turned are measured with a protractor. The ratio of the second angle to the first is then calculated. The process is repeated for a number of different positions of the mirror, and the results are given in tabular form.

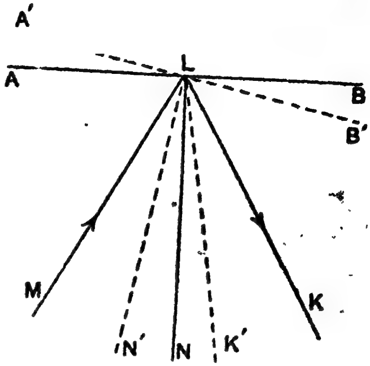


FIG. 128.—Rotation of a Mirror.

Show also that if the direction of the *reflected* ray is kept constant, the objects viewed when the mirror is used first in one position, then when rotated through an angle θ, are along directions which subtend an angle 2θ at the axis of rotation of the mirror.

THE SEXTANT

The **Sextant** is an instrument for measuring the angular separation between two distant objects; that is, the angle between the lines joining the eye of the observer to the two objects. It is used in navigation for measuring the altitude of the sun, or of a star.

EXPT. 96. **Examination and Adjustment of the Sextant.**—Examine the sextant, and notice that the instrument consists of a graduated arc AB, of about 60°, connected with two fixed radial arms CA and CB. A third radial arm CD is movable about the centre C of the arc, and carries an index and vernier at D. A clamp fitted with a tangent screw is provided to give a slow motion of this arm. At C is a plane mirror, called

the **index glass**, attached to the arm and turning with it; the plane of this mirror should be perpendicular to the plane of the arc. At E is fixed a piece of plate glass, whose lower half only is silvered. The plane of this glass also is perpendicular to the plane of the arc. It is called the **horizon glass.** T is a telescope pointing towards the horizon glass. Usually the instrument is provided with a number of coloured glasses for diminishing the intensity of the sun's light.

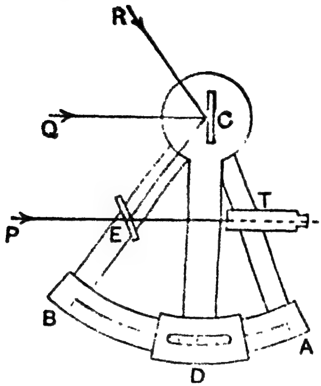


FIG. 120.—The Sextant.

When the index glass is exactly parallel to the horizon glass, rays from a distant object can reach the telescope by separate paths. One set of rays passes through the unsilvered part of the horizon glass and enters the telescope without deviation. Another set of rays is reflected from the mirror to the silvered portion of the horizon glass, and enters the telescope in the same direction as the first set. The parallel rays are brought to a focus in the focal plane of the object glass, and give rise to a single image of the distant object. When this is the case the index arm D should be at the zero of the graduated arc. If it be not, the reading of D should be taken; it is called the **zero reading.** If now the arm D with its mirror be turned through a small angle, the rays reflected by the mirror will enter the telescope at a different angle, and the image formed by these rays will be displaced with regard to the image seen directly.

Suppose it is required to measure the angle between two objects in the directions EP and CR respectively. The sextant is supported so that the telescope is pointed directly towards the object in the direction EP, the rays passing through the unsilvered glass. The mirror C is turned till rays coming in the direction RC are reflected along CE, and fall on the silvered glass which reflects them into the telescope. Then the angle between the directions of the two objects is the angle RCQ which is twice the angle ACD, through which the arm CD has turned from the zero position. This is seen from the foregoing experiment, the direction of the ray reflected from C being the same (CE) in each case.

To save calculation, the arc AB is usually graduated so that each degree is numbered as 2 degrees; and the required angle can therefore be read directly from the graduations. The difference between the present reading and the zero reading gives the angle RCQ required.

In order to obtain accurate results the following conditions must be satisfied :—

(1) The plane of the index glass must be perpendicular to the plane of the graduated arc.

(2) The axis of the telescope must be parallel to the plane of the arc.

(3) The zero reading must be taken for each pair of objects used; it varies if the distance of the objects from the sextant is altered.

Adjusting screws are provided for making the necessary adjustments, but we shall assume that this has already been attended to by the instrument-maker.

Expt. 97. **Measurement of Azimuth with the Sextant.** – Use the sextant to measure the angle θ between two objects in the same horizontal plane. Two candles or two incandescent lamps may conveniently be employed. The sextant must be held in the same horizontal plane as the objects. Measure the distance to each object, and calculate the distance between the objects from these distances and the angle θ. Confirm the result by measuring the distance between the objects directly.

Expt. 98. **Measurement of Altitude with the Sextant.** —Use the sextant to measure the angular elevation of a distant object. In this measurement the foot of the object should be on the same level as the sextant. Calculate its height from the angle measured and the horizontal distance from the foot of the object to the sextant. If possible, confirm the result by measuring the height. A better method of finding the altitude of a distant object is to measure the angle between the object seen directly and its image formed by a horizontal mirror, such as the surface of mercury. The angle thus measured is twice the required altitude.

§ 3. Refraction at Plane Surfaces

THE LAWS OF REFRACTION

When a ray of light passes from one medium into a second, its direction is in general changed, and the direction of the

refracted ray is found to obey, for all isotropic[1] media, the two laws of refraction here stated.

Law I. The incident ray, the normal to the surface, and the refracted ray lie in one plane.

Law II. The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for any two particular media and for light of a particular colour. This constant is called the **refractive index** (μ) in passing from the first medium into the second.

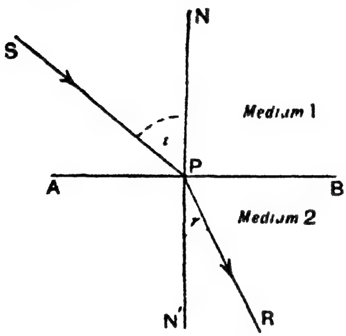


FIG. 130. Law of Refraction.

Thus if AB (Fig. 130) represent the surface of separation between the two media, SP a ray incident at the point P, and NPN′ the normal to the surface, the refracted ray PR lies in the plane containing SP and NPN′, and

$$\frac{\sin i}{\sin r} = \text{a constant} = {}_1\mu_2$$

where i is the angle of incidence, *i.e.* the angle SPN, and r is the angle of refraction, *i.e.* the angle RPN′.

The absolute refractive index of a substance is the value of the constant when a ray passes into the substance from a vacuum; this is nearly the same as the value obtained for a ray passing into the substance from air.

EXPT. 99. **Verification of the Laws of Refraction.**—Place a rectangular glass block on a large sheet of drawing-paper, and mark its position by drawing a fine pencil line round it. Then set up two pins on one side of the block, so that the line joining them may represent the direction of a ray incident obliquely on the glass face. The two pins should be *at least* 10 cm. apart. Look through the glass from the opposite side and move the head until both pins can be seen. Then, using one eye only, move the head until the two pins appear

[1] An isotropic medium is a substance which shows no differences of quality in different directions.

to be in one straight line. Set up two more pins between the block and the eye, so that these also appear in line with the two pins on the far side of the block. The second pair of pins should be *at least* 10 cm. apart. Note that when the eye is just on a level with the surface of the paper the points where the four pins pierce the paper appear to be in one straight line. Since the block is rectangular, the normal to

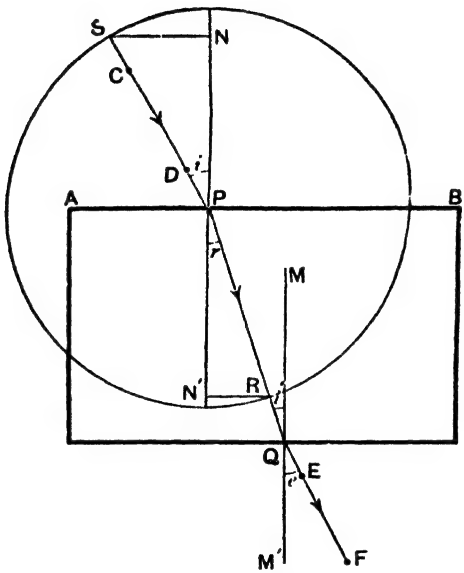


FIG. 131.—Refraction through Glass Block.

the refracting surface lies in the plane of the paper, so that the first law of refraction is verified.

Mark the positions of the pins (C, D, E, F) and remove pins and block from the paper. Join CD and produce the line to meet the first surface of the block at P. Join EF and produce the line to meet the second surface of the block at Q. CD is evidently the direction of the incident ray and EF that of the emerging ray ; hence the ray entered the glass at P and emerged at Q. Join PQ ; then PQ represents the direction of the ray of light through the glass. Verify that the emergent ray EF is parallel to the incident ray CD. Draw the normals at P and at Q.

The angle of incidence at the first surface is SPN. Call this angle i.

The angle of refraction at the first surface is N'PQ. Denote this by r.

The corresponding angles, MQR and M'QR at the second surface, are denoted by i' and e respectively.

To find the ratio of $\sin i$ to $\sin r$ two methods may be employed:

(1) Measure the angles i and r with a protractor, and look up the values of the sines in the tables (p. 606). Calculate the value of $\dfrac{\sin i}{\sin r}$.

(2) **A Graphic Method.**—Describe a circle with P as centre and a radius of at least 10 cm. Find the point S where the incident ray cuts the circle, and also the point R where the refracted ray PQ (produced if necessary) cuts the circle. From S and R draw perpendiculars SN and RN' to the normal at P. Measure carefully the lengths of these perpendiculars.

Then $$\frac{\sin i}{\sin r}=\frac{SN}{RN'}\Big/\frac{SP}{RP}=\frac{SN}{RN'}.$$

Calculate the value of $\dfrac{SN}{RN'}$.

In order to verify the second law of refraction the determination must be repeated for *at least two* other positions of the incident ray. The values of $\dfrac{\sin i}{\sin r}$ obtained for different values of i should be in close agreement. The mean of these values may be taken to represent the refractive index of the glass.

A similar determination may be applied for the refraction at the second surface of the glass, showing $\dfrac{\sin i'}{\sin e}$ is a constant.

The first constant determined, $\left(\dfrac{\sin i}{\sin r}\right)$, is the refractive index from air to glass.

The second constant is the refractive index from glass to air.

They may be denoted by ${}_a\mu_g$ and ${}_g\mu_a$ respectively. It will be found that ${}_g\mu_a = 1/{}_a\mu_g$.

This can be deduced from the facts that the block of glass is parallel-sided and the emerging ray is parallel to the incident ray, *i.e.* $i=e$ and $r=i'$.

Hence $${}_a\mu_g = \frac{\sin i}{\sin r} = \frac{\sin e}{\sin i'} = \frac{1}{{}_g\mu_a}.$$

Since $i = e$, there is no *deviation* produced by refraction through a parallel-sided block of any medium, the deviation at the one surface being exactly reversed at the other.

REFRACTION THROUGH A PRISM

When a ray passes through a prism of glass, or of some optically dense medium, as in Fig. 132, the deviation at the first surface is usually followed by a deviation *in the same direction* at the second surface. Even if this be not the case, there is *on the whole* a deviation produced when a ray of light passes through a prism, the ray being bent towards the base of the prism. The angle between the direction of the emergent

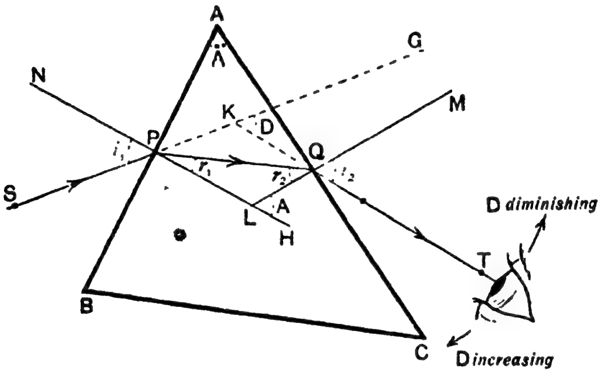


FIG. 132. Refraction through a Prism.

ray QT, and the direction of the incident ray SP, is called the **angle of deviation.** It is the angle marked D in Fig. 132. The amount of deviation produced by a given prism varies with the angle of incidence. It can be shown both by theory and by experiment, that the angle of deviation is least when the ray passes through the prism symmetrically; that is, when the direction of the ray in the glass (PQ) makes equal angles with the sides of the prism. The prism is then said to be in the **position of minimum deviation.**

In this case it can be shown that if i and r are the angles of incidence and refraction, the deviation $D = 2i - 2r$, and the angle of the prism $A = 2r$.

This gives $i = \frac{1}{2}(A + D)$ and $r = \frac{1}{2}A$.

Consequently

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A}.$$

To find the refractive index of the glass of a prism w measure the refracting angle A of the prism, and the angl D of minimum deviation, and calculate μ from the formula

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A}.$$

EXPT. 100. **Refraction through a Glass Prism, using Pins.**—Place a large glass prism on a sheet of drawing-paper with its refracting edge vertical, marking its position with a fine pencil line. Place a pin very close to one face of the prism and another pin about 10 cm. away from the first. Look into the other side of the prism, moving the eye until the two pins when seen through the refracting angle of the prism appear to be one behind the other. Place two pins between the eye and the prism, in line with these first pins as viewed through the prism. Draw the incident and emerging rays, produce them until they meet, and find the angle of deviation produced: find also the angle of emergence. Keep the first pin in the same position in contact with the prism side, but place the second pin so that the incident ray considered makes a different angle of incidence with the prism face; find the corresponding emerging ray and angle of deviation.

Repeat this for several angles of incidence, altering this by 5° at a time, and plot a curve showing the variation of the angle of deviation with different angles of incidence.

This curve will have a minimum value corresponding with a certain angle of incidence. Show that for this value the angle of incidence and the angle of emergence are equal.

EXPT. 101. **Determination of the Angle of Minimum Deviation for a Prism (Pin Method).**—Place the prism on the drawing-board as before, with a pin in contact with one of the sides including the refracting angle, and a second pin about 10 cm. from this.

Observe these pins by looking through the refracting angle from the other side of the prism, placing the eye so as to see the two pins, one behind the other. Rotate the prism about the pin in contact with the side, moving the eye so as to keep the pins one behind the other in all positions of the prism.

The prism when rotated in one direction will require the eye to move in the direction *in which the refracting angle points*; reversing the direction of rotation will require a motion of the eye in the opposite direction (Fig. 132). In the first case, the motion of the prism reduces the deviation of the light, and as this is what is desired, the prism must be rotated so that the eye, when sighted on the pins, moves in the direction in which the refracting angle points.

When the prism has been rotated through some angle in this manner, the pins will appear to be stationary for a little while, although the prism is being moved slowly all the time. A further rotation of the prism requires the eye to *reverse* its former motion. This means that the deviation is beginning to *increase* again. The prism must be rotated back again slightly until the eye is as near the dotted line GK as possible.

When the prism is in the position of minimum deviation, set up two pins to mark the direction of the emergent ray, and trace the outline of the prism on the paper marking the refracting angle A. The prism and the pins may now be removed, and the diagram completed by producing the incident and the emergent rays so as to show the angle of minimum deviation, D. As a check on the accuracy of the setting, notice whether the path of the ray through the glass makes equal angles with the faces of the prism.

To determine the refractive index of the glass by means of the formula

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A}$$

two methods may be employed :—

(1) **With the Protractor.**—Measure the angles A and D with the protractor, and find the values of $\sin \frac{1}{2}(A + D)$ and $\sin \frac{1}{2}A$ by means of the tables.

(2) **A Graphic Method.**[1]—The angle of minimum deviation having been marked on the paper as already described, the prism is placed on the paper so that one edge coincides with the direction of the emergent ray, and the vertex of the prism coincides with

[1] For this method we are indebted to our colleague, Dr. W. Wilson.

the point of intersection of the incident and emergent rays. The angle of the prism is then marked on the paper by drawing a line along the second edge of the prism, as in Fig. 133. With K as centre and a radius of 10 or 15 cm. a circle is described, cutting the three lines in X, Y, and Z.

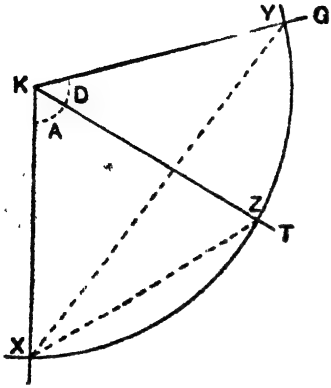


FIG. 133.—To find μ.

Then from the geometry of the figure

$$\frac{\sin \frac{1}{2}(A+D)}{\sin \frac{1}{2}A} = \frac{XY}{XZ},$$

and therefore

$$\mu = \frac{XY}{XZ}.$$

Measure the lengths of XY and XZ and calculate the value of μ.

TOTAL INTERNAL REFLECTION AND THE CRITICAL ANGLE

When a ray of light passes from a denser into a rarer medium, it is bent *away from* the normal, *i.e.* $\frac{\sin i}{\sin r}$ is < 1.

Thus the angle of refraction increases more rapidly than the angle of incidence.

For a certain angle of incidence C, the emerging ray is parallel to the surface, *i.e.* the angle of refraction is 90°, and thus $\frac{\sin C}{\sin 90}$ = the refractive index from the denser to the rarer medium.

But $\sin 90° = 1$; therefore this refractive index is equal to $\sin C$.

Light incident at an angle *greater than* this angle C cannot emerge, since the angle of emergence would require to have a sine greater than unity, which is impossible; the whole of the light in this case would be reflected *totally internally*.

This limiting angle C, which is the angle of incidence for grazing emergence, or which is the smallest angle of incidence

for which **total internal reflection** occurs, is called the **critical angle** for this pair of media.

If the rarer medium is air, sin C is the reciprocal of the refractive index for the denser medium (p. 238), since it is the refractive index *from the dense medium into air.*

EXPT. 102. **Determination of the Critical Angle.**—A thin film of air is enclosed between two parallel plates of plate glass which are kept separate by a thin india-rubber ring, or an annulus cut out of tinfoil. This apparatus is fixed to a vertical spindle parallel to the faces of the plates, so that the whole may be rotated about a vertical axis. The angle of rotation can be measured by means of a co-axial circular scale, AA′ (Fig. 134).

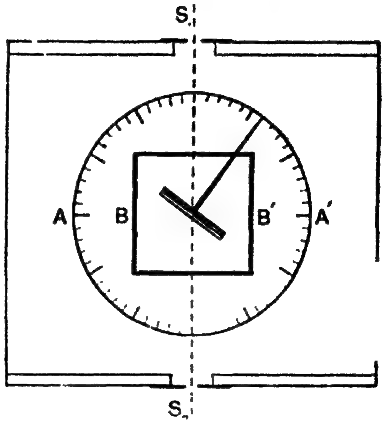


FIG. 134.—Determination of Critical Angle.

The glass plates with the enclosed air film dip into the liquid of which the critical angle is to be determined. The liquid is contained in a cubical glass box BB′, the sides of which are of plate glass.

A beam of light is passed through the liquid in a direction at right angles to one pair of sides. In order to obtain a definite beam, two narrow slits S_1 S_2 are employed, so that the eye looking through one slit receives the beam from a source of light beyond the other slit.

When the air film is perpendicular to the path of the beam of light, the light passes through it. As the spindle is rotated, the angle of incidence of the light passing from the liquid to air increases until the critical angle is reached. If the spindle is rotated through a greater angle, total reflection occurs, and no light is transmitted. The position on the graduated circle at which this occurs is noted. The cell is then rotated back so that the light is transmitted again, the rotation being continued until the light is extinguished once more. The angle through which the cell has been turned is equal to twice the critical angle for the liquid.

The refractive index of the liquid is then obtained from the formula

$$\mu = \frac{1}{\sin C}.$$

Determine in this way the critical angle and the refractive index for water.

The extinction takes place when the light is incident *in glass* to the air film between the glass plates, *i.e.* when the angle of incidence *in the glass* is the critical angle for glass. The critical angle obtained, however, is the critical angle for water. This is shown as follows:—

If the angle of incidence in the water is C, the critical angle for water, and the angle of incidence to the glass-air surface is C', then

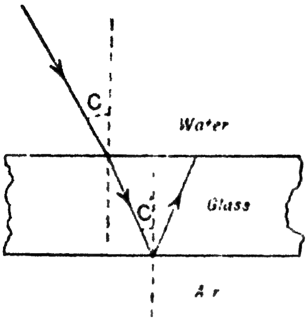


Fig. 135.—Critical Angle.

$$\frac{\sin C}{\sin C'} = \mu \text{ between water and glass}$$

$$= \frac{\mu \text{ for glass}}{\mu \text{ for water}}$$

i.e. $$\frac{\sin C}{\sin C'} = \frac{\mu_g}{\mu_w}$$

Now $$\sin C = \frac{1}{\mu_w}$$

$$\therefore \sin C' = \frac{1}{\mu_g}$$

Hence, if the light is incident to the water-glass surface at the critical angle for water, the refracted light will strike the glass-air surface at the critical angle for glass. Thus total internal reflection will occur at the glass-air surface when the angle of incidence to the water-glass surface is the critical angle for water.

REFRACTIVE INDEX BY APPARENT THICKNESS

To an observer looking vertically into a pond the depth of the water appears less than it actually is. In the same way the thickness of a slab of glass appears smaller than its real thickness to an observer looking through it. This is a direct consequence of the bending or refraction of the light in passing from the water or the glass into the air.

Let P be a point from which rays of light pass to emerge at the surface SS′ separating the two media (Fig. 136). Two rays PS, PS′ equally inclined to the normal PO will be refracted in directions SQ, S′Q′ as shown. The directions of these refracted

rays, if produced backwards, meet in a point P′. An observer looking into the optically denser medium and receiving these

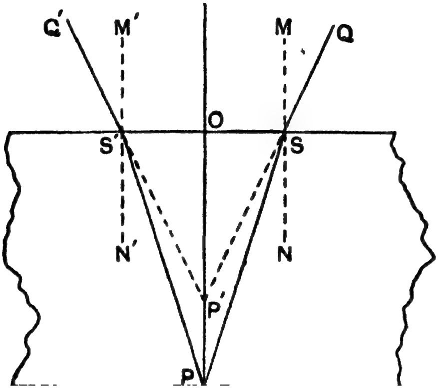


FIG. 136.—Apparent Thickness.

two rays would infer that the point P′ represented the position of the object.

If μ be the refractive index in passing into the denser medium,

$$\mu = \frac{\sin QSM}{\sin PSN} = \frac{\sin SP'O}{\sin SPO}$$

$$= \frac{OS/SP'}{OS/SP} = \frac{SP}{SP'}.$$

When the observer is looking normally into the slab, the angles OPS and OP′S are very small, so that SP becomes practically the same as OP, and SP′ as OP′.

Then $$\mu = \frac{OP}{OP'} = \frac{\text{Real thickness}}{\text{Apparent thickness}}.$$

If then we measure the real thickness and also the apparent thickness we can determine at once the refractive index of the material.

EXPT. 103. **Refractive Index for Water by Apparent Depth.**—Place a small white object at the bottom of a can or beaker. A pointed piece of white paper weighted by a

coin will serve admirably. The bottom of the can must be blackened, or the beaker must stand on black paper or a dark bench. Fill the vessel with water, and place it at such a height that the observer can look down into it. Then fix a second paper pointer in a stand so that its height above the surface of the water can be adjusted. On looking down into the water the first paper can be seen without difficulty, and a *reflected* image of the second paper formed by reflection at the water surface can also be distinguished, *provided the lower face of the second paper is well illuminated.* The height of the second paper must now be adjusted until there is no parallax between the two images in question. The reflected image and the refracted image are now coincident; but the image formed by reflection is at the same distance below the water surface as the paper pointer is above that surface. Consequently the apparent depth is equal to the distance of the second paper above the water surface. Measure the apparent depth and the real depth and calculate the refractive index.

EXPT. 104. **Refractive Index for Glass by Apparent Thickness.**—Use a large rectangular block of glass placed on a sheet of white paper across which a straight line has been drawn. On looking down from above the whole of the line can be seen, but part of it is viewed through the glass, part of it through air alone. The part of the line seen through the glass is apparently raised. Its apparent position is determined by raising or lowering a horizontal pin placed parallel to the line, with its point in contact with the side of the block, and finding when there is no parallax between the point of the pin and the line seen through the glass. The pin for this purpose should be mounted in a stand capable of vertical adjustment.

Measure the distance from the point of the pin to the upper surface of the glass, and also the real thickness of the block of glass. Calculate the refractive index of the glass.

This method is suitable only for thick blocks of glass. For thin sheets (2 cm. thick or less) a vernier microscope with a vertical adjustment is often employed.

EXPT. 105. **Refractive Index by using a Microscope.**—The microscope is focussed (*a*) on the paper, or some other flat surface, when there is no glass, (*b*) on the paper through the glass, and (*c*) on the upper surface of the glass, care being taken that there is no parallax between the image seen and the cross-hairs of the microscope in each case.

From the readings of the vernier scale in these positions both the real thickness and the apparent thickness of the glass plate are easily deduced. The refractive index is then calculated as before.

The method with the vernier microscope is also applicable in the case of liquids of which only small quantities are available. The microscope is focussed on the base of the containing vessel when it is empty and also when it is full, and again on the upper surface of the liquid, a floating speck of lycopodium being used for focussing on the upper surface.

§ 4. Caustic Curves

In the elementary theory of reflection and refraction at plane and spherical surfaces, it is assumed that a pencil of rays proceeding from a given point will, after reflection or refraction, converge to or diverge from a second point—the **conjugate focus.** In general, this is only approximately true. Two consecutive rays may intersect one another, but the point of intersection need not exactly coincide with the point of intersection of two neighbouring consecutive rays; all the rays touch a certain curve called the **caustic curve.**

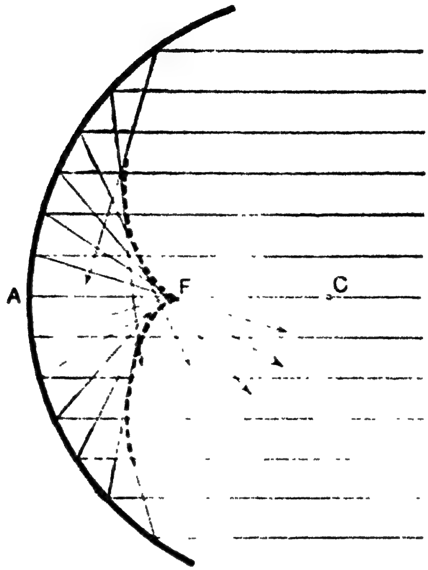
As an example, consider reflection at a concave hemispherical mirror when the incident rays are all parallel to the principal axis. Fig. 137 shows that only rays near the axis pass through the principal focus half-way between C and A. The other reflected rays touch a caustic curve which is symmetrical about the axis and has a **cusp** at the point F which is the principal focus of the mirror.

Expt. 106. **Caustic by Reflection.** Make an accurate drawing to scale in the note-book showing the caustic curve when parallel rays fall on a hemispherical mirror.

Draw a semicircle to represent a section of the mirror. Then draw any ray parallel to the axis CA. The corresponding reflected ray may be found by a simple construction. Draw a circle with centre C so as to touch the incident ray. Draw another tangent to this circle from the point where the incident ray meets the mirror. This tangent represents the reflected ray. (Prove this.)

Repeat the construction for several rays parallel to the axis, and draw a curve to represent the locus of intersections

of consecutive reflected rays. This curve is a section of the caustic by reflection at a concave hemispherical mirror for incident rays parallel to the axis.



[illegible] by R[illegible]

The caustic curve formed by refraction through a glass may be traced experimentally by means of pins.

EXPT. 107. **Caustic by Refraction.**—Lay the glass block on a sheet of drawing paper and fix a pin about 2 cm. from one corner on one of the longer sides as at A (Fig. 138). Mark off a number of points P_1, P_2, P_3, [illegible] on the opposite side of the block half a centimetre apart. Place a pin at one of these points, P_1 say, and determine where another pin P_1' must be placed, so that on looking through the block the three pins appear in a straight line. Repeat the observation for each point P. When all the points have been found, remove the glass block, the trace of which should be marked on the paper. Join the points P_1 P_1' by a line and produce it in both directions. Join the points P_2 P_2' in the same way. These two lines will meet in a point which represents the virtual image of the pin A as seen by an eye near P_1'. Join the remaining pairs of points in the same way. If the work has been done

carefully it will be found that all the lines touch a caustic curve having a well-defined cusp.

In order to obtain both branches of the caustic curve, it may be necessary to shift the glass block sideways to the position indicated by the dotted lines. To an eye looking normally

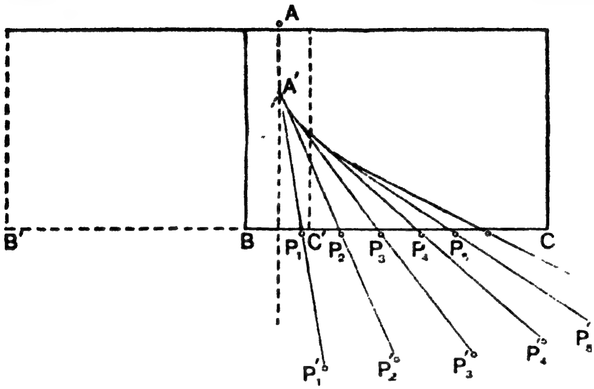


FIG. 138.—Caustic by Refraction.

through the block, the virtual image would appear at the cusp of the caustic. The diagram should accompany the student's notes or be reproduced in the note-book.

Next calculate the refractive index of the glass by the formula proved on p. 245.

$$\mu = \frac{\text{Real thickness of block}}{\text{Apparent thickness of block}}.$$

The apparent thickness is the distance from the face of the block nearest the observer to the cusp of the caustic curve. The real thickness should be measured with a pair of callipers.

NOTE.—The position of the point of the cusp A′ cannot be determined with great accuracy, and the value of μ obtained by this graphic construction is therefore not very exact.

CHAPTER II

SPHERICAL MIRRORS

§ 1. Introductory Theory

The **mirror** usually considered in elementary work is a polished surface having the form of a portion of a sphere. The centre of the sphere of which the mirror forms a part is called the **centre of curvature** of the mirror. If the polished surface faces the centre of curvature the mirror is **concave**, if the polished surface faces away from the centre of curvature the mirror is **convex**. The centre of curvature must be distinguished from the centre of the face of the mirror, which is usually called the **pole**. The **axis** of the mirror is the line joining the centre of curvature to the pole. The angle subtended by the diameter of the face of the mirror at its centre of curvature may be called its **aperture**; the aperture of a mirror is usually small.

If a pencil of rays parallel to the axis fall on a spherical mirror, they will either converge to (in the case of a concave mirror) or diverge from (in the case of a convex mirror) a point on the axis, called the **principal focus** of the mirror.

The principal focus of a spherical mirror is situated half-way between the pole of the mirror and the centre of curvature.

If a point source of light be placed at the principal focus of a concave mirror, the emergent pencil will consist of rays parallel to the axis. If a convergent pencil be directed towards the

principal focus of a convex mirror, the emergent rays will be parallel to the axis.

Definite conventions must be adopted with regard to the

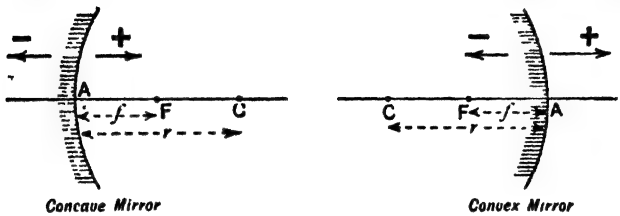


FIG. 139.—Concave and Convex Mirrors

signs to be attached to distances measured on the axis of a mirror. The following are usually employed:

(1) All distances are to be measured *from the pole* of the mirror.

(2) Distances on one side of the mirror are considered positive, on the other side negative.

(3) Distances are to be considered positive when measured in a direction towards the source of light.

With these conventions the radius of curvature and the focal length of a concave mirror are reckoned positive, while the radius of curvature and the focal length of a convex mirror are reckoned negative (Fig. 139).

Two points on the axis are said to be **conjugate foci** when a pencil of rays diverging from one point and reflected from the mirror either converge to or diverge from the second point. One point may be called the geometrical image of the other point.

In the case of a mirror, radius of curvature r, focal length f, the distance u of the object from the pole and the distance v of the image from the pole are connected by the formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{r}.$$

The **curvature** of a spherical surface may be measured by **the reciprocal of the radius of the sphere.** It is easy to see that the surface of a sphere of large radius has small curvature,

while the surface of a sphere of small radius has large curvature. Thus if r denote the radius of a sphere, the curvature of the surface may be denoted by R, where $R = 1/r$. Opticians use a special unit for measuring curvature, the **DIOPTRE**. This unit represents **the curvature of a sphere having a radius of 1 metre.**

Hence the curvature in dioptres

$$= \frac{1}{r\,(\text{metres})} = \frac{100}{r\,(\text{cm.})} = \frac{39{\cdot}37}{r\,(\text{in.})}.$$

The following table should be studied in order to form definite ideas as to curvatures expressed in this way :—

Curvature in dioptres	1	2	2·5	[illegible]	4	5	10	20	25	50	100
Radius of curvature in cm. . . .	100	50	40	33·3	25	20	10	5	4	2	1

The Curvature of a Small Circular Arc is Proportional to the Sagitta of the Arc.—If AMB be the chord of an arc APB, the distance PM on the diameter bisecting the chord at right angles is called the **sagitta** or **sag** of the arc.

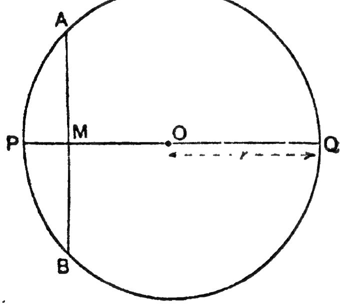


FIG. 140.—Curvature of an Arc.

Now $PM \cdot MQ = MA^2$, so

$$PM = \frac{MA^2}{MQ},$$

or approximately

$$PM = \frac{MA^2}{2r} = \frac{MA^2}{2} \cdot R.$$

Thus in the case of a number of small circular arcs having the same chord, the curvature of any arc is proportional to the sagitta. The sagitta is the distance measured by an ordinary spherometer, and it is possible to construct a spherometer which shall determine the curvature of a surface directly in dioptres. Simple instruments based on the same principle are used by opticians for measuring the curvature of the surface of a spectacle lens.

§ 2. Formation of a Real Image by a Concave Mirror

Image and Object Coincident.—If a small bright source of light be placed at the centre of curvature of a concave mirror, all the rays of light will fall on the face of the mirror normally. Consequently each ray will be reflected back along its original path, and all the reflected rays will pass through the centre of curvature. Thus an image of the source will be formed at the point of intersection of the rays at the centre of the sphere. Hence image and object coincide in position when the object is at the centre of curvature, and an *inverted* image will be formed.

Expt. 108. **Determination of the Radius of Curvature of a Concave Mirror.**—The position of the centre of curvature can be found readily by setting up a small object such as a pin, and finding, by the method of parallax, the position in which the object and the image coincide. The mirror may be fixed with its face vertical, in which case it should be set up on a table, or it may be fixed with its face horizontal, in which case it should be placed on a stool of convenient height so that the observer can look down into it from above. On looking into the mirror the observer will see the reflection of his own face. Using one eye only, the head should be moved until the reflection of the open eye is seen in the middle of the mirror. Then the eye and its image are in the axis of the mirror.

Now take a pin and place it so that its point lies on the axis. When this is the case the point of the pin will appear to overlap the image of the eye seen in the mirror. If this adjustment is correct an image of the pin should be seen in the mirror, and, provided the pin is not too near the mirror, the image should be inverted. The important point to be observed for success, in practically all optical experiments using pins, is that the observer should get as far away as convenient from the mirror or lens, and the object-pin should also be at a considerable distance.

Fig. 141.—Coincidence of Pin Point and its Image.

The adjustment so far made secures that the point of the pin lies on the axis of the mirror, but the object and image do not necessarily coincide. The pin must be moved so that, to

the eye looking along the axis, the point of the pin and the point of the image coincide. To test for such coincidence the method of parallax is employed (see p. 229).

When no parallax can be detected the point of the pin is at the centre of curvature of the mirror. Measure the radius of curvature, *i.e.* the distance from the pole of the mirror to the point of the pin.

Calculate the curvature of the surface in dioptres.

To check the result, the radius of curvature may be measured by means of the spherometer, but it must be remembered that in this case it is the front surface of the mirror that is employed, while in the optical measurement it is usually the back surface that is made use of. Many so-called concave mirrors are actually converging lenses mounted with a plane mirror behind them, or they may be lenses silvered on the back surface of the lens.

Conjugate Foci.—When an object is placed between the principal focus and the centre of curvature of a concave mirror, a **real**, inverted image is formed at a distance from the mirror greater than the radius of curvature. Such an image can be received upon a screen, as the rays forming the image actually intersect.

EXPT. 109. **Determination of the Positions of Conjugate Foci of a Concave Mirror, and Deduction of the Focal Length.**—Determine the radius of curvature of the mirror by the method described in Expt. 108. The principal focus is midway between the centre of curvature and the pole of the mirror. Place the pin, with its point still on the axis of the mirror, between the centre of curvature and the principal focus but not far from the centre of curvature *at first*. For this position of the pin an image will be formed real, inverted, and magnified, at a distance from the mirror greater than the radius of curvature.

To find the image, the observer should get at a considerable distance from the mirror, with his eye on the axis of the mirror (see p. 253). An inverted image of the pin will then be seen. For convenience, the object-pin may be provided with a small flag of paper to facilitate the recognition of its image.

Now take a second pin and set it up with its point along the axis of the mirror and adjust it by the parallax method, until it is in the continuation of the image of the first pin. Having

found the correct position for the second pin, measure, as accurately as possible, the distance from the pole of the mirror to the point of the first pin (u), and the distance from the pole of the mirror to the image, *i.e.* to the point of the second pin (v).

Repeat the experiment for three or four positions of the object, moving the object nearer to the principal focus for each successive experiment. Note that the image moves further from the mirror as the object approaches the mirror.

The focal length of the concave mirror is now calculated in each case from the formula

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f}.$$

Signs in Optical Formulae.—In using any formula for mirrors or for lenses, the signs in the formula must never be altered. The values of the various quantities observed, u, v, etc., should be written down at the side, with the proper sign (+ or −) prefixed according to the conventions on p. 251. They should then be substituted in the formula, and no 'rectification' of signs should be made until the actual numerical values have been substituted in this manner. Failure to observe this rule is certain to lead to errors, especially in the more complicated expressions which are used in dealing with lenses.

§ 3. Formation of a Virtual Image by a Spherical Mirror

When a real object is placed in front of a convex mirror, or between the pole and the principal focus of a concave mirror, the image formed is **virtual**. The directions of the reflected rays, but not the rays themselves, intersect; consequently such an image cannot be received upon a screen.

Expt. 110. **Determination of the Focal Length of a Convex Mirror by the Pin Method.**

Method I.—Place a pin in front of a convex mirror. The image is always behind the mirror, but its position can be found if a large pin is placed behind the mirror, and adjusted until its head, as viewed over the top of the mirror, gives no parallax with the virtual image of the first pin as seen in the mirror. Owing to spherical aberration, the adjustment cannot be carried out very accurately if the aperture of the mirror is large. Sometimes a small area of the silvering is removed

from the centre of the mirror and the pin behind the mirror is viewed through the transparent hole left.

Observe the position of the reflected image for several positions of the object; note that as the object approaches the mirror the image also is brought nearer to the mirror. Measure u and v in each case.

Calculate the focal length of the mirror for each pair of distances obtained, taking special precautions as to the signs of these quantities (see note above).

For other methods suitable for use with a convex mirror see pp. 264-267.

EXPT. 111. **Determination of the Focal Length of a Concave Mirror by means of a Virtual Image.**—Carry out a similar experiment with a *concave* mirror, placing the object-pin between the principal focus and the pole of the mirror, and locating the virtual image either by looking over the top of the mirror, or through a hole in the middle.

CHAPTER III

LENSES

§ 1. Introductory Theory

The **lens** considered in elementary work is bounded by two surfaces each of which forms a portion of a sphere. The lens is supposed to be *thin*, that is, the distance between the two surfaces is small compared with the radius of curvature of either surface. Since there are two surfaces there must be **two centres of curvature** and **two radii of curvature.** If one surface of the lens is plane the corresponding radius of curvature is infinite. The line joining the two centres of curvature is called the **axis** of the lens.

Lenses may be divided into two classes, **converging** and **diverging.**

A converging or, as it is often called, a convex lens is thicker in the middle than it is at the edges.

A diverging or concave lens is thinner in the middle than it is at the edges.

Lenses (unlike mirrors) possess **two principal foci** and **two focal lengths,** which are, however, equal to one another numerically. The focal length is the distance from the lens to a principal focus.

The **first principal focus** is the position of the object (point) for which the image is at infinity, or the point corresponding to parallel emergent rays.

The **second principal focus** is the position of the image

(point) when the object is at infinity, or the point corresponding to parallel incident rays.

A plane drawn perpendicular to the axis through the point where the axis meets the lens is called the **principal plane** of the lens; planes drawn perpendicular to the axis through the focal points are termed the **focal planes.**

The **optic centre** of a *thin* lens is the point where the axis meets the lens.

The angle subtended by the diameter of the lens at a principal

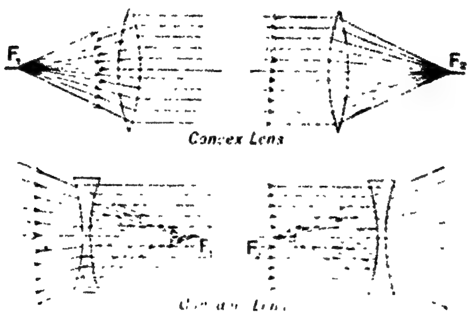


Fig. [illegible] Convex and Concave Lenses.

focus may be called the aperture of the lens: the aperture of a lens is usually small.

As in the case of mirrors, definite conventions must be used with regard to the **signs** of all distances measured along the axis. The usual conventions are as follows:—

(1) All distances are to be measured *from the centre* of the lens.

(2) Distances on one side of the lens are considered positive, on the other side negative.

(3) Distances are to be considered positive when measured in a direction towards the source of light.

In speaking of the focal length of a lens, it is customary to consider the distance from the lens to the *second* principal focus as **the focal length.** With the conventions adopted, it follows that the focal length of a *convex* lens is to be reckoned *negative*, and the focal length of a *concave* lens *positive.*

In the case of a lens of focal length f, the distance u of the object from the lens and the distance v of the image from the lens are connected by the formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}.$$

If we put $\frac{1}{v}=\mathrm{V}$, $\frac{1}{u}=\mathrm{U}$, and $\frac{1}{f}=\mathrm{F}$, the formula may be written

$$\mathrm{V}-\mathrm{U}=\mathrm{F}.$$

In this equation U measures the curvature of the incident wave-front, V the curvature of the wave-front after passing through the lens.

The quantity F, which is the reciprocal of the focal length of the lens, is called **the focal power** of the lens.

From the point of view of the Wave Theory of Light, this formula expresses the fact that the change produced in the curvature of the wave-front by the lens is equal to the focal power of the lens. These curvatures, and also the focal power of the lens, are measured in **dioptres** (p. 252). The focal power of a lens is 1 dioptre[1] when its focal length is 1 metre.

§ 2. Simple Experiments with Lenses

Expt. 112. **Determination of the Character of a Lens.**— A simple, yet delicate, test for distinguishing between a convex and a concave lens is to hold the lens just in front of the eye, and to move it from side to side while viewing a distant object through it. If the object appear to move in the *opposite* direction to the lens, the lens is *convex*; if the object appear to move in the *same* direction as the lens, the lens is *concave*.

Test a number of thin lenses in this way, and separate the converging from the diverging lenses. Then combine the lenses in pairs and determine whether the combination is converging or diverging.

Examine the image formed by a convex lens of focal length 20 or 30 cm. When the lens is close to the eye, the image is erect and magnified. If the object is distant the image seen will be blurred, but objects at a smaller distance will give

[1] Spectacle-makers call the focal power of a convex lens positive, and that of a concave lens negative. This convention is the opposite of that adopted above.

distinct virtual images. If, when viewing a distant object, the lens is moved away from the eye, the image becomes more and more blurred, until at a certain distance it is impossible to distinguish the character of the object at all. If, however, the lens is moved still further from the eye an inverted image will be seen. This is a real image formed between the lens and the eye.

Examine, in a similar way, the image formed by a concave lens. The image is always erect, diminished and virtual.

METHODS OF DETERMINING THE FOCAL LENGTH OF A CONVEX LENS

METHOD I. **By finding the Image of a Distant Object.**—If rays from a very distant source of light fall on a convex lens, they are rendered convergent and brought to a focus at the principal focus of the lens. The distance from the lens to this point is the focal length of the lens.

> EXPT. 113. **Determination of the Focal Length of a Convex Lens. I.**—A simple method of finding the principal focus of a convex lens consists in forming a real image of a distant object on a screen by means of the lens. If direct light from the sun cannot be used, employ a distant lamp or window as the object. Adjust the lens so that a sharp image is focussed on the screen, and measure the distance from lens to screen. This is approximately the focal length. It is essential that the distance of the object should be great compared with the focal length of the lens under test.

METHOD II. **By using a Combination of Plane Mirror and Lens.**—If a point source of light be placed at the principal focus of a convex lens the rays emerging from the lens will be parallel to one another. If now a plane mirror be placed at right angles to the emergent rays, they will be reflected back along their original path and after passing through the lens will form a real image at the point from which they started.

This fact may be made the basis of a simple method of finding the position of the principal focus of a convex lens or of any converging system of lenses.

> EXPT. 114. **Determination of the Focal Length of a Convex Lens. II.**—Place a piece of plane mirror face upwards on a table; on it place the lens to be tested. Support a pin in a stand so that its point is vertically above the centre of the face of the lens. A paper flag attached to the pin may

be used with advantage in finding the *real inverted* image.[1] The observer must get as far away from the lens as convenient (see the caution on p. 253).

The pin must now be adjusted till the point of the pin and the point of this real image coincide, that is until there is no parallax between them. The method of adjustment is exactly the same as that already described in the process of finding the radius of curvature of a concave mirror (p. 253). When this position has been found, measure the distance from the upper surface of the lens to the pin, and also the distance from the lower surface of the lens (*i.e.* the surface of the mirror) to the pin. The mean of these distances gives the focal length of the lens.

The same method may be applied when the lens and mirror are supported with their faces vertical instead of horizontal.

METHOD III. **By finding the Positions of Conjugate Foci.**—In this method a pin is set up on one side of the lens so as to give

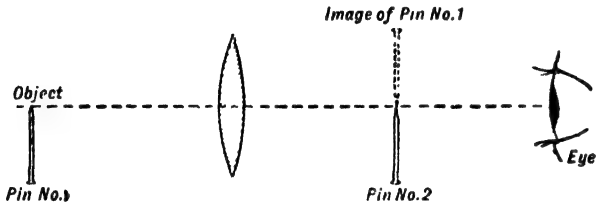


FIG. 143.—Conjugate Foci.

a real image on the other side of the lens, and a second pin is adjusted so as to coincide in position with this real image.

For this to be possible two conditions must be satisfied. The object must be at a distance from the lens greater than the focal length. The distance between the two pins must be not less than four times the focal length.

EXPT. 115. **Determination of the Focal Length of a Convex Lens. III.**—The adjustment presents no difficulty if the pin be placed some considerable distance from the lens, and the observer place his eye along the axis of the lens, on the side remote from the pin, and at a considerable distance

[1] When the pin is at a sufficient distance from the lens the image is real and inverted. If the pin is lowered the image becomes blurred, and if it is lowered still further the image is virtual and erect. The action of the combination of lens and mirror is similar to that of a concave mirror.

from the lens. The object-pin should be supplied with a small flag as before. When the inverted image of pin No. 1 is seen, set up a second pin (pin No. 2 in Fig. 143) and adjust it till its point coincides in position with the point of the image, using the method of parallax.

Measure as accurately as possible the distance from the lens to the object (u) and the distance from the lens to the image (v) and calculate the focal length by means of the formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}.$$

Remember to take into account the proper signs for u and v when substituting the numerical values in the formula (see note on p. 255).

Repeat the observations for two other positions of the object-pin, and take the mean of the values obtained for f to represent the focal length. Calculate also the focal power of the lens in dioptres.

METHODS OF DETERMINING THE FOCAL LENGTH OF A CONCAVE LENS

METHOD I. **By Use of a Distant Object.**—If rays from a very distant source of light fall on a concave lens, they are rendered divergent and appear to proceed from the principal focus of the lens.

EXPT. 116. **Determination of the Focal Length of a Concave Lens. I.**—Set up a retort stand at a well-lighted window and at a distance of several metres from the concave lens. Place the lens vertically in a holder, and view the retort stand through the lens. An erect virtual image of the retort stand will be seen, this image being on the side of the lens remote from the observer. By placing a pin on the same side as the retort stand, but close up to the lens, the position of this image can be found. The pin is adjusted until, as viewed over the top of the lens, there is no parallax between it and the image of the retort stand viewed through the lens. The distance from the lens to the pin is the focal length of the lens when this condition is satisfied.

METHOD II. **By Determination of Conjugate Foci.**—In the case of a concave lens a real object gives rise to a virtual image on the same side of the lens as the object.

EXPT. 117. **Determination of the Focal Length of a Concave Lens. II.**—Set up a pin at a distance of about 1 metre from the lens and adjust a second pin to coincide with the image of the first pin formed by the lens, in the same way as described in Method I. for a distant object. Measure the distances of the object and image from the lens.

Repeat the experiment for several positions of the object and calculate the focal length of the lens from the formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

taking due note of the signs of the quantities involved.

NOTE.—The image seen through the lens will be distorted in consequence of spherical aberration, and the adjustment of the pin seen over the top, to the position where there is no parallax, is only approximate.

METHOD III. **By combining with it a Suitable Convex Lens.**—When two thin lenses are placed in contact the focal power of the combination is equal to the algebraic sum of the focal powers of the two component lenses. That is

$$F = F_1 + F_2,$$

or, since the focal power is inversely proportional to the focal length

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}.$$

In these formulae F, f refer to the combination, F_1, f_1 to the first component, F_2, f_2 to the second component.

EXPT. 118. **Determination of the Focal Length of a Concave Lens. III.**—If with the given concave lens a convex lens of sufficient power (or of shorter focal length) is combined, a combination is obtained which acts like a convex lens. The focal length of this combination may be measured by any of the methods already given for a convex lens. The focal length of the single convex lens may be measured in a similar way. The focal length of the concave lens may then be calculated by substituting the values obtained in the formula above.

Care must be taken with regard to signs. According to the conventions adopted, the focal length of a convex lens or of a converging combination of lenses is to be considered negative.

CHAPTER IV

FURTHER EXPERIMENTS WITH MIRRORS AND LENSES

§ 1. Radius of Curvature of a Spherical Mirror

Some simple methods of finding the radius of curvature of a spherical mirror have been described in Chapter II. When the image formed is real, its position may be found accurately by the method of parallax. When the image is virtual, the results obtained by this method are less accurate.

RADIUS OF CURVATURE OF A CONVEX MIRROR

Method II. **By using a Plane Mirror.**[1]—When an object is placed in front of a convex mirror, the image formed is necessarily virtual and lies between the principal focus and the pole of the mirror. The following method, which is more generally applicable than the pin method already described, may be used to find the position of the virtual image.

Expt. 119. **Radius of Curvature of a Convex Mirror. II.**—Set up a pin some distance in front of the convex mirror. On looking into the mirror from a position behind the pin the virtual diminished image can be seen easily. Now set up a piece of plane mirror between the pin and the convex mirror, so that its upper edge is on a level with the centre of the face of the convex mirror whilst its plane is perpendicular to the axis. Then on looking into the mirror as before from a position behind the pin, only the upper half of the convex mirror will be seen, the lower half being covered by the

[1] For Method I. see p. 255.

plane mirror. Two images of the pin will be seen, one in the convex mirror, the other in the plane mirror, the former being smaller than the latter, as shown on the right of Fig. 144. The plane mirror must now be adjusted until there is *no* parallax between these two images; that is, until the image in the convex mirror appears to be a continuation of that in the plane mirror when the eye is moved into different positions from side to side. The image in the convex mirror should

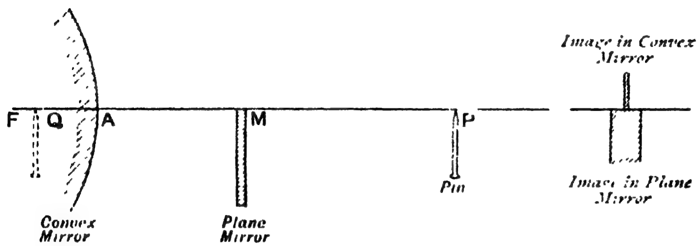


FIG. 144.—Convex Mirror and Plane Mirror.

remain accurately in the *centre* of the plane-mirror image; when this is the case, the image Q formed of the pin P by the convex mirror at A coincides in position with the image of P formed by the plane mirror M. Measure the distances AP, AM, and MP. (Verify the accuracy of the measurements by seeing that AP = AM + MP.) Since M is a plane mirror we know that MP = MQ.

Hence find AQ which is the difference between MQ and AM, *i.e.* between MP and AM. We now know the *numerical* values of u, the distance of the object from the pole, and of v, the distance of the image from the pole. We can therefore determine the radius of curvature r, or the focal length f, by means of the formula

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f}.$$

Pay special attention to the signs of the quantities considered. Several different determinations should be made, varying the positions of the plane mirror and of the pin.

METHOD III. **By using a Convex Lens.**—When a suitable convex lens is placed between a pin and a convex mirror, it is possible to adjust the lens and the pin so that a real image of the pin is formed in coincidence with the pin itself.

When the point of the pin coincides with its image, rays starting from the pin must retrace their original path after falling on the convex mirror. The condition necessary is that all the rays should fall on the mirror normally, or, in other words, that the rays after passing through the lens should be directed towards the centre of curvature of the mirror (Fig. 145). If then we can find the point to which the rays of light converge after passing through the lens we can find the centre of curvature of the mirror.

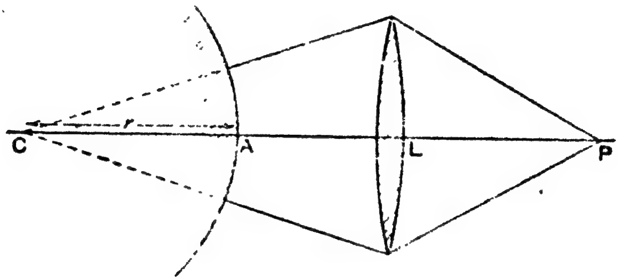


FIG. 145.—Convex Mirror and Lens.

EXPT. 120. **Radius of Curvature of a Convex Mirror. III.**—Set up a pin some distance in front of the convex mirror. Between the pin and the mirror set up a convex lens so that its axis coincides in direction with the axis of the mirror. By properly adjusting the lens and (if necessary) the pin, a *real inverted* image of the pin can be formed and made to coincide in position with the pin itself. This coincidence can be tested by the method of parallax. Measure the distance LA between the mirror and the lens.

Remove the mirror altogether, *but be careful not to alter the position of the lens and the pin.* Then set up another pin so that its point may coincide in position with the image of the first pin formed by the lens. Test the coincidence by the method of parallax. Measure the distance LC between the lens and the second pin. Since the pin is now in the position that was occupied by the centre of curvature of the mirror, the radius of curvature is found at once by subtracting the first measurement from the second :

$$r = LC - LA.$$

NOTE.—A lens of suitable focal length must be chosen. The distance LC must be greater than the focal length of the lens, and the distance PC must be at least four times the focal length.

The results of all these experiments may be checked by measuring the radius of curvature with the spherometer, but it must be noted that the spherometer gives the radius of the front surface while the optical method gives the apparent radius of the back surface in the case of a glass mirror.

Calculate also the curvature of the surface in dioptres.

EXPT. 121. **The Radius of Curvature of a Concave or Convex Mirror by means of a Turn-table.**—A convenient method of determining the radius of curvature is by the use of a turn-table, consisting of a flat table which can turn about a vertical axis. The mirror is mounted on the table so that its axis is parallel to the table. A low-power telescope is used to view an ink-spot or a speck of lycopodium at the pole of the mirror, and the position of the mirror on the table is adjusted till no motion of this spot is observed when the table is rotated. This point must then lie on the axis about which the table is rotated.

The telescope is now used to view the reflection of some distant object in the mirror, and the mirror is adjusted again on the turn-table till no motion of this reflected image can be observed when the table is rotated. This will be the case when the centre of curvature of the mirror lies on the axis of rotation of the turn-table; for then the result of rotation is merely to substitute one portion of the spherical surface of the mirror for another, so that no displacement of the reflected image takes place.

The distance between the two positions of the mirror on the table is equal to its radius of curvature.

§ 2. FOCAL LENGTH OF A LENS

THE TELESCOPE OR RANGE-FINDER METHOD OF TESTING LENSES

By the following method very accurate determinations of the focal length of a lens can be made. The method is particularly interesting as the actual position of the principal focus is determined both for a convex and a concave lens.

Light diverging from the first principal focus of a convex lens emerges from the lens as a parallel beam (Fig. 142).

If a parallel beam fall upon a concave lens it is rendered divergent and appears to proceed from a point which is the principal focus of the lens (Fig. 142).

In the case of thin lenses the distance between the lens and the principal focus is called the focal length of the lens.

The apparatus required consists of a needle with a sharp point mounted in a stand and a telescope with an eye-piece of high magnification.

EXPT. 122. **Determination of the Focal Length of a Convex Lens.**—The telescope must be focussed carefully for parallel light. If the telescope is provided with cross-wires, first focus the eye piece till the cross wires are seen distinctly, and then focus on a very distant object (seen through an open window) *so that there is no parallax between* the image on the cross-wire and the cross-wire. When once this adjustment has

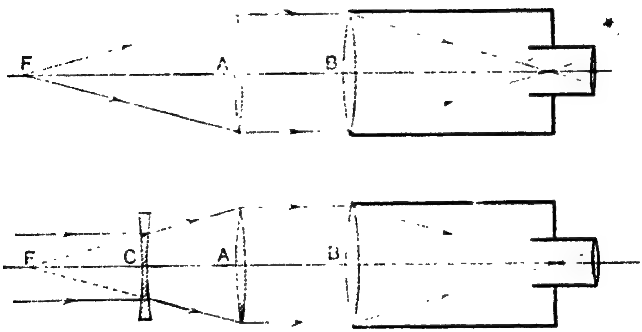


FIG. 146.—Focal Lengths by Telescope.

been made it must not be altered during the course of the experiment.

Set up the telescope on the table with the axis horizontal, and place the convex lens, the focal length of which is to be measured, in front of the object glass, taking care that the centre of the lens is on the axis of the telescope. Mount the needle so that its point is at the same height as the centre of the lens and move it about in front of the lens till an image is seen in the field of the telescope.

Adjust the position of the needle carefully so as to give the sharpest possible image with the point of the needle in the centre of the field of view. There must be no parallax between the cross-wire and the image of the pin as seen through the telescope. The point must then be at the principal focus of the lens. For as the telescope is focussed

for parallel light, the light falling on the object glass must form a parallel beam in order to give a clear image. Hence the light must diverge from F, the principal focus of the lens. Measure the distance AF, the focal length of the convex lens.

EXPT. 123. **Determination of the Focal Length of a Concave Lens.** The focal length of the concave lens must be less than that of the convex lens just found for the following method to be applicable. First find the principal focus F of the convex lens A as in the previous experiment. Then place the concave lens C between A and F, and move it about until objects at a great distance are focussed clearly. When this is the case F must also be the principal focus of the concave lens. The path of the rays is shown in Fig. 146. Parallel light from a distant object falling on the concave lens is rendered divergent and appears to come from the principal focus of the lens. This divergent light is rendered parallel by the convex lens. Hence the principal focus of the convex lens must coincide with the principal focus of the concave lens. Measure the distance CF, which is the focal length of the concave lens

In finding the position of the point F in this case we may dispense with the needle point and move the concave lens till a point on its surface is focussed sharply. This point will be at F, and the focal length is the distance CF between the two positions of the lens.

A pair of lenses suitable for this experiment may be selected without difficulty; they will form a diverging combination when placed in contact.

§ 3. DETERMINATION OF REFRACTIVE INDICES

EXPT. 124. **Determination of the Refractive Index of a Liquid, using a Concave Mirror.**—Place the mirror with its face horizontal at a convenient level, so that the observer can look down into it from above. Adjust a pin so that the image of its point coincides in position with the point itself as tested by the method of parallax. The process of adjustment has been described already on p. 253, where it was shown that the point must be at the centre of curvature of the mirror. Measure the distance from the pole of the mirror to the centre of curvature.

Place a small quantity of the liquid, the refractive index of which is sought, on the face of the mirror so as to form

a shallow pool from 5 to 10 cm. in diameter. Again adjust the pin so that its point coincides in position with the real image formed by reflection in the mirror, and measure the distance from the pole of the mirror to the point. Then the refractive index of the liquid is found by dividing the first distance (the radius of curvature of the mirror) by the second.

The truth of this statement can be verified by drawing a figure showing the path of a ray from the point to the mirror in the second case (Fig. 147).

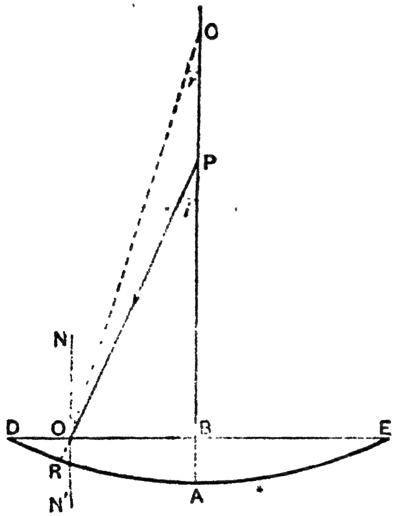


FIG. 147.—Refractive Index of a Liquid, using a Concave Mirror.

Let C be the position of the pin in the first experiment, P that in the second. Then C is the centre of curvature of the mirror. The ray PO, starting from the point P, is refracted on striking the surface of the water at O. The refracted ray after striking the surface of the mirror at R must retrace its original path in order that it may return to the point P. The condition for this is that the ray OR in the water should strike the mirror at right angles. Hence OR being a normal to the mirror must pass through the centre of curvature C.

The angle of incidence i is the angle PON, which is equal to the angle OPA. The angle of refraction r is the angle RON', which is equal to the angle OCA.

Thus $$\mu = \frac{\sin i}{\sin r} = \frac{\sin \text{OPA}}{\sin \text{OCA}}$$

$$= \frac{\text{OB}}{\text{OB}} \cdot \frac{\text{OP}}{\text{OC}} = \frac{\text{OC}}{\text{OP}}.$$

When the angle of incidence is very small, this becomes approximately AC/AP, which proves the statement made above.

REFRACTIVE INDEX OF THE MATERIAL OF A LENS

The focal length (f) of a lens depends upon the refractive index (μ) of the material, and upon the radii of curvature r and s of the two surfaces, the formula connecting these quantities being

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r}-\frac{1}{s}\right).$$

If by experiment the values of f, r, and s are determined the value of μ can be deduced.

EXPT. 125. **Determination of the Refractive Index of the Material of a Lens.**—The focal length of the lens may be found by any of the methods already described, but for the present determination, if a convex lens be used, it may be found by the pin method No. III. on p. 261.

The values of r and s might be found by an optical method regarding the surfaces of the lens as portions of spherical mirrors (see pp. 253 and 264). It is, however, sometimes convenient to determine these quantities by means of the spherometer as described on p. 47. It is important that attention should be paid to the conventions as to signs when substituting the numerical values for f, r, and s in the formula (p. 255).

The refractive index of a liquid, when only a small quantity is available, may be found by using the liquid to form a lens the focal length of which is then measured.

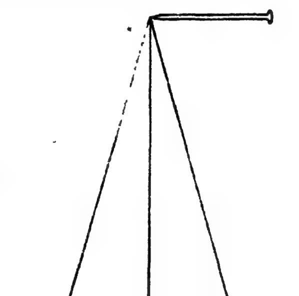


FIG. 148. Refractive Index of a Liquid, using a Lens and a Plane Mirror.

EXPT. 126. **Determination of the Refractive Index of a Liquid, using a Lens and a Plane Mirror.**—Select a convex lens having a focal length between 10 and 15 cm. Lay the lens on a plane horizontal mirror, and find the position of a pin point on the axis such that the point and its real image coincide. The distance from the centre of the lens to the pin is then equal to the focal length of the lens (Method II. p. 260). Now place a little of the

liquid between the lower surface of the lens and the plane mirror. This will form a plano-concave lens of the liquid, the radius of curvature (r) of the upper surface of the liquid being the same as that of the lower surface of the glass lens.

If the focal length of this liquid lens be f_a,

$$\frac{1}{f_a} = (\mu_a - 1)\frac{1}{r},$$

where μ_a is the refractive index of the liquid.

Now determine by the pin method the focal length of the compound lens of glass and liquid. Let this be F_A.

Then $$\frac{1}{F_A} = \frac{1}{f} + \frac{1}{f_a}, \text{ or } \frac{1}{f_a} = \frac{1}{F_A} - \frac{1}{f}.$$

By means of this formula, f_a may be calculated, and the value thus found may be used in the previous formula in order to determine μ_a. The value of r may be found by using the spherometer.

The foregoing method may be used to *compare* the refractive indices of two liquids, *without requiring a knowledge of the radius of curvature* r. If a second liquid, of refractive index μ_b, be substituted for the first, the corresponding equations will be

$$\frac{1}{f_b} = (\mu_b - 1)\frac{1}{r}, \text{ and } \frac{1}{f_b} = \frac{1}{F_B} - \frac{1}{f}.$$

Hence $$\frac{\mu_a - 1}{\mu_b - 1} = \frac{1/f_a}{1/f_b}$$

$$= \frac{1/F_A - 1/f}{1/F_B - 1/f}.$$

Consequently if f, F_A, F_B be measured by the method described, we can compare the *refractivities* of the two liquids. If the refractive index of one liquid be known, then that of the second can be calculated.

EXPT. 127. **Comparison of the Refractive Indices of Two Liquids, using a Lens and a Plane Mirror.**—It is convenient to use water ($\mu = 1{\cdot}333$) as one of the liquids. Glycerin or anilin may be used as the other liquid. Measure the focal lengths f, F_A, F_B by the method of Expt. 126, calculate $1/f_a$, $1/f_b$, and the refractive index of the liquid employed.

CHAPTER V

THE OPTICAL BENCH

§ 1. CONSTRUCTION OF THE OPTICAL BENCH

THE **optical bench** (Fig. 149) is used for carrying out accurate measurements in connection with mirrors and lenses or other optical apparatus. It consists of a long beam with guides like a lathe-bed, on which can slide a number of carriages to support the optical fittings. In this way the distance apart of the fittings

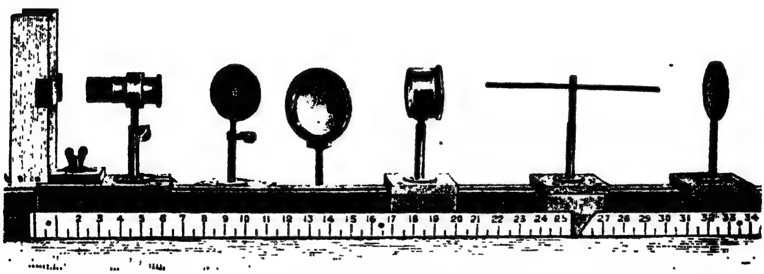


FIG. 149.—Optical Bench.

can be varied without any transverse motion taking place. In some cases provision is made for a transverse motion also. In order to measure the distance between two particular fittings (*e.g.* between a mirror and the screen used to receive the image) the bench is generally fitted with a graduated scale. The determination of the distance is made by using a measuring rod of known length mounted on a carriage on the optical bench. The measuring

rod is adjusted till one end of it (A) touches the first fitting, and the position of its carriage on the bench is read. Then the carriage is moved till the other end (B) touches the second fitting, and the position of the carriage is again read. The difference between the two readings gives the distance through which the carriage has been moved. The distance between the two fittings is equal to this distance plus the length of the measuring rod.

In some cases it is more convenient to make one end (A) of the rod touch the two fittings in succession. Then the distance between the fittings is simply the distance through which the carriage has been moved.

In experiments with mirrors and lenses on the optical bench, a real image of some object is formed usually on a white screen. The object may consist of a piece of wire gauze or two wires stretched at right angles across an aperture. The object should be illuminated by a source of light placed behind it. The flame of an oil or gas lamp may be used for this purpose; it is sometimes more convenient to employ a small incandescent electric lamp mounted behind a short focus lens arranged to throw a nearly parallel beam of light along the optical bench, *i.e.* the distance between the lens and the lamp must be made equal to the focal length of the lens.

It is important in all experiments with the optical bench that *all the lenses and mirrors used should lie on the same axis, which must be parallel to the bench itself.*

§ 2. Experiments with the Optical Bench

Expt. 128. **Optical Bench: Determination of the Focal Length and Radius of Curvature of a Concave Mirror.**—Set up the mirror in its holder on the optical bench so that it faces a wire gauze which is illuminated by a lamp. Use a screen with a small hole at its centre placed between the gauze and the mirror. Adjust the gauze and screen so that the light passing through the gauze goes through the hole in the screen and falls on the mirror. For this adjustment it is necessary that the source of light, the centre of the gauze, the

centre of the hole, and the pole of the mirror should be in a straight line.

By altering the position of the mirror it can be made to throw a distinct image on the screen close to the hole.

When the image is focussed sharply on the screen, measure by means of the measuring rod the distance from the mirror to the object, u, and the distance from the mirror to the image, v. Write down the values of u and v with the proper signs affixed, and calculate the values of r and f by means of the formula

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f}.$$

Repeat the determination, using at least three different positions for the screen.

Lastly, adjust the mirror so that it throws on the screen a distinct image of a wire stretched across the hole in the screen. The mirror must be rotated slightly about a vertical axis, so that this image may be as close to the hole as possible. In this case $u=v$, and therefore $r=u$ or $f=r/2$. Tabulate the results obtained from u, v, r, and f.

Draw a diagram showing the paths of rays of light by which a real image is formed by a concave mirror.

EXPT. 129. **Optical Bench: Determination of the Focal Length of a Convex Lens.**—Set up the lens in its holder on the optical bench between the illuminated object and an adjustable screen. Adjust the height of the lens so that its axis passes through the centre of the object. A blurred image of the gauze will probably be seen on the screen. The position of the lens and the screen must now be altered until a sharp well-defined image is formed on the screen.

In carrying out the adjustment two points should be borne in mind:

1. In order that the image formed by the lens should be real and not virtual, the distance from the lens to the object must be greater than the focal length; the lens must therefore be put some distance from the illuminated gauze.

2. In order that the real image should be formed on the screen, the distance between the object and the screen must be at least equal to four times the focal length. The screen should be put far enough away and gradually brought nearer until the image is focussed sharply.

METHOD I.—Let

u = distance from the lens to the object,
v = distance from the lens to the image,
f = focal length of the lens.

Then (p. 259) $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $V - U = F$.

Hence if u and v are measured, F and f may be calculated.

Measure the distances u and v by using the measuring rod fixed on a sliding carriage, and calculate the value of F and that of f, being careful to attach the proper signs to the numerical quantities.

Repeat the observations, using at least three different positions, and tabulate the results obtained.

u	v	U	V	F	f

The mean value of f should be calculated, and the focal power expressed in dioptres.

Draw a diagram showing the paths of rays by which the real image is formed by a convex lens.

A Graphic Construction.—An interesting graphic construction for determining the value of the focal length is associated with the name of Sir Howard Grubb. Two axes at right angles to one another are taken, and the values of u are marked off along one axis, the corresponding values of v being marked off along the other axis. In the present case since the values of v are negative the corresponding axis is drawn downwards. Corresponding points on the axes are then joined by straight lines, and if the construction is carried out accurately all such lines should intersect in a single point. The distance of this point from each of the axes is equal to f, *i.e.* OM and MF in Fig. 150 are each equal to the focal length f.

METHOD II.—When a convex lens is used to form a real image of an object on a screen, there are in general *two* positions of the lens corresponding with a given distance between the object and the screen. In one of these positions the lens gives a magnified.

in the other a diminished image. The distance from the lens to the object in the first case is equal to the distance from the lens to the screen in the second case.

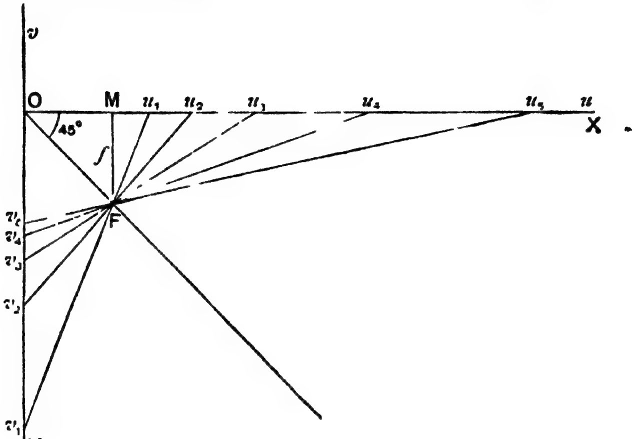


FIG. 150.—Graphic Construction for Focal Length.

Let d be the distance between the object and the screen, and a the distance between the two positions of the lens. Then

$$u = \frac{d-a}{2}, \quad v = \frac{d+a}{2},$$

and on substituting in the formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we obtain
$$f = \frac{d^2 - a^2}{4d}.$$

Set up the lens between the object and the screen so that a real image is formed on the screen. Keeping the object and the screen fixed, move the lens so as to find the second position in which it can form a real image. Measure the distance through which the lens has been moved, and also the distance between the object and the screen, and calculate the focal length.

As a particular case of the above, find by trial the position for which $a = 0$. In this case d has its minimum value and

$$f = \frac{d}{4}.$$

EXPT. 130. **Optical Bench: Determination of the Focal Length of a Concave Lens.**—Since it is impossible to obtain a real image using a single concave lens, it is necessary to combine a convex lens of suitable focal length with the given concave lens in order to carry out the foregoing methods. The combination consisting of the convex and concave lenses should act like a convex lens, and the focal length of the combination (f) may be found as in Expt. 129. The focal length (f_1) of the single convex lens employed may be found by the same method. Then the focal length of the concave lens (f_2) may be calculated by means of the formula

$$\frac{1}{f} \quad \frac{1}{f_1} \quad \frac{1}{f_2}$$

Care must be taken to give the proper signs to each quantity in the formula.

EXPT. 131. **Measurement of the Radii of Curvature of the Surfaces of a Double Convex Lens.**—First find the focal length of the lens by one of the methods already described. Next find the distance from the lens at which an object must be placed so that it may coincide in position with the image formed by reflection from the *second* face of the lens.

The way in which this image is formed is shown in Fig. 151. A ray starting from P and refracted by the first surface of the lens

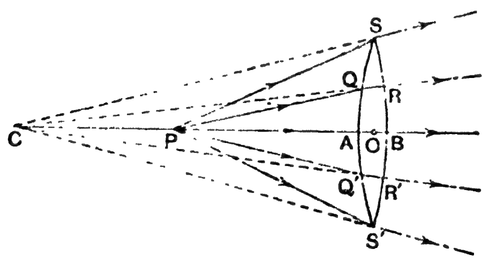


FIG. 151.—Reflection from the Second Face of a Lens.

will retrace its original path after reflection from the second surface provided it meet that surface at right angles. Consequently the refracted ray QR must pass through C, the centre of curvature of the second surface. Some of the light is transmitted through this surface as shown by the dotted lines on the right. Thus C is the

virtual image of P formed by rays passing *through* the lens. If OP $= d$ and OC $= s$, then in the equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \quad u = d, \ v = s.$$

Hence
$$\frac{1}{s} - \frac{1}{d} = \frac{1}{f},$$

so that
$$s = \frac{df}{d+f}.$$

In this formula f is to be considered as having its *algebraic* value.

> The experimental determination of d may be carried out by using the method of parallax with a pin as the object, but as the image formed by reflection is faint it is easier to carry out the measurement on the optical bench, using a screen with a circular aperture crossed by wires at right angles to receive the image of the illuminated aperture. Since object and image are then at the same distance from the lens, the distance from the lens to the screen is equal to d.
>
> In a dark room the experiment can be done with a pin fitted with a small well-illuminated flag, or in the laboratory the lens may be floated on mercury to intensify the reflection.
>
> The radius of curvature r, of the other face of the lens, may be found by the same method after reversing the lens in its holder.

Knowing the values of f, r, and s, the refractive index of the lens can be found from the formula

$$\frac{1}{f} = \mu - 1 \left(\frac{1}{r} - \frac{1}{s}\right).$$

The quantities f, r, and s must all be prefixed with their proper signs. To do this for the two radii, the lens may be imagined to be set up for use; one side is considered as the *incident* side, and all distances measured on this side are positive. Thus, if either surface of the lens is *convex* in this direction, its radius of curvature is negative since its radius would be measured in the opposite direction, and so on.

CHAPTER VI

OPTICAL INSTRUMENTS

§ 1. Magnifying Power of a Simple Lens

The apparent size of an object depends on the angle which the object subtends at the observer's eye, that is, it depends on the linear dimensions of the object and on its distance from the eye. By bringing the object nearer to the eye its apparent size is

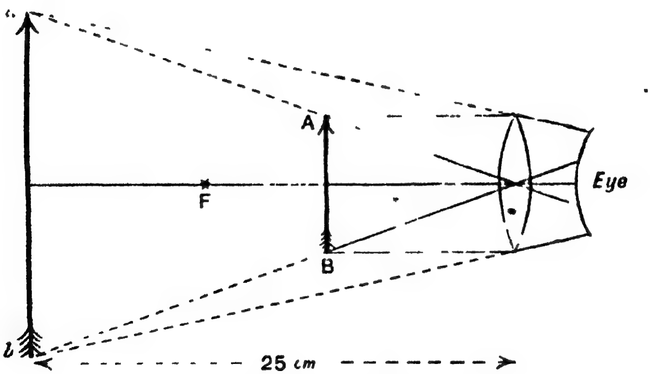


Fig. 152. Magnifying Power of Lens.

increased, but when it is brought too close it cannot be seen distinctly. The **least distance of distinct vision** for a normal eye is taken usually as 25 cm.

When a single lens is used as a simple microscope, it is placed close to the eye, and the distance of the object is adjusted so that

a virtual image is formed about 25 cm. away. Thus, if AB is an object at a distance from the convex lens less than its focal length, a virtual image is formed at ab which should be 25 cm. from the eye (Fig. 152).

The **magnifying power** of a lens or microscope is defined as the ratio of the angle subtended at the eye by the virtual image to the angle which would be subtended at the eye by the object *when placed 25 cm. away.* In speaking of a telescope the expression 'magnifying power' is used in a different sense.

When the angles are small we may replace them by their tangents and write

$$\text{Magnifying power M} = \frac{ab/25}{\text{AB}/25},$$

$$= \frac{ab}{\text{AB}}.$$

Relation of Magnifying Power to Focal Length of Lens.—Let the focal length of the lens be f cm. Let the distance of AB from the lens be u cm.

Then
$$\frac{1}{25} - \frac{1}{u} = \frac{1}{f},$$

$$\therefore \frac{1}{u} = \frac{1}{25} - \frac{1}{f}.$$

But the magnifying power $\text{M} = \dfrac{ab}{\text{AB}} = \dfrac{25}{u} = 1 - \dfrac{25}{f}.$

Hence, if f be known, the magnifying power can be calculated. Here f has its algebraic value; for a convex lens it is negative.

EXPT. 132. **Determination of Magnifying Power of a Simple Lens.**

METHOD I.—Measure the focal length of the lens by placing it between two pins, and adjusting the distances of the pins so that the image of one coincides with the other. Measure the distances (u and v) of the pins from the lens, and substitute in the formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

Remember that distances are positive only when measured opposite to the direction in which the light is travelling. Knowing the focal length, the magnifying power is given by

$$M = 1 - \frac{25}{f}.$$

METHOD II.—Place a millimetre scale on the table, and support another scale about 20 cm. above it, and parallel to it. View the upper scale with the lens, and arrange so that the lower scale can be seen directly at the same time with the other eye. Adjust the lens and scales so that both scales can be seen distinctly, the upper through the lens and the lower directly. Then count the number of millimetres on the lower scale which appear equal to two or to three millimetres on the upper scale. Let N_1 mm. on the upper scale equal N_2 on the lower scale.

Then $$M = \frac{N_2}{N_1}.$$

§ 2. THE MICROSCOPE

CONSTRUCTION AND MAGNIFYING POWER OF A MICROSCOPE

The essential parts of a **compound microscope** are two convex lenses of short focal length :

(1) The **object-glass** or **objective.**

(2) The **eye-piece** or **eye-lens.**

The distance between the object and the objective is a little greater than the focal length of the objective. Consequently a real inverted magnified image is produced on the other side of the lens. In Fig. 153 AB is the object, A′B′ is the real magnified inverted image formed by the object glass at O. This real image is then viewed through the eye-lens, which acts in exactly the same way as a magnifying glass.[1]

The distance between the real image and the eye-lens is less than the focal length of the lens ; consequently a magnified virtual image is produced. The lens is adjusted so that this virtual image is

[1] If the experiment on the determination of the magnifying power of a simple lens has not been done, it should be worked through before going further with the compound microscope.

formed at the least distance of distinct vision, usually taken as 25 cm. from the eye.

A′B′ is the real image formed by the objective, A″B″ is the virtual image formed by the eye-piece E.

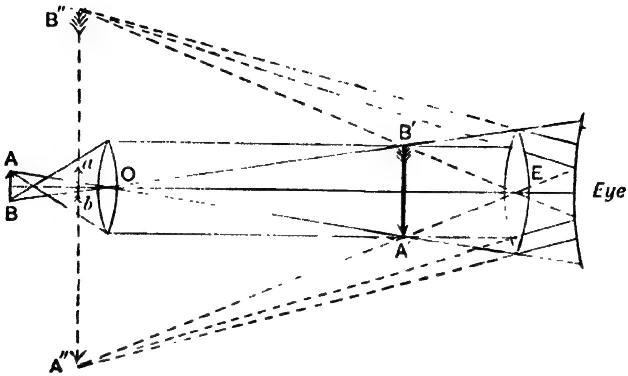


FIG. 153.—Magnifying Power of Microscope.

The magnifying power of the instrument is defined as the ratio of the angle subtended at the eye by the image A″B″ to the angle subtended at the eye by the object when placed at the least distance of distinct vision.

Hence the magnifying power

$$= \frac{\text{Angle subtended at E by A B″}}{\text{Angle subtended at E by object at 25 cm.}},$$

$$= \frac{\text{Angle subtended at E by A″B′}}{\text{Angle subtended at E by } ab} \text{ (where } ab = \text{AB)},$$

$$= \frac{\text{A″B″}}{ab}, \text{ approximately.}$$

EXPT. 133. **Construction of a Microscope.**—1. As the object use a small piece of squared paper, or a short clearly-marked millimetre scale, placed on the horizontal base of a retort stand.

2. Find approximately the focal length of the short-focus lens [1] to be used as objective, and support the lens above the squared paper at a distance a little greater than this.

3. Above the lens place a small horizontal platform with a circular hole in it, and arrange matters so that the axis of the

[1] The focal length should be less than 5 cm., preferably about 2 or 3 cm.

lens passes through the centre of the hole. A second piece of squared paper is fixed on the platform.

4. Above the platform arrange on the retort stand a metal ring carrying cross-wires. On looking down from above, the real magnified image A'B' will be seen. Adjust the height of the cross-wires so that there is no parallax between them and the lines of the image. When this is the case the cross-wires must be in the same horizontal plane as the image formed by the object-glass.

5. Place the eye-piece[1] in position so as to magnify the image formed by the object-glass.

6. Find the position of the **eye-ring**, that is the position that the pupil of the eye must occupy so as to include the greatest number of rays passing through the eye-lens. When the eye is in this position the field of the eye-lens should appear filled with the magnified image of the squared paper. The position of the eye-ring may be marked by a metal ring if desired.

7. Adjust the platform till it is 25 cm. from the eye-ring.

EXPT. 134. Magnifying Power of a Microscope.

METHOD I.—Observe the squared paper on the platform directly with one eye while looking through the microscope at the magnified image of the first paper with the other eye. With practice it is possible (assuming the vision of both eyes to be normal) to see the two images at the same time with the magnified squares overlapping those seen directly. If any difficulty is found in seeing the two images together, try opening and shutting the eyes alternately for a time so as to see separate images, and then open both eyes at once to get the superposed images. Observe the number n of divisions seen directly which correspond with a number m seen through the microscope. Then the magnification is n/m, for in this case $A''B''/ab = n/m$.

In this determination the eye should be placed at the eye-ring.

METHOD II.—Determine separately the magnification M_o, produced by the objective, and M_e, that produced by the eye-lens; then the magnifying power of the microscope is $M = M_o \times M_e$.

Determination of M_o.—Place a small piece of squared paper on the ring supporting the cross-wires and arrange it so that the divisions are alongside the divisions of the real image A'B'. Find the number n' of divisions on this small piece of paper which correspond with a number m' of the image A'B'.

[1] The focal length of the lens for the eye-piece should be less than 7 cm., preferably about 4 or 5 cm.

Then $$M_o = \frac{n'}{m'}.$$

Determination of M_e.—To find the magnifying power of the eye-piece, place a small piece of squared paper on the ring supporting the cross-wires and arrange it so that it covers over the real image A′B′. Then place the eye at the eye-ring and compare the divisions on this piece of paper with the divisions on the squared paper on the platform seen with the other eye. The process is exactly that employed in finding the magnifying power of a simple lens. Note the value of M_e thus found and calculate

$$M = M_o \times M_e.$$

METHOD III.—Determine M_o and M_e separately by calculation, and calculate $M = M_o \times M_e$.

To calculate M_o :—

$$M_o = \frac{\text{Size of image A'B'}}{\text{Size of object AB}},$$

$$= \frac{\text{Distance of image A'B' from O}}{\text{Distance of object from O}}.$$

Measure these distances and calculate M_o. Note that the distance of the object AB from O is very nearly equal to the focal length of the object-glass, while the distance of the image A′B′ from O is about the length of the microscope tube.

To calculate M_e : —

Assuming the eye-ring to be very near the eye-lens the magnifying power of the eye-lens is given by the formula

$$M_e = 1 - \frac{25}{f},$$

where f stands for the algebraic value of the focal length of the eye-lens. Find f, and calculate M_e, and hence determine

$$M = M_o \times M_e.$$

If the distance between the eye-ring and the eye-lens is not small, let it be denoted by e. The virtual image is formed at 25 cm. from the eye-ring, not from the eye-lens. The magnifying power is now

$$M_e = 1 - \frac{25 - e}{f}.$$

Since e is nearly equal to f this gives as an approximate formula

$$M_e = -\frac{25}{f}.$$

§ 3. The Telescope

CONSTRUCTION AND MAGNIFYING POWER OF A TELESCOPE

The essential parts of an astronomical telescope are two convex lenses:

(1) The **object-glass** or **objective** of long focal length.

(2) The **eye-piece** or **eye-lens** of short focal length.

The lens of long focal length forms a real inverted image of a distant object. If the object be very distant, as is the case with the astronomical telescope, this image is formed in the focal plane of the object-glass.

In Fig. 154, rays coming from a point of the distant object

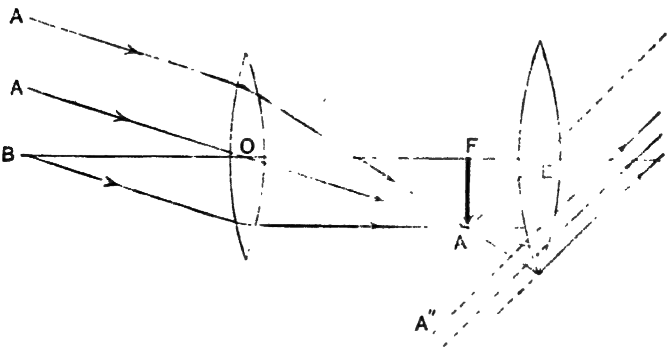


Fig. 154.—Telescope in Normal Adjustment.

in a direction parallel to the axis are brought to a focus at F, the principal focus of the object-glass O. A pencil of parallel rays coming from some other point of the distant object in a direction parallel to AO is brought to a focus at A′, a point in the focal plane of the object-glass.

The real inverted image thus formed is then magnified by the eye-lens, which forms a virtual image on the same side of the eye-lens as this real image.

When the telescope is in **normal adjustment,** the eye-lens is placed at a distance equal to its focal length beyond the real image formed by the object-glass. Consequently, in this case

the rays emerging from the eye-piece are parallel, the final virtual image being formed at an infinite distance from the eye. The direction of the parallel rays is found by joining A' to E, the centre of the eye-lens.

If the eye, accommodated for distant vision, be placed behind the eye-lens, the parallel rays will be focussed on the retina and a magnified image will be seen.

The **magnifying power** of an optical instrument is defined as **the ratio of the angle subtended at the eye by the image to the angle subtended by the object.**

In order to make this definition complete, it is necessary to specify the exact position of the image and also that of the object. In dealing with the magnifying glass or the microscope, it is usual to assume that, in making the comparison, both the image and the object are placed at the least distance of distinct vision, that is about 25 cm. from the eye.

In the case of the astronomical telescope such an assumption would obviously be absurd, and instead we assume both image and object are at an infinite distance from the eye.

Thus for a telescope in normal adjustment the magnifying power

$$= \frac{\text{Angle subtended by image}}{\text{Angle subtended by object}},$$

$$= \frac{A'EB}{AOB} = \frac{A'EF}{A'OF},$$

$$= \frac{A'F/EF}{A'F/OF}, \quad \text{regarding the circular measure as equal to the tangent of the small angle;}$$

$$= \frac{\text{Focal length of object-glass}}{\text{Focal length of eye-lens}}.$$

The simple telescope is often used to view terrestrial objects the distance of which may be considerable but still far from infinite. In such a case the telescope is not in normal adjustment, and the final image may be formed at any distance convenient to the observer. Thus, the observer may adjust the eye-lens so that

the final image is at the same distance from the eye as the object, or he may prefer to have the final image at the least distance of distinct vision.

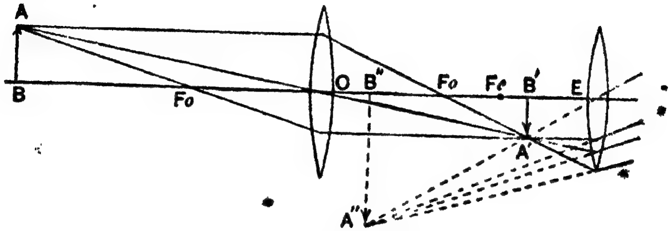


FIG. 1[illegible].—Telescope not in Normal Adjustment.

The magnifying power can then be calculated as follows:—

$$\text{Magnifying power} = \frac{\text{Angle subtended by image}}{\text{Angle subtended by object}},$$

$$= \frac{A''EB''}{AOB} = \frac{A'EB'}{A'OB'},$$

$$= \frac{OB'}{EB'}, \text{ approximately.}$$

Thus the magnifying power

$$= \frac{\text{Distance of real image from object-glass}}{\text{Distance of real image from eye-lens}}.$$

This expression holds good whether the telescope is or is not in normal adjustment, and whatever be the distance of the final virtual image from the eye.

EXPT. 135. **Construction of a Simple Telescope.**—Set up a graduated scale at a considerable distance to serve as the object. In default of a scale, a brick wall forms a convenient object on which to make observations. Choose two convex lenses, one having as long, the other as short, a focal length as possible. Set up the long focus lens to serve as the object-glass so as to produce a real image of the distant scale. This real image can be seen by an eye placed at a sufficient distance behind it. Set up a pin so that it coincides in position with the real image of one of the divisions of the scale. This will be the case when there is no parallax between the pin and the image.

Then arrange the short focus lens as an eye-piece to magnify the real image, so that the divisions of the scale are clearly visible.

EXPT. 136. **Magnifying Power of a Telescope.**—With one eye look through the telescope, with the other look at the scale directly. The two eyes are thus being used independently at the same time, and this fact may cause difficulty on a first trial. The difficulty is reduced by arranging the eye-lens so that the *accommodation* is the same for both eyes, *i.e.* that the final virtual image is formed at the same distance from the observer as the scale itself. Focus the telescope by moving the eye-lens, with the idea clearly in mind that the image is situated at the actual distance of the scale. If the adjustment is correct, and both eyes are made use of, a slight movement of the head should not cause relative movement of the two images.

Fix the attention on a certain definite number m of divisions seen through the telescope, and notice the number n seen *directly*, which correspond with these. Then the magnifying power $= n/m$.

Verify this by measuring the distance from the object-glass to the pin, and dividing it by the distance from the eye-lens to the pin.

Also determine the focal lengths of the two lenses and calculate the ratio of the focal length of the object-glass to that of the eye-lens. This gives the value of the magnifying power for the telescope in normal adjustment.

§ 4. THE OPTICAL LANTERN

The **optical lantern** is used for projecting a magnified image of an object—usually a photographic transparency—on a screen. It comprises two lenses, or lens systems, the **projection lens (or objective)**, and the **condenser.** The former is a corrected achromatic system which gives a real, magnified image of an object placed at a point somewhat beyond the first principal focus. The condenser usually consists of two plano-convex lenses mounted near together and forming a converging system. Its object is to concentrate the divergent light from the source, so that as much light as possible may pass through the middle of

the projection lens. This causes the image to be distorted as little as possible and also gives the largest field.

The **linear magnification** produced by the projection lens

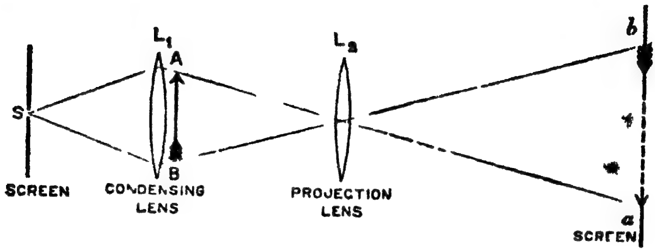


FIG. 1[illegible]0.—Diagram showing Construction of Optical Lantern.

is the ratio of the linear dimensions of the image to the corresponding linear dimensions of the object. This ratio is considered positive for an erect, negative for an inverted, image. The general formula for the linear magnification m is

$$m = \frac{v}{u},$$

for the length of the image is to that of the object as the distance v of the image from the lens is to the distance u of the object from the lens.

But $\qquad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for a lens of focal length f.

Hence $\qquad 1 - \frac{v}{u} = \frac{v}{f}$ or $1 - m = \frac{v}{f}$.

Thus the focal length $\qquad f = \frac{v}{1 - m}$.

*EXPT. 137. **Construction of an Optical Lantern.**—To illustrate the action of the optical lantern select two lenses of large aperture, one having a focal length of about 25 cm., the other about 15 cm. A source of light of small dimensions is required. This may be secured by placing the flame of a candle or lamp behind a hole of about 0·5 cm. diameter in a metal screen. As object a metal arrow, or a scale ruled on glass, may be employed. If an attempt is

made to obtain a magnified image on a distant screen, using the lens of longer focal length as the projection lens, it will be found that the image is very faint, and that only the central portions appear at all. Now place the lens of shorter focal length behind the object, and adjust the screen with the hole in it so as to form an image of the hole at the place occupied by the projection lens, *i.e.* so that the hole and the projection lens are at conjugate foci with regard to the condenser or short focus lens. For example, the apparatus may be adjusted so that the distance between the hole and the projection lens is a minimum (p. 277) and equal to four times the focal length of the condenser. The conjugate foci in this case are said to be the **symmetric points** of the lens.

If the object is just in front of the condenser, the image on the screen should now be uniformly illuminated and all parts of the object (assumed smaller than the aperture of the condenser) should be represented on the screen. Examine the effect of moving the position of the screen with the hole in it, and notice that there is only one position which gives uniform illumination.

EXPT. 138. **Measurement of the Magnification and Determination of the Focal Length of the Projection Lens.**—Measure the distance between two well-defined points on the object, and the corresponding distance between the image points. Calculate the linear magnification, which in this case is considered negative as the image is inverted.

Measure the distance from the lens to the screen and deduce the value of the focal length f from the formula

$$f = \frac{v}{1 - m},$$

being careful to give the proper signs to v and to m

CHAPTER VII

SPECTRA AND THE SPECTROMETER

§ 1. Formation of the Spectrum

When white light is passed through a prism, as in the celebrated experiment of Sir Isaac Newton, the light is dispersed giving rise to a coloured band known as the **spectrum.** To produce a pure spectrum, in which there is no overlapping of the images of different colours, it is necessary to use a very narrow slit, and also to pass parallel rays through the prism when the latter is in the position of minimum deviation.

Expt. 139. **Projection of a Spectrum on a Screen.**—A powerful source of white light is required, such as lime light or the electric arc, but in a perfectly dark room a gas or oil lamp may be used. The light of the source should be focussed on a narrow vertical slit in a metal plate by a convex lens used as **a condenser.**

A second convex lens is placed on the other side of the slit, and is adjusted till an image of the slit is focussed sharply on a white screen. The prism is then placed, with its refracting edge vertical, in the path of the light issuing from the lens. On placing a sheet of white paper in the path of the emergent rays, a coloured band will be seen, and the screen will, in general, require to be moved from its first position, so that this band of colour may fall upon it. Rotate the prism about a vertical axis, and note whether the coloured image moves towards or away from the original undeviated image. Turn the prism until the position of minimum deviation is reached, *i.e.* until the spectrum is formed as near as possible to the undeviated image. It will probably be necessary to focus the

image of the slit on the screen again, after the position of minimum deviation has been found. This may be done by placing a small piece of plane mirror in the path of the light

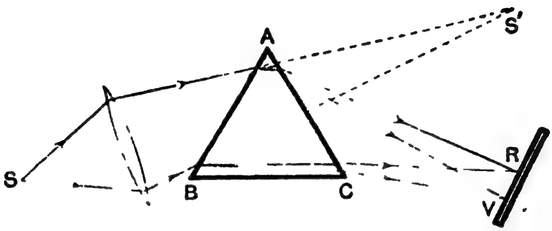


FIG. 157. Projection of Spectrum.

issuing from the lens, turning the mirror till a white image of the slit is formed on the screen close to the spectrum, and then moving the lens till this image is focussed sharply. In this way a moderately pure spectrum is projected upon the screen.

Since the rays passing through the prism belong to a converging beam the spectrum produced is not strictly a pure spectrum. To satisfy the condition that monochromatic rays should be parallel, when passing through the prism, the distance between the lens and the slit must be equal to the focal length of the lens. An eye

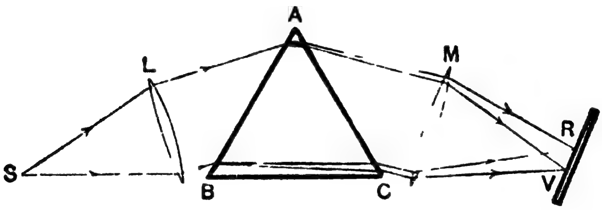


FIG. 158.—Projection of Pure Spectrum.

placed to receive the beam emerging from the prism would then see a virtual pure spectrum. In order to project the pure spectrum on a screen, a second convex lens must be placed in the path of the light, so that its distance from the screen is equal to its focal length. This arrangement for projecting a pure spectrum is sometimes useful, as, for example, when it is desired to photograph the spectrum by replacing the screen by a photographic plate. The same principle is employed in the construction of the spectrometer.

§ 2. The Spectrometer

The **spectroscope** is an instrument for producing dispersion of rays of light so as to form a spectrum, and for observing the spectrum so formed.

The **spectrometer** is a similar instrument provided with suitable arrangements for *measuring* the deviation of the dispersed rays.

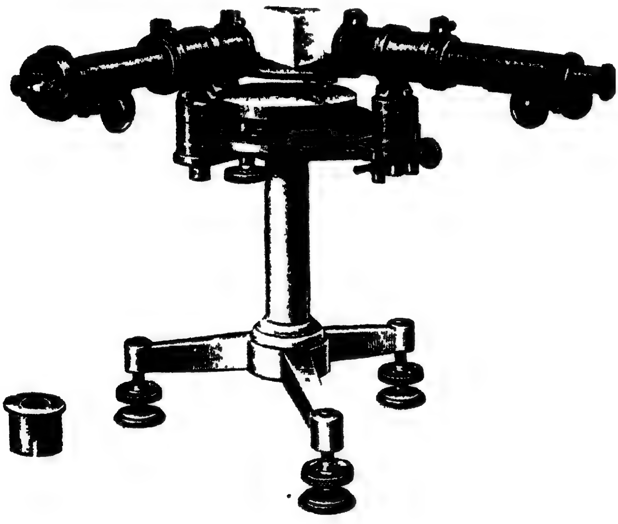


Fig. 159.—Spectrometer

The essential parts of such an instrument are:

(1) The **collimator**, which is an apparatus for securing a pencil of parallel rays.

(2) The **prism** (or diffraction grating), for dispersing the rays, mounted on a revolving table.

(3) The **telescope**, for viewing the spectrum.

The instrument is also provided with graduated circles (with verniers) for determining accurately the positions of the prism

and the telescope. Figs. 159 and 160 show the most important features of the instrument.

The **collimator** is a tube which carries at one end a narrow adjustable slit S, and at the other an achromatic convex lens L. The slit is illuminated by the source of light the spectrum of which is to be examined. For many purposes a flame impregnated with a salt of sodium, and showing a characteristic yellow coloration, is a convenient source, since the light is approximately monochromatic. The distance between the slit and the lens can be adjusted so that the slit is at the focus of the lens and the light emerges from the lens as a parallel beam.

The **prism** ABC rests on a circular table D, which is capable of rotation about a vertical axis. The table is provided usually with

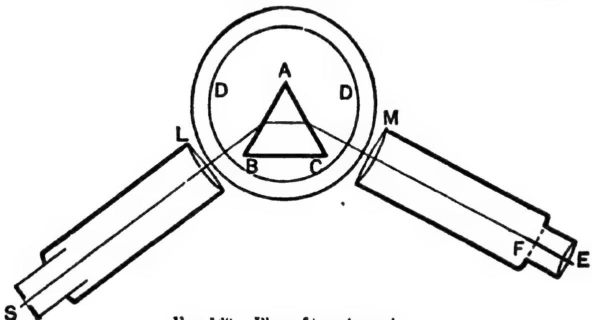


FIG. 160.—Plan of Spectrometer.

a clamp so that it may be fixed in any desired position, and sometimes with a tangent screw to give it a slow motion.

Parallel light emerging from the prism falls on the lens M, and is thereby brought to a focus at F, the principal focus of the lens, so that a real image of the slit is formed in the focal plane. Another lens E (or more commonly a compound eye-piece) is used to give a magnified virtual image of this real image. The two lenses M and E, mounted in a tube, together constitute a **telescope**. The telescope can be rotated about the same vertical axis as the prism table, and, like the latter, is usually provided with a clamp and a tangent screw.

EXPT. 140. **Adjustments of a Spectrometer.**—The exact adjustment of a spectrometer is a process which requires considerable care. It may be assumed that the mechanical adjustments have been made by the instrument-maker, and only the principal optical adjustments will be described.

The Telescope.—The eye-piece of the telescope is used to give a magnified image of an object placed at a certain small distance from the field-lens. It is made to slide in and out of the telescope tube. Turn the telescope towards a uniformly illuminated surface, such as a light wall, and slide the eye-piece in and out of the tube until the spider-line or cross-wires fixed in the tube can be seen distinctly. **The eye-piece is** now said to be **focussed on the cross-wires.** In consequence of the power of accommodation of the eye there is, however, a certain amount of latitude in this adjustment. It is next necessary **to focus the telescope for parallel rays,** *i.e.* to make the distance of the object-glass from the cross-wires equal to the focal length of the lens. The simplest way of making this adjustment is to focus the telescope on a very distant object.

When the adjustment has been made, the observer looking through the telescope should be able to see clearly both the distant object and the cross-wires without altering the accommodation of the eye. To test the accuracy of the adjustment the method of no parallax should be used, that is, the eye should be moved from side to side *behind the eye-piece* and any relative motion of the spider-line and the distant object should be noted. The adjustment is correct when no such motion can be observed.

The Collimator.—Set up a sodium flame (p. 295) so that the brightest part of the flame is opposite the slit of the collimator, and turn the telescope so that the axes of the tubes are in the same straight line.

On looking through the telescope the yellow light passing through the slit should be seen, but the image of the slit will, as a rule, be badly defined. *The collimator must now be focussed* by altering the distance between the slit and the lens until the image of the slit is seen with clear and well-defined edges.

When the adjustment is correct there should be no parallax between the cross-wires and the edges of the slit. Since the telescope already has been focussed for parallel light, the collimator must now be giving parallel light from the slit.

EXPT. 141. **Measurement of the Angle of the Prism of the Spectrometer.**—Open the slit fairly wide so as to allow plenty of light to pass through the collimator. Place the prism on the table of the spectrometer with the angle to be measured turned towards the lens of the collimator. Parallel light from

this lens will now fall on both the faces AB, AC of the prism, which contain the angle to be measured. Some of the light falling on each face will be reflected as shown by the continuous lines, and it is easy to prove that the angle between the two reflected beams is equal to twice the angle of the prism. By moving the eye in the horizontal plane of the axis of the collimator, and looking at one face AB of the prism, the direction of the reflected beam can be found. Turn the telescope

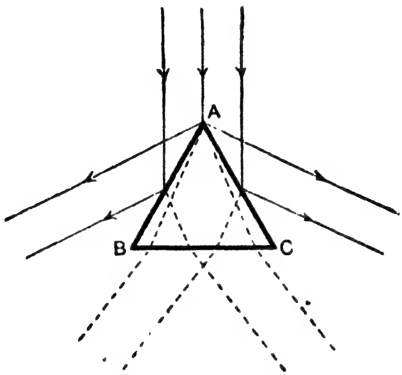


FIG. 161.—Measurement of Angle of Prism.

to point in this direction; then on looking through it the image of the slit should be seen. *When the slit has been brought into the field of view, the width of the slit must be made very small* and the telescope turned till the intersection of the cross-wires coincides with the image of the narrow slit.

Read the position of the telescope by the vernier or verniers provided. To do this it may be necessary to reflect the light of a gas or electric lamp upon the verniers by means of a small mirror. Without moving the prism or the table, turn the telescope to view the image formed by reflection from the second face AC of the prism, and again read the position of the telescope. Find the angle between the two positions of the telescope, and deduce the angle of the prism, A.

It often happens that *four* slit images can be seen as the telescope is moved round in a horizontal plane. Two of these are reflection images, the two of which the positions are required; the other two are images formed by *refraction* through the back surface of the prism, and care must be taken to use neither of these

by mistake. This can be avoided readily if the images are first obtained by the unaided eye, and the telescope brought into position without moving the eye, it being easy to see from which surface the light is coming.

The bogus images correspond with the refracted rays shown dotted in Fig. 161. This trouble can be avoided entirely if the back surface BC is covered with paper, or if a prism with a *matte* face is used.

Sometimes the reflected images can be seen plainly with the unaided eye, but cannot be seen in the telescope. This is due to the table of the spectrometer not being properly levelled, the light being reflected either upwards or downwards so that it strikes the inside of the telescope tube. It will be observed in such a case that when the telescope is swung into position after finding the images with the eye unaided, the eye-piece of the telescope is not level with the eye. The table must be levelled with the screws fitted to it until the eye when viewing the reflected images unaided is on the same level as the eye-piece. Then finally adjust the level till the image of the slit occupies the same position in the telescope field when viewed by reflection in either face, and also when viewed directly with the telescope and collimator in line, and no prism on the table.

EXPT. 142. **Measurement of the Angle of Minimum Deviation.**—Place the prism on the table of the spectrometer so that the angle A already measured may serve as the refracting angle. Then the light from the collimator should be incident on the face AB and emerge from the face AC to enter the telescope. The positions of the different parts of the apparatus when light is refracted through the prism are shown in plan in Fig. 160. *In setting up the prism be careful to place it in such a position that the maximum amount of light available from the collimator may be utilised and enter the telescope.* This is best achieved if the *refracting edge* of the prism is placed over the *centre* of the table.

In order to find the direction in which the telescope must point, turn it to one side, and, using one eye only, look into the face AC of the prism, moving the eye until an image of the slit formed by refraction through the prism is found. Have the slit wide open in looking for this image. When the proper direction has been found, turn the telescope to point in this direction without moving the head. On looking through the telescope the image of the slit should now be within the field of view.

The light has been deviated in passing through the prism;

the angle of deviation being the *acute* angle between the direction of the collimator and that of the telescope. This angle is a minimum when the light passes through the prism symmetrically.

To find the position of minimum deviation, look through the telescope and rotate the table carrying the prism so that the image of the slit moves towards the axis of the collimator produced. It may be necessary to move the telescope so as to keep the image within the field of view. A position will be found in which the image of the slit is as near as it can possibly be to the axis of the collimator. This is the position of minimum deviation.

The telescope is adjusted until the slit is approximately in the centre of the field of view when the prism is in this position, and the telescope must then be clamped. The slit is now made as narrow as possible, and the prism is rotated slowly backwards and forwards through the minimum position several times. The telescope is moved by the slow motion screw until, as the prism is rotated, the slit moves up from one side until it is bisected by the vertical cross-wire, and then moves away again *to the same side* without ever passing beyond this position. The position of the telescope must now be read by means of the verniers and the graduated circle.

Now remove the prism from the table of the spectrometer, and turn the telescope to point directly towards the collimator, so that the undeviated rays may enter the object-glass and form an image of the slit on the cross-wires. Clamp the telescope in this position and make the final adjustment with the slow-movement screw. Again read the position of the telescope by means of the verniers and the graduated circle.

The difference between the reading in this position and that already obtained in the position of minimum deviation gives the angle of minimum deviation D.

The refractive index of the material of the prism may now be calculated by means of the formula

$$\mu = \frac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}}.$$

The refractive index of a liquid may be determined by the same method, using a hollow prism with *parallel* worked glass faces to contain the liquid.

MAPPING SPECTRA

The position of a line in the spectrum may be determined by finding the position of the telescope when the line is on the cross-wires, or by finding the reading of the line on a scale which is reflected into the field of the telescope from the second face of the prism. In either case the prism is supposed to be kept fixed. In some instruments, known as **Constant Deviation Spectrometers,** the telescope is kept fixed, and the prism is rotated so as to bring one line after another upon the cross-wires. The angle through which the prism is turned then serves to fix the position of a particular line.

If a curve be plotted showing the relation between the wave-length of the lines and their position as defined above, the graph is called a **map** of the spectrum, or a **calibration curve** of the spectrometer. Such a map may be used to find the wave-length of any line of which the position can be determined.

Wave-lengths are usually expressed in Ångström Units (A.U.) or Tenth Metres (10^{-10} metre or 10^{-8} cm.). Occasionally, however, they are expressed in terms of a unit ten times larger, viz. the micromillimetre ($1\mu\mu = 10^{-6}$ mm. $= 10^{-7}$ cm.).

EXPT. 143. **Mapping Spectra.**—Adjust the spectrometer as in Expt. 140, and using a sodium flame as the source of light, arrange that the prism may be in the position of minimum deviation as in Expt. 142. Clamp the prism in this position.

When a photographic scale fixed in a separate collimator tube is used to determine the position of the lines, this must be set up so that the scale, illuminated by a small lamp, gives rise to an image in the focal plane of the telescope, by reflection from the face of the prism. When this method is not employed, the position of the telescope must be determined by reading the vernier attached to it.

The position of the sodium line should be taken as a standard, and the position of other lines determined with reference to it. The sodium line, when examined by a spectroscope of sufficient resolving power, is found to consist of two lines close together, known as the D lines.

Determine the position of a number of lines in the flame

spectra of metallic salts, the spectra being produced by volatilising salts of the metals in a Bunsen flame. Introduce the salts into the flame on a platinum wire fused into a glass tube as a handle, cleaning the wire between each experiment by immersing it, while incandescent, in hydrochloric acid. Suitable salts for this purpose are lithium chloride, thallium chloride, potassium chloride. (See Appendix, p. 599.) In the case of the potassium salt two lines can be found, one in the red, the other in the extreme violet. The latter can only be found by turning the telescope far enough into the violet, and observing *immediately* after the salt is introduced into the flame. Two experimenters are required, one to introduce the salt into the flame, the other to observe the line in the telescope. Nitre may be used for this line. Strontium chloride gives a strong line in the blue at 4607 A.U. Barium and calcium chloride give a number of lines which can be identified *after* the calibration curve has been drawn.

Spark spectra may be observed by passing the discharge of an induction coil (see Expt. 219 in Electricity) between terminals of the metal to be examined. The inner and outer coats of an insulated Leyden jar should be connected to the terminals of the spark-gap.

The spectra of gases may be observed by passing the discharge of an induction coil through 'vacuum tubes' containing the rarefied gases.

Absorption spectra may be observed by illuminating the slit with white light, and introducing the absorbing substance in the path of the rays travelling towards the slit. Observe in this way the characteristic spectrum due to a dilute solution of blood, and the spectrum due to an alcoholic solution of chlorophyll. The vapour of iodine, obtained by heating a few crystals in a glass tube in front of the slit, gives rise to fine dark absorption lines.

By reflecting sunlight into the collimator, the dark Fraunhofer lines, due to absorption in the atmospheres of the sun and the earth, may be observed.

Plot a curve on squared paper, on a large scale, showing the relation between the scale-reading and the wave-length corresponding with certain suitable lines. This curve is an **Interpolation Curve** for the particular prism used. From this curve may be determined the wave-lengths of bright lines or the limits of absorption bands.

CHAPTER VIII

PHOTOMETRY

§ 1. General Principles

The branch of Physics which deals with the estimation of the **light-giving power** or luminous intensity of a source of light is called **Photometry**. The **candle-power** is taken as the unit of illuminating power; and the luminous intensity of any source is expressed in terms of the number of **standard candles** which would give out the same quantity of light.

The standard candle is defined as a sperm candle $\frac{7}{8}$ inch in diameter, weighing six to the pound, and burning at the rate of 120 grains an hour. It is an unsatisfactory standard, and usually some other standardised source, such as a Pentane lamp, is used. The most convenient standard is perhaps an incandescent electric lamp, working at a certain constant voltage. The unit of luminous intensity based on such standards is called the International Candle.

The **illumination**, as the **intensity of illumination** of a surface is usually called, is measured in terms of a unit called the **lux**. The illumination on a surface is 1 lux when the surface is illuminated normally by a point source of unit intensity at a distance of 1 metre.

In Great Britain the **candle-foot** is commonly used as the unit of intensity of illumination. It is the illumination on a surface illuminated normally by one standard candle at the distance of one foot.

The term **luminous flux** is used in photometry to denote the emission of light per unit time. The unit of light flux is the flux emitted per unit solid angle from a source of unit intensity; this is called **1 lumen.**

The eye is unable to make direct estimate of luminous intensity with any approach to accuracy, on account of the variation of the diameter of the iris, and also for other reasons, mainly physiological and psychological. To compare luminous intensities, therefore, some form of apparatus is used to assist the eye. Any such apparatus is called a **Photometer.**

The use of a photometer depends on the adjustment of two surfaces to have equal intensity of illumination, this equality being judged by the eye of the observer. With lights of the same *colour* the adjustment can be made with practice to about 0·5 per cent; but if the surfaces are illuminated by lights of different colours the accuracy is not nearly so great. In this case it will be found much easier to make the comparison between the two surfaces if the eyes are half-closed. It is impossible to retain any accurate idea of the intensity of illumination of a surface even for so short a time as a second, and therefore the two surfaces to be compared must be viewed simultaneously or interchanged rapidly as in the Flicker Photometer. If the surfaces are separated by a band whose intensity of illumination is different from that of the surfaces to be compared, the estimation is rendered much less accurate, and therefore the surfaces must really be *contiguous* parts of the *same* surface illuminated simultaneously.

The illumination of a surface due to a small source of light varies *inversely as the square of the distance* of the surface from the source. If, therefore, a source of luminosity of candle-power, I, is placed d cm. from a surface, the intensity of illumination is measured by I/d^2.

If two sources of candle-powers, I_1 and I_2 respectively, illuminate two parts of a surface equally, when at distances d_1 and d_2 cm., the illuminating powers are related by the equation

$$\frac{I_1}{d_1^2} = \frac{I_2}{d_2^2};$$

so that if I_2 is known and d_1 and d_2 are measured I_1 can be calculated, for

$$I_1 = I_2 \frac{d_1^2}{d_2^2}.$$

§ 2. Photometric Measurements

RUMFORD'S PHOTOMETER OR THE SHADOW PHOTOMETER

In this apparatus the surface illuminated may be either a white opaque sheet of paper viewed from the same side as the two sources of light, or it may be a translucent screen viewed from the side remote from the sources. In either case, one part of the surface is screened from one of the sources by a rod placed between the screen and the source, the rod being so situated that shadows

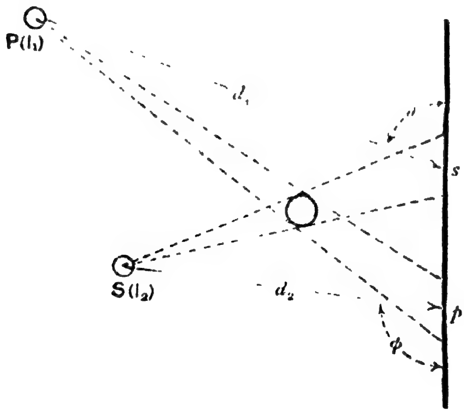


Fig. 162.—Rumford's Photometer.

of the rod, one thrown by each source, lie side by side on the screen. The shadows must neither overlap nor should they be separated by a bright band illuminated by both sources. In Fig. 162 the shadows are separated to simplify the diagram.

The shadow thrown by each source is of course illuminated by light from the other, and when the shadows are equally intense, the intensities of illumination due to the two sources are equal.

EXPT. 144. **Rumford's Photometer.**—Set up in a dark room a vertical rod in front of the screen of the photometer. Compare the luminosity of a gas (or electric) light with that of a wax candle by adjusting the distances of the sources from the screen, until the shadows are of the same intensity. Care must be taken that the lines joining the sources to the rod are equally inclined to the screen, *i.e.* the angles θ and ϕ in Fig. 162 must be approximately equal. Measure the distances, d_1, d_2, from the sources *to the screen*, and calculate the candle-power of the source under test. Repeat the determination several times with the sources at different distances from the screen, and take the mean of the results obtained.

BUNSEN'S PHOTOMETER OR THE GREASE-SPOT PHOTOMETER

In this form of photometer, an opaque white screen (of paper) is rendered translucent over a portion of its surface by means of a spot of clean white paraffin wax.[1] It is illuminated from one side

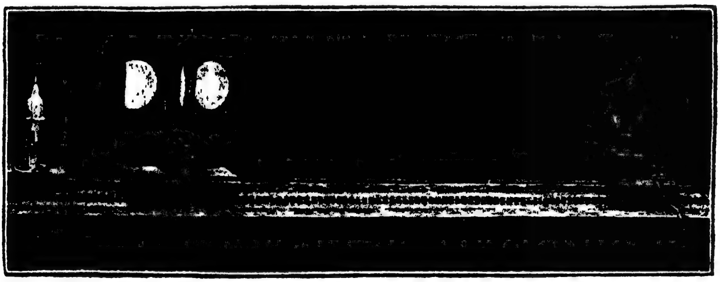


FIG. 163.—Bunsen's Photometer.

by the standard source, and from the other by the source of which the luminosity is to be measured.

The opaque portions of the screen are assumed to *reflect* the whole of the incident light, while the translucent portions reflect a definite fraction, say $1/n$, of the light falling on them, transmitting the remainder. If the intensity of illumination from one side,

[1] "A piece of good homogeneous paper is uniformly warmed on a plate. In the centre of this a small circlet or annulus is described with a little melted stearin on a fine brush. This ring is allowed to cool; there is a free unwaxed spot within the boundary thus made, which must now be filled with melted wax, well pressed into the paper. The previously formed boundary secures a well-defined spot."—Sheppard, *Photo-chemistry*, p. 31.

is I_1/d_1^2 and from the other side is I_2/d_2^2, the grease-spot will appear to be of the same brightness as the rest of the screen, when

$$\frac{I_1}{d_1^2}=\frac{1}{n}\frac{I_1}{d_1^2}+\left(1-\frac{1}{n}\right)\frac{I_2}{d_2^2},$$

i.e. when

$$\frac{I_1}{d_1^2}=\frac{I_2}{d_2^2}.$$

According to this simplified description the grease-spot should disappear when viewed from either side. There is, however, a certain fraction of the light *absorbed* in passing through the translucent grease-spot, and although it may be possible to make the spot almost invisible when viewed from one side, the appearance on the other side is always quite different. In practice, the adjustment should be made until the grease-spot appears *the same amount darker* than the rest of the screen on *both* sides.

Two plane mirrors are usually attached to the screen so as to make an angle of about 60° with it on either side. By means of these mirrors both surfaces of the screen may be viewed at the same time.

An alternative method is to adjust the 'unknown' source until the grease-spot is invisible when viewed from the *standard* side, and to observe the value of d_1 which gives this result; then to readjust the position until the grease-spot is invisible when viewed from the *unknown* side, keeping the standard source and the screen fixed, *i.e.* d_2 being kept constant.

If these distances are d_1 and d_1' their mean may be taken as the true value of d_1, or the expression

$$I_1 = I_2\frac{d_1^2+d_1'^2}{2d_2^2}$$

may be used to calculate I_1.

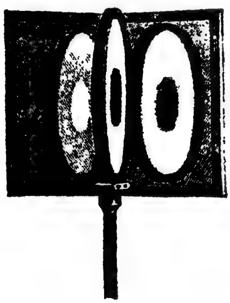


Fig. 164.—Bunsen's Photometer Head.

This second method is easier to use, there being no trouble such as is involved in the first method in estimating when the grease-spot is the same amount darker than the rest on both sides.

Expt. 145. **Bunsen's Photometer.**—Use the photometer for comparing the illuminating power of an electric lamp with that of a candle, and that of a gas flame with that of a candle. Check the results of the observations by comparing the gas flame and the electric light directly

If possible use a stand on which three candles can be mounted close together and make the comparisons with one, two, and three candles, estimating the percentage error possible in the measurement.

JOLY'S PHOTOMETER

Two rectangular blocks of paraffin, about 5 × 2 × 1 cm., are placed together, with two of the larger faces separated by a sheet of tinfoil. The blocks are placed between the two sources of light to be compared, so that one block is illuminated by one source and the second block by the other source. The observer views the blocks from the side (Fig. 165), and adjusts the position until the two faces separated by the tinfoil as dividing line appear equally bright. The eyes of the observer should be protected by suitable screens from the direct rays of the lights used.

Fig. 165.—Joly's Paraffin Wax Photometer.

Expt. 146. **Joly's Photometer.**—Set up the photometer on a long optical bench and use it to compare the candle-power of an incandescent gas lamp with that of an electric lamp. When the correct positions have been found, measure the distances from the photometer to the two sources and calculate the ratio of the illuminating powers. Repeat the observation several times for different positions of the sources, and take the mean of the results. Make an estimate of the percentage error possible in the measurement.

LUMMER-BRODHUN PHOTOMETER

The essential part of this instrument is the same as that in Swan's prism photometer (1859). The two sources send beams of

light to two mirrors at $22\frac{1}{2}°$ to the paths of the beams, and at $45°$ to each other. After a second reflection the beams strike a block of glass consisting of two right-angled prisms cemented together at the middle parts of their faces with Canada balsam, but separated by an air film at their edges (Fig. 166).

The observer views the base of one of the prisms through a telescope at C. The light from the source A is transmitted by the balsam, but is totally reflected by the air film. The light from B

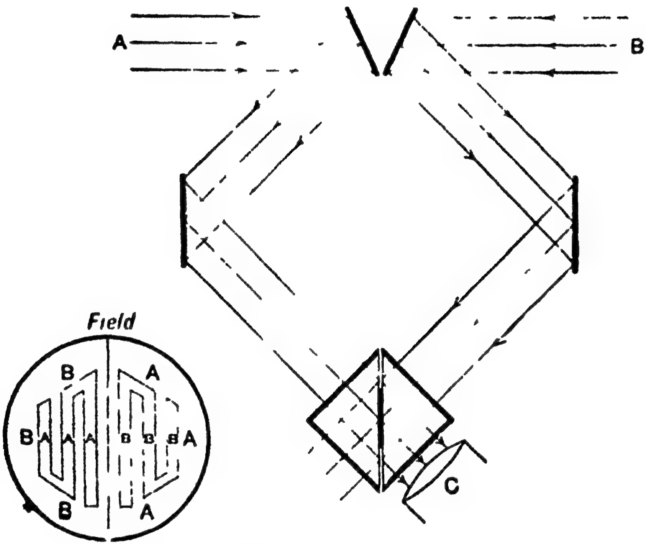


FIG. 166.—Lummer Brodhun Photometer

is totally reflected by the air film so that it enters the telescope parallel to the transmitted light from A. Thus the telescope receives a composite beam of light, the edges being composed of light from B, and the centre of light from A. The apparatus is usually arranged to give a somewhat elaborate 'field pattern.'

By using a balsam of the same refractive index as that of the two prisms, there is no reflection at the inter-face and no absorption of the transmitted light, so that the trouble which arose in Bunsen's photometer is entirely obviated. Very great accuracy can be secured by this form of apparatus, and it is very largely used in photometric laboratories.

EXPT. 147. **Lummer-Brodhun Photometer.**—Set up the Lummer-Brodhun photometer on the optical bench, and use

it to find the candle-power of a gas flame and of an electric lamp. Confirm the results by comparing directly the gas flame and the electric lamp. Estimate the percentage error possible in the measurement.

§ 3. MEASUREMENT OF ILLUMINATION

The illumination of a surface can be measured by means of an illumination photometer. This is a portable instrument provided with a screen which may be placed in the position where the illumination is to be measured. A neighbouring surface, viewed at the same time, is illuminated only by a standardised source, usually a small electric lamp supplied by an accumulator. The illumination of this surface can be varied until equality of illumination is secured by various methods, *e.g.* by tilting the surface. The scale of such an instrument must be calibrated empirically.

PART III

ADDITIONAL EXERCISES IN LIGHT

1. When a pin is fixed between two parallel mirrors a number of images can be seen. Trace the path of the rays by which the third image in one mirror is seen.

2. Set up two plane mirrors so that the angle between them is 72°. Find the positions of the images of a pin placed in the angle between the mirrors.

3. Plot a curve showing how the lateral displacement of a ray of light passing obliquely through a plate of glass depends upon the angle of incidence.

4. A cubical glass tank is filled with water, and a vertical pin is placed inside it. Plot the caustic curve for the rays refracted into the air through one side of the tank.

5. Plot the caustic curve formed by rays reflected from a cylindrical mirror, using a pin as the object and two other pins to determine the reflected rays. Do this both for convex and concave mirrors.

6. Plot the caustic curve formed by rays refracted into air from a cylindrical beaker containing water, using a pin inside the beaker as the object and two other pins to determine the refracted rays.

7. Plot the caustic curve formed when parallel rays are refracted through a cylindrical lens. (One-half of a lantern condenser may be used instead of a cylindrical lens.)

8. A pin is fixed in a vertical position inside a cylindrical beaker of water. Trace the paths of rays from the pin into the air. Deduce the position of the image seen by an eye viewing the pin from the side of the vessel nearest to the pin.

9. Trace the paths of parallel rays of light through a convex lens, and deduce the focal length.

10. Trace the paths of parallel rays of light through a concave lens, and deduce the focal length.

11. Find the focal length of the given convex lens in three different ways.

12. Place the given convex lens so as to form on a screen an image three times the size of the object. Measure the distance from the object to the screen, and deduce the focal length of the lens.

13. Plot a curve showing how the distance of the image from the given convex lens depends upon the distance of the object, using pin-sights and the method of parallax.

14. Find the shortest distance between an object and its image formed by the given convex lens. Deduce the focal length of the lens.

15. The given lens is fixed at a distance of 40 cm. from a screen. Find at what distance from the lens an object must be placed to give a sharply defined image on the screen. Determine the linear magnification of the image.

16. Find the focal length of the lens formed by filling a watch-glass with the given liquid.

17. Set up the two given convex lenses so that parallel rays passing through the first meet again at the principal focus of the second.

18. Measure the focal length of the combination formed by two given lenses, (*a*) in contact, (*b*) separated by a distance of 2 cm.

19. Determine the radii of curvature of the surfaces of the given concave lens.

20. Determine the radii of curvature of the surfaces of the given convex lens.

21. Set up a convex lens to form a real image on a screen. Between the lens and the screen introduce a concave lens with a plane mirror behind it. Adjust the position of this lens so that an image is formed coincident with the object. Deduce the focal length of the concave lens.

22. Find the centre of curvature of a concave mirror. Set up a concave lens between the mirror and its centre of curvature. Adjust the position of a pin so that it may coincide with its reflection formed by rays passing through the lens. Deduce the focal length of the lens. In what case does this method fail? Is the method applicable for a convex lens?

23. Arrange a slit, prism, and lenses to project a pure spectrum on a screen.

24. Adjust a prism on the table of a spectrometer to be in the position of minimum deviation. Measure the angle at which light is incident on the prism by finding the direction of the rays reflected from the first face.

25. Plot a curve showing how the angle of deviation for the given prism varies with the angle of incidence.

26. Measure the angle of the prism of a spectrometer, *keeping the telescope fixed*, and turning the prism so that the image of the slit is observed by reflection first from one face, and secondly from the other face. (The angle so measured is the supplement of the angle of the prism.)

27. Compare the refractive indices of two liquids, using a spectrometer and a hollow prism of small angle.

28. Map the flame spectra of calcium, strontium, and barium.

PART IV
HEAT

CHAPTER I

THERMOMETRY

§ 1. Introductory

To define **a scale of temperature** we may use any property of a body which varies continuously with temperature. If this property have values X_0 at the **Freezing Point**, and X_{100} at the **Boiling Point** of water under standard pressure, we define one centigrade degree as that change in temperature which causes a change $(X_{100} - X_0)/100$ in this property.

If the value of the property be X_t when the body is in certain surroundings, the temperature of its surroundings is given by

$$t^\circ \text{ C.} = \frac{X_t - X_0}{X_{100} - X_0} \times 100$$

on the particular scale which depends on this property X.

For most practical purposes we take as our scale the scale depending on the position of the top of a thread of mercury in a glass tube. Its position is observed at the Freezing Point and again at the Boiling Point; and the thermometer stem between these points is divided into 100 equal divisions, each being one centigrade degree. Two mercury-in-glass thermometers will agree only if similar kinds of glass are used and the bore of each is quite regular.

Mercury-in-glass thermometers are used chiefly on account of their convenience. The standard thermometer for scientific purposes is a constant volume thermometer (p. 337) filled with hydrogen gas.

The efficiency of a laboratory depends on the care taken of apparatus *when in use*, and the student should use all reasonable precautions in the handling of fragile apparatus such as thermometers. A thermometer should never be raised to a higher temperature than that for which it is constructed, and the thermometer should be returned to its case when finished with.

In reading a thermometer any error due to parallax must be avoided; that is, the line joining the eye to the top of the mercury thread must be at right angles to the stem of the instrument, so that the divisions of the scale may not be displaced relatively to the top of the thread. The student should practise estimating the reading of the thermometer to the tenth part of a centigrade degree.

It must not be forgotten that a thermometer registers its *own* temperature; and therefore in using it to determine the temperature of any substance it must be brought into intimate contact with that substance and must be left there a sufficient length of time to acquire its temperature.

EXPT. 148. **Effect of Stem Exposure.**—Place a thermometer in a hypsometer (p. 318) so that the whole stem is enclosed in the steam up to the 100° C. mark. Note the reading of the thermometer when the water is boiling gently. Raise the thermometer until the stem is exposed from the 70° mark upwards, leave it for a few minutes and take the reading again, the water being kept boiling gently the whole time. Repeat the observations with the stem exposed from 40° C. upwards, and again from 10° C. upwards, and note the effect the exposure of the stem produces on the *reading*, although the temperature of the bulb is maintained the same throughout.

Bear this in mind in all thermometric measurements.

§ 2. FIXED POINTS OF A THERMOMETER

Two fixed points are necessary in order to define a scale of temperature.

The **lower fixed point** is defined as the temperature of fusion of ice from pure distilled water; that is, it is the

temperature at which ice and water can exist together in equilibrium. This is called the Freezing Point or Zero Point and is marked 0° on the centigrade scale. The effect of pressure on the melting point of a substance is so small that it can be disregarded in defining the freezing point for all practical purposes.

The **upper fixed point** is defined as the temperature of steam rising from pure distilled water boiling under normal atmospheric pressure. This pressure corresponds to a barometric height of 760 mm. of mercury. The upper fixed point is called the Boiling Point and is marked 100°. Thus, on the centigrade scale, the interval between the freezing point and the boiling point is divided into one hundred degrees.

The temperature of steam from boiling water is independent of the nature of the vessel in which the water is boiled, and of the impurities in the water, but varies with the atmospheric pressure. The variation of the boiling point with the pressure was carefully studied by Regnault, who found that in the neighbourhood of 760 mm. an increase of pressure of 26·8 mm. produced an elevation in the boiling point of 1° C. For small variations the change in boiling point may be taken proportional to the difference in pressure. The graph (Fig. 168) is drawn on this assumption. This should be copied in the student's note-book.

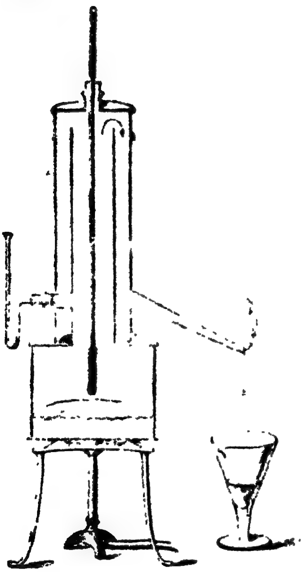
It is found that the glass of the thermometer changes gradually with time producing small changes in the fixed points. It is therefore necessary to redetermine these from time to time, so that corrections may be applied for the errors. In this country it is usual to mark the lower fixed point first.

EXPT. 149. **Determination of the Fixed Points of a Thermometer.**—(i.) **Freezing Point.**—Fill a suitable vessel nearly to the top with ice in small lumps, and allow the spaces between the lumps to become filled with ice-cold water. It is better not to drain away the water from the melting ice, but too much water must not be allowed to accumulate. The whole must be kept well stirred.

Place the thermometer carefully in the ice so that the bulb

is in the centre of the vessel, and the zero point is just above the surface of the ice. Read the lowest point reached by the top of the mercury column (estimating to one-tenth of a degree) while the mercury column is still surrounded by the ice. If the reading be above zero, the error is called positive; if below, negative. If the *error* be positive, the *correction* to be applied to the reading, to give the true temperature, is negative.

(ii.) **Boiling Point.**—To determine the boiling point the thermometer is placed in a metal vessel called a **hypsometer.**



Hypsometer.

This is a boiler fitted with a double-walled steam jacket above it. The thermometer is supported by a cork fitted into the top of the hypsometer in such a way that the upper fixed point is just visible above the cork. Care must be taken that the thermometer does not fall into the hypsometer, as this would probably result in breakage of the bulb. A loop of wire through the hole at the top of the stem will prevent such an accident. The thermometer should remain in the steam about ten minutes before the reading is taken. The water must not be made to boil too violently or the pressure of the steam in the hypsometer will exceed the atmospheric pressure. Read the top of the mercury column to a tenth of a degree.

Correction for Pressure.—Read the height of the barometer in millimetres and determine from the graph (Fig. 168) the boiling point corresponding to the observed atmospheric pressure.

Enter in the note-book this, the true boiling point, and also the boiling point recorded by the thermometer under test. Calculate the error of the thermometer at the boiling point.

In order to determine the correction required at any temperature between the freezing point and the boiling point, use a graphic method. Take intervals along a horizontal axis to

represent intervals on the thermometer scale, and distances

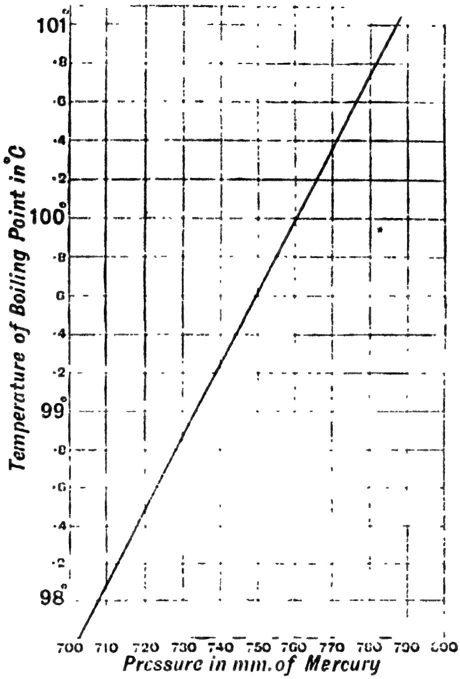


FIG. 168.—Variation of Boiling Point with Pressure.

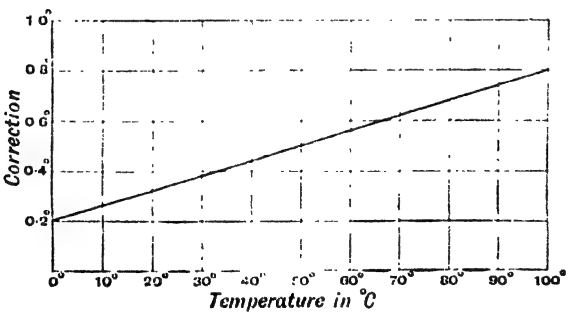


FIG. 169.—Correction for Thermometer.

along a perpendicular axis to represent the correction required.

In the diagram (Fig. 169) the correction at the freezing point is supposed to be + 0·2° C. and the correction at the boiling point + 0·8° C.

The correction is the amount that has to be *added* to the reading to give the true temperature.

§ 3. Calibration and Graduation of a Thermometer

To obtain equal values for the temperature differences indicated by equal movements of the mercury along the tube, it is essential that the bore should be uniform: this is rarely or never found to be the case.

To correct for this the bore must be calibrated by moving a thread of mercury along it, and measuring the length of the thread in different parts of the tube.

Expt. 150. **Calibration of the Bore of a Thermometer.**—A *small* flame is allowed to play on the tube at a point approximately 10 degrees away from the end of the mercury column. This detaches a thread of mercury by boiling the mercury just at the point where it is heated, and this thread can be used to calibrate the tube. It should be very nearly 10 degrees in length when detached. The thermometer stem is allowed to cool, and the thread is moved by gentle shaking till one end is approximately at the 0 C. mark, the bulb being cooled with ether to prevent the thread from joining up to the rest of the mercury. The position of each end of the thread is then observed with a travelling microscope,[1] the position being estimated on the scale of degrees of the thermometer. This is done by measuring the length of one degree in cm. on the scale of the microscope and measuring the *fraction* of a degree from the end of the thread to the preceding mark also in cm. The position is expressed to $\frac{1}{100}$ of a degree; *e.g.*

Microscope	Scale reading	on 9th degree division	=	12·36 cm.
,,	,,	,, 10th ,, ,,	=	14·08 cm.
,,	,,	,, end of thread	=	14·00 cm.

[1] The same kind of estimation can be made if a micrometer eye-piece is used, without using the travelling scale; there is no need to standardise the micrometer scale in this case, as only relative values are required.

The end of the thread is therefore at

$$9+\frac{1\cdot 64}{1\cdot 72} \text{ degrees, } i.e.\ 9\cdot 95(3).$$

The thread is then moved along until its 'lower' end is approximately where the 'upper' end was in the first measurement, and the positions of the two ends are again noted. It is then adjusted to a third position between 20° and 30° and measured again, this being repeated until the upper end is at the B.P.

The correction is worked out as in the following numerical example :—

1st position of thread from			− 0·03° to 9·79°,	thread length	9·82
2nd	,,	,,	9·85° to 19·93°	,,	10·08
3rd	,,	,,	19·88° to 29·94°	,,	10·06
4th	,,	,,	29·98° to 40·12°	,,	10·14
5th	,,	,,	40·00° to 49·78°	,,	9·78
6th	,,	,,	49·82° to 60·00°	,,	10·12
7th	,,	,,	59·95° to 69·90°	,,	9·95
8th	,,	,,	70·00° to 80·00°	,,	10·00
9th	,,	,,	80·03° to 90·17°	,,	10·14
10th	,,	,,	90·06° to 99·92°	,,	9·86

Mean length of thread = 9·995,

i.e. this mass of mercury would occupy 9·995 degrees anywhere up the scale, if the bore and scale were accurately uniform, or would occupy 9·995 mean degrees.

Imagine the thread starting at 0° C. ; its upper end would be at 9·82° very nearly. It *should be* at 9·995° C. if the bore were uniform.

The correction to be added to the reading 9·82 is thus + 0·175° C. Call this δ_{10}. If an identically similar thread were then joined on to it, the two together would reach to 9·82 + 10·08. They ought to reach to 2(9·995), *i.e.* the correction is 19·99 − 19·90 or + 0·09° C. Call this δ_{20} ; it is the correction required in the vicinity of 20° C.

Similarly at about 30° the correction is 3(9·995) − (9·82 + 10·08 + 10·06) = + 0·025° C. ; and so on. Thus we get

$$\begin{aligned}
\delta_{10} &= +0\cdot 175\\
\delta_{20} &= +0\cdot 09\\
\delta_{30} &= +0\cdot 025\\
\delta_{40} &= -0\cdot 2\\
\delta_{50} &= +0\cdot 095\\
\delta_{60} &= -0\cdot 03\\
\delta_{70} &= +0\cdot 015\\
\delta_{80} &= +0\cdot 01\\
\delta_{90} &= -0\cdot 135\\
\delta_{100} &= 0\cdot 00
\end{aligned}$$

The last value must be zero of course.

From these observations a correction curve can be drawn, giving the amount that has to be added at each point in the scale to correct for unevenness of bore and scale.

GRADUATION OF A THERMOMETER WITH AN ARBITRARY SCALE

In some cases the scale engraved on the stem of a thermometer may be entirely arbitrary, so that the readings are not obtained directly in degrees centigrade. For example, the stem might be marked with a scale of millimetres, yet such a thermometer can be used to find the temperature on the centigrade scale. For this purpose the thermometer must first be standardised by finding the two fixed points by the methods described in the previous section. Thus it might be found that at the freezing point the mercury stands at a point 24 mm. from the bottom of the scale, while at the boiling point the mercury stands at a point 184 mm. from the bottom of the scale. If the reading of the barometer at the time of the determination were 733 mm. the boiling point would be 99° C. instead of 100° C. Consequently the point 24 mm. from the bottom of the scale corresponds to 0° C., and the point 184 mm. from the bottom corresponds to 99° C. Thus a distance of 160 mm. on the scale corresponds to an interval of 99 centigrade degrees. It is then easy to calculate the interval on the centigrade scale for 1 mm. on the thermometer: in this case 99/160 degrees correspond to 1 mm.

Suppose this thermometer is employed for finding the temperature of a liquid in a calorimeter (p. 313), and that the mercury stands at 64 mm. from the bottom of the scale. Then the mercury stands at 40 mm. above the freezing point, and the corresponding temperature on the centigrade scale is $40 \times \frac{99}{160} = 24{\cdot}75^\circ$ C.

The relation between the scale of the given thermometer and the centigrade scale can be shown graphically, taking the readings of the thermometer as abscissae and the readings of the centigrade scale as ordinates.

Expt. 151. **Graduation of a Thermometer with an Arbitrary Scale.**—Standardise a thermometer provided with an arbitrary scale, in the manner described, and use it to determine the temperature of the room, and also the temperature of the water supply.

§ 4. Melting Points and Boiling Points

Expt. 152. **Determination of the Melting Point of a Solid.**—To determine the melting point of a solid such as paraffin wax, draw out a piece of glass tube in the flame of a

blowpipe so as to form a thin-walled capillary tube. Cut off with a file, or glass-cutter's knife, a piece of this tube a few centimetres long. The tube must now be filled with the

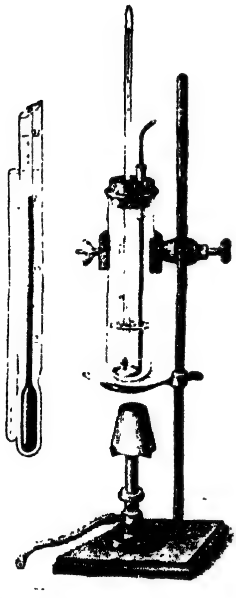


Fig. 170.—Melting Point of a Solid.

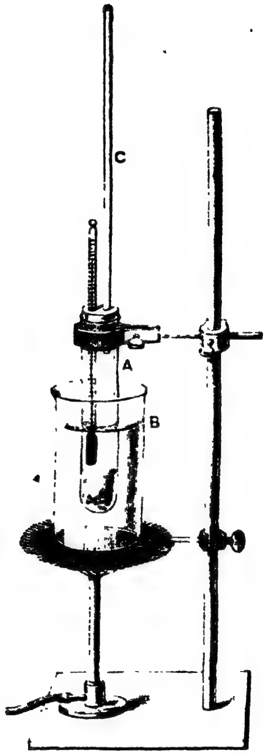


Fig. 171.—Boiling Point of a Liquid.

material under investigation by dipping one end into the liquid formed by heating a small quantity of the solid in a suitable vessel. In most cases the liquid will be drawn into the tube by capillary action. The tube must be sealed off at the bottom after filling, otherwise the substance will run out when melted, or the water will rise up the tube and the solidifying point cannot be observed.

The tube containing the solid substance is now attached to

the bulb of a thermometer by elastic bands or fine threads, and the bulb is heated carefully in a water bath (Fig. 170). The reading of the thermometer is noted at the instant when the solid in the small tube assumes the liquid state, and another reading may be taken by allowing the water-bath to cool and noting when the solid reforms. The temperature taken for the *melting* will be somewhat above the true melting point, and the temperature for the *solidification* will be too low. The mean must be taken as the true melting point. There is, however, a possibility of *super-cooling* the liquid and in such an event the true melting point is not obtained by this type of experiment.

In some cases the capillary tube may be dispensed with and a thin film of the solid may be formed round the bulb of the thermometer, which is then heated carefully as before.

See also the experiment on the curve of cooling when a liquid solidifies, p. 359.

EXPT. 153. **Determination of the Boiling Point of a Liquid.**—For this determination, place the liquid in a test-tube fitted with a cork provided with two holes. A thermometer passes through one hole and a glass tube to carry off the vapour through the other. The test-tube is heated carefully by a small flame or a water bath till the boiling point of the liquid is reached. In order to prevent *boiling with bumping*, a few glass beads, or short pieces of thin walled capillary tubing (made by drawing out a glass tube in the flame of the blowpipe), should be placed in the liquid. The position of the thermometer in the test-tube will depend on the liquid under test.

(*a*) In the case of a pure liquid, the thermometer is used to register the temperature of the vapour only, and the bulb of the thermometer should not dip into the liquid (Fig. 171).

(*b*) In the case of a solution, the temperature of the liquid differs appreciably from that of the pure solvent. In order to determine the boiling point of the *solution*, the bulb of the thermometer must be immersed in the liquid. Notice the difference between the readings when the thermometer bulb is in the solution, and when it is in the vapour above the liquid. The solution must be boiling *very gently* to avoid superheating.

CHAPTER II

COEFFICIENTS OF EXPANSION

§ 1. The Coefficient of Linear Expansion

The increase in length of a rod produced by raising its temperature one degree is small compared with the length of the rod, and is found to be nearly constant for different temperatures.

The **coefficient of linear expansion** of a solid may be defined as **the ratio of the increase in length to the original length for a rise in temperature of one degree.**

Thus, if the original length of the bar is l_0 and its length becomes l_1 when its temperature is raised 1°, the coefficient of linear expansion a is given by

$$a = \frac{l_1 - l_0}{l_0}.$$

If the length of the bar becomes l when its temperature is raised $t°$ we may write

$$a = \frac{l - l_0}{l_0 t}. \qquad (1)$$

Hence $$l - l_0 = l_0 a t$$

or $$l = l_0(1 + at). \qquad (2)$$

It is sometimes convenient to take the original temperature of the bar as 0° C.; then l_0 represents the length at 0° C. and t represents the temperature of the bar in degrees centigrade corresponding to the length l.

Since the change in length actually observed is small, it is in practice convenient to assume that the original temperature is

that of the room, and that l_0 represents the length of the bar at that temperature. In this case it must be noted that t represents the *rise* of temperature; that is, the difference between the final temperature and that of the room.

We see from equation (1) that a determination of the coefficient of linear expansion involves the measurement of three quantities, the original length, the rise of temperature, and the *increase in length*. The only measurement presenting any difficulty is the last. Since the probable error in this measurement is large, it is useless attempting great accuracy in the measurement of the other two quantities. (See p. 7.) The original length of the bar should first be determined to within 1 part in 1000, and the temperature at which the measurement is made be noted. To measure the small increase in length when the bar is heated to a known temperature several methods may be employed:—

1. The increase in length may be magnified by means of a mechanical, or an optical, lever (Lavoisier and Laplace) in a known proportion.

This first method is not at all accurate, the magnification factor being unknown to within 2 or 3 per cent.

2. The increase in length may be measured directly by means of a micrometer screw. An ordinary spherometer can be employed for the purpose.

3. The increase in length may be measured directly by employing two micrometer or vernier microscopes, one focussed on each end of the bar under test.

This method has the advantage over the other two in the fact that observations are made on both ends of the bar, and no assumption is made that one end of the bar remains fixed throughout the experiment. This is essentially the method of Roy and Ramsden.

EXPT. 154. **Determination of the Coefficient of Linear Expansion.**—The following apparatus is an example of the second method. The bar to be experimented upon is placed inside a steam jacket consisting of a hollow metal (or glass) tube through which a current of steam can be passed. The ends of the bar project slightly beyond the ends of the jacket, the joints being made steam-tight by cork or rubber tubing. A thermometer is provided to measure the temperature at each end of the bar. One end of the bar remains in contact with a fixed metal stud; the other end is free to expand. A micrometer screw with a divided head (spherometer) is arranged at this end so that the axis of the screw coincides in direction

with that of the bar. Contact between the end of the screw and the end of the bar may be detected by the sense of touch, or a *ratchet* micrometer may be used, so as to slip as soon as contact takes place, but it is preferable to use a simple electrical device to indicate the position of contact.

One pole of a voltaic cell is put in connection with the micrometer screw; the other pole is connected through a simple galvanometer with the stud against which the fixed end of the bar rests. As soon as the point of the micrometer screw touches the end of the bar, the circuit is completed and the galvanometer needle is deflected.

Set up the apparatus and determine at the ordinary temperature the reading of the micrometer screw when contact takes place with the end of the bar. This adjustment should be repeated several times.

Now *turn the micrometer screw back several turns to allow for expansion.* Heat the bar by passing a current of steam from a boiler through the jacket, and wait until the bar has had time to acquire a steady temperature. Note the readings of both thermometers. Again adjust the micrometer screw to give contact, and take the reading. The reading should be repeated several times. The difference between this and the former reading gives the increase in length of the bar.

Calculate from the observations the coefficient of linear expansion of the bar.

The third method may be employed to find the coefficient of expansion of a metal tube.

EXPT. 155. **Determination of the Coefficient of Linear Expansion of a Metal Tube.**—Make two transverse scratches, one near each end of a metal tube about one metre long. Measure the distance between them at the temperature of the room by setting up two travelling microscopes as in the comparison of the Yard and the Metre, Expt. 4. It is desirable to set up the microscope stands on a slab of slate, so that the distance between them may not be affected when the tube is heated. Focus the microscopes on the scratches. Pass a current of steam through the tube. Adjust the tube so that the scratch at one end coincides with the cross hair of the first microscope, and measure the distance through which the second microscope must be moved to give coincidence at that end of the tube. This gives the increase in the length of the tube. Calculate the coefficient of linear expansion on the assumption that the tube is heated to 100° C.

§ 2. THE COEFFICIENT OF EXPANSION (DILATATION) OF A LIQUID

The coefficient of expansion of a liquid may be defined in two distinct ways:—

I. **The Zero Coefficient of Expansion.**—The coefficient of expansion of a liquid is the ratio of the increase in volume to the volume at 0° C. produced by a rise of temperature of 1° C.

Thus, if V_1 be the volume at 1° C., V_0 the volume at 0° C., and a the coefficient of expansion,

$$a = \frac{V_1 - V_0}{V_0}.$$

If we assume that the substance expands uniformly with rise of temperature; that is, that equal changes of volume correspond to equal changes of temperature, the volume V_t at any temperature t is given by

$$a = \frac{V_t - V_0}{V_0 t}$$

or

$$V_t = V_0 (1 + at).$$

II. **The Mean Coefficient of Expansion between two Temperatures.**—The mean coefficient of expansion between any two temperatures is the ratio of the increase in volume to the original volume per degree rise of temperature. Thus, if a rise of temperature of $t°$ change the volume from V to V′ the mean coefficient of expansion is

$$\frac{V' - V}{Vt}.$$

Note that no reference is made here to the original temperature being 0° C.

In the case of a substance like water, which does not expand uniformly, this definition is necessary.

EFFECT OF A CHANGE OF TEMPERATURE ON THE DENSITY OF A LIQUID

Let V_0, ρ_0, denote the volume and density of a given mass of liquid at 0° C. Then the mass of the liquid is $V_0 \rho_0$.

Let V, ρ, denote the volume and density at any other temperature $t°$ C. Then the mass of the liquid is $V\rho$. But the mass is the same at both temperatures.

Hence $$V\rho = V_0\rho_0,$$

or $$\frac{V}{V_0} = \frac{\rho_0}{\rho}.$$

But $$\frac{V}{V_0} = 1 + \alpha t,$$

so it follows that $$\frac{\rho_0}{\rho} = 1 + \alpha t,$$

or $$\rho_0 = \rho(1 + \alpha t).$$

The difference should be noticed between this equation and that containing V. The effect of a rise in temperature is, in general, to increase the volume but to diminish the density.

The coefficient of expansion is given by

$$\alpha = \frac{\rho_0 - \rho}{\rho t}.$$

In the same way, the *mean* coefficient of expansion between two temperatures t_1 and t_2 may be shown to be

$$\alpha = \frac{\rho_1 - \rho_2}{\rho_2(t_2 - t_1)},$$

where ρ_1 is the density at t_1, and ρ_2 is the density at t_2.

COEFFICIENT OF EXPANSION OF WATER OVER VARIOUS RANGES OF TEMPERATURE

In the case of a liquid, it is much easier to determine the variation in *density* than to determine the variation in the volume of a given mass of liquid. The method usually adopted is to fill a specific gravity bottle up to the mark with liquid at various temperatures, and to weigh the quantity of liquid present.

EXPT. 156. **Expansion of Water by Specific Gravity Bottle Method.**—In this case the density of the liquid is proportional to the weight of liquid filling the bottle. Dry and weigh a

specific gravity bottle of about 100 c.c. capacity. Fill the bottle to the mark with water at a temperature between 2° C. and 7° C. Weigh the bottle and water.

Empty the bottle, place it in a bath of water and raise the temperature to about 20° C. Fill the bottle with water from the bath, adjusting the level of the water to the mark on the neck, while the bottle is still in the bath. Take the temperature of the bath. Remove the bottle and water from the bath, carefully dry the outside of the bottle, and weigh it.

Repeat the experiment, adjusting the temperature to 40° C., 60° C., and 80° C., and weighing the bottle when full of water to the mark at each of these temperatures. In weighing, the bottle and water will cool considerably and the liquid surface will fall below the mark in the neck. No notice need be taken of this. The liquid in the bottle is the amount of liquid which filled the bottle to the mark at the temperature of the bath, its *mass* is not altered by its contraction.

It is necessary, however, at the high temperatures to weigh as quickly as possible to avoid evaporation. There may also be an error due to the upward convection current past the hot bottle, and it is therefore advisable to cool the bottle rapidly under the cold-water tap before weighing.

The mass in gm. of the water filling the bottle at the first temperature t_1 (between 2° C. and 7° C.), may be taken as numerically equal to the volume of the vessel V_1 at that temperature, the density of the water being one gram per c.c. within the limits of accuracy of experiment over this range of temperature.

Calculate the capacity of the vessel V at each of the other temperatures taken, using the expression

$$V = V_1(1 + \beta(t - t_1)),$$

β being the coefficient of cubical expansion of glass: β is approximately 0·000025 per 1° C.

Find the density of the water at each temperature by dividing the mass of water in the bottle by the volume of the bottle at that temperature as calculated above. Tabulate the quantities: temperature, mass of liquid in bottle, volume of bottle (calculated), and density of liquid.

From the densities at 20° and t_1° C. calculate the mean coefficient of expansion of the water between these two temperatures—

$$\text{Mean } \alpha \ (t_1 \text{ to } 20°) = \frac{\rho_1 - \rho_{20}}{\rho_{20}(20 - t_1)}$$

Calculate also the mean coefficients of expansion from 20° C. to 40° C., 40° C. to 60° C., and 60° C. to 80° C., by a similar method.

Plot a curve showing the variation of density with temperature and also showing the variation of the coefficient of expansion with temperature.

The mean coefficient of expansion from 20° C. to 40° C. is practically the same as the coefficient of expansion at 30° C., and so on.

DENSITY OF WATER AT VARIOUS TEMPERATURES BY MEANS OF A GLASS SINKER

The variation of the density of water with the temperature may be found by observing the weight of a glass sinker in water at different temperatures.

Let V_0 denote the volume of the glass bulb at 0° C., and β the coefficient of cubical expansion of glass. Then the volume of the bulb at any temperature $t°$ C. will be $V = V_0(1 + \beta t)$. The value of β for ordinary glass is about 0·000025.

If ρ_t denote the density of water at t C., the weight of water displaced by the sinker when completely immersed is $V\rho_t = V_0(1 + \beta t)\rho_t$. But this is equal, in accordance with the principle of Archimedes, to the *loss* of weight in water = W, say.

Thus
$$V_0(1 + \beta t)\rho_t = W$$
and
$$\rho_t = \frac{W}{V_0(1 + \beta t)}.$$

The value of V_0 may be determined indirectly by finding the loss of weight when the sinker is immersed in water whose temperature is approximately 4° C. For temperatures not far from 4° C. the density of water may be considered as 1 gm. per c.c., so that the volume of the sinker at this temperature is found readily.

EXPT. 157. **Determination of the Density of Water at various Temperatures by Means of a Glass Sinker.**—A convenient form of sinker consists of a glass bulb containing lead shot. The quantity of shot must be adjusted before the bulb is sealed finally so that the sinker is sufficiently heavy to sink in water. The bulb is suspended by a fine wire from one arm of a sensitive balance. If a chemical balance with a closed case is employed, a small hole must be provided in the floor of the balance-case through which the wire can pass. Another hole must be made in the shelf on which the balance-case rests so that the wire passes freely

through the two holes. The sinker is attached to the lower end of the wire, and can be immersed completely in a large vessel of water which can be heated to any desired temperature. To diminish the effects of surface tension at the point where the wire passes through the surface of the water, the diameter of the wire should not exceed 0·1 mm.

The sinker is first counterpoised in air, then it is immersed completely in the water in the vessel and weighed again. The difference between the two weighings gives the loss of weight in water. The first observation may be made when the water is cooled to a temperature of about 4° C. Then heat the bath to 70° or 80° C. and allow it to cool slowly. It is easier to regulate the temperature when the bath is cooling, and to maintain it at a steady value while the process of weighing is being carried on. The flame of a Bunsen burner should be regulated carefully by altering its size or its distance below the bath so as to keep the temperature steady during the observation. Care must be taken to stir the water thoroughly between the observations, so that the temperature is uniform throughout the mass. Observations of the loss of weight and of the temperature, should be made at intervals of 10° or 15° C.

A table should be drawn up giving the density of water at different temperatures, and the results should be plotted on squared paper.

Calculate the mean coefficients of expansion of water between the consecutive pairs of temperatures taken.

THE WEIGHT THERMOMETER

The **Weight Thermometer** is a cylindrical glass bulb, with a neck drawn down into a fine tube. This tube is bent round so that its open end may dip into a vessel of liquid. The apparatus is used for finding the coefficient of expansion of a liquid. It is simplest to regard it as an instrument for comparing the densities of the liquid at two specified temperatures.

Let V_0 = volume of the thermometer at 0° C.

m = mass of liquid filling it at 0° C.,

ρ_0 = density of liquid at 0° C.

Let V_t, m_t, ρ_t denote the corresponding quantities at t° C. Then, if β is the coefficient of cubical expansion of glass,

$$V_t = V_0(1 + \beta t).$$

From the definition of density it follows that

$$m_0 = V_0\rho_0 \text{ and } m_t = V_t\rho_t.$$

Hence
$$\frac{V_0\rho_0}{V_t\rho_t} = \frac{m_0}{m_t},$$

or
$$\frac{\rho_0}{\rho_t} = \frac{m_0}{m_t} \times \frac{V_t}{V_0}$$

$$= \frac{m_0}{m}(1 + \beta t)$$

But it has been proved (p. 329) that

$$\frac{\rho_0}{\rho_t} = 1 + at,$$

where a is the coefficient of absolute expansion of the liquid.

Hence
$$1 + at = \frac{m_0}{m_t}(1 + \beta t).$$

Solving this equation for a, it is found that

$$a = \frac{m_0 - m_t}{m_t t} + \beta\frac{m_0}{m_t}.$$

Notice that no approximations whatever have been made in obtaining this result.

If the expansion of the thermometer bulb be ignored, $\beta = 0$, and the **coefficient of apparent expansion** of the liquid is

$$\frac{m_0 - m_t}{m_t t}.$$

Expt. 158. ***Determination of the Coefficient of Expansion of Glycerin by the Weight Thermometer.***—Find the mass of the empty thermometer. Fill the thermometer with glycerin by heating the bulb cautiously with a Bunsen flame, and letting the nozzle dip into a vessel containing some warm glycerin. As the bulb cools, glycerin will be drawn into it. By repeated heating and cooling the bulb should be filled completely with glycerin. When the bulb has cooled to the temperature of the room, surround it with a vessel containing crushed ice, still keeping the nozzle in the glycerin. While

the bulb is cooling to 0° C., weigh a small cup or crucible. Remove the thermometer from the ice, placing the cup so as to catch the liquid that escapes. Weigh the thermometer and the cup together, and determine the mass of glycerin filling the thermometer at 0° C.

Next place the thermometer in a beaker of water and heat to the boiling point, allowing the glycerin that is forced ou to escape. Remove the thermometer, and let it cool to the temperature of the room. The liquid will contract, but the mass is still the mass of liquid filling the thermometer at 100° C. Weigh the thermometer again, and determine the mass of glycerin.

Calculate the coefficient of apparent expansion of glycerin. Calculate also the coefficient of absolute expansion, assuming the coefficient of expansion of glass to be known.

THE VOLUME DILATOMETER

The **Dilatometer** consists of a cylindrical bulb to which is attached a straight graduated tube. If the volume of the bulb up to the first division on the stem is known, and also the volume corresponding to a division of the tube, the apparatus may be used to determine the coefficient of apparent expansion of a liquid.

EXPT. 159. **Determination of the Coefficient of Apparent Expansion of a Liquid by Means of the Dilatometer.**—First weigh the empty dilatometer. Then fill it to the first division on the stem with a liquid of known density, and weigh again. From the mass of liquid thus found, calculate the volume of the bulb. Fill the dilatometer to a mark near the top of the stem and weigh again. Find the mass of liquid filling a definite length of the stem, and calculate the volume of this length of the stem. Deduce the volume corresponding with one scale division.

To find the coefficient of apparent expansion of a liquid, fill the bulb and part of the stem with the liquid, and cool the whole to 0° C. by immersing in ice. Read the position of the liquid in the stem. Then heat to a known temperature in a water bath, and again read the position of the liquid in the stem. Calculate the volumes corresponding to these readings. Calculate the coefficient of apparent expansion from the formula $V_t = V_0(1 + at)$.

§ 3. Expansion of Gases

THE EXPANSION OF AIR AT CONSTANT PRESSURE

When a given mass of a gas expands under a constant pressure, in consequence of a rise of temperature, the relation between the volume and the temperature is expressed by the equation

$$V_t = V_0(1 + at),$$

where V_t represents the volume of the gas at $t°$ C., V_0 the volume at 0° C., and a is called **the coefficient of expansion, or the coefficient of increase of volume at constant pressure.**

The equation expresses in symbolic form the **Law of Charles** which states that **when a fixed mass of gas expands under constant pressure, the volume increases by a definite fraction of the volume at 0° C. for each degree rise of temperature.**

Expt. 160. **Determination of the Coefficient of Expansion of Air at Constant Pressure.**—A flask of 300 or 400 c.c. capacity is provided with a well-fitting rubber stopper through which passes a *short* length of glass tubing. The lower end of the tube should be flush with the bottom of the stopper and the upper end should not project more than 2 or 3 cm. above the stopper. A piece of rubber tubing about 5 cm. long is fitted to the projecting glass tube.

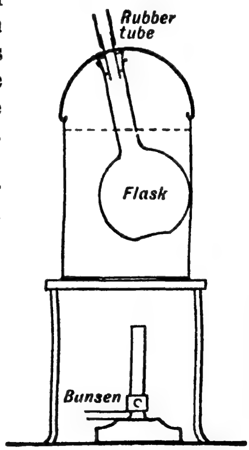


Fig. 172.—Flask heated to the Boiling Point.

The flask, stopper and tube must be dried thoroughly. This drying may be done by washing out with methylated spirit, and blowing a current of air through the apparatus. The weight W_1 of the dry flask is next found.

The flask, with the stopper inserted, is then placed in a can of water which is heated gradually to the boiling point. If the can be fitted with a wire handle, the latter will serve to hold the flask immersed in the water (Fig. 172). The flask must be left in the water for at least

five minutes after the boiling point has been reached, so that the air inside may reach the temperature of the boiling water, which we shall assume to be 100° C. The rubber tubing is then pinched firmly between the thumb and finger, and the flask is *quickly* lifted out of the can and turned upside down in a large vessel containing cold water (Fig. 173). As soon as the stopper is under the surface of the cold water, the rubber tube may be released so that cold water may enter the flask. The flask must be immersed completely for several minutes, neck downwards, so that the contents may come to the temperature of the water. Let this temperature be $t°$ C. The flask is then raised *till the level of the water inside the flask is the same as the level outside, i.e.* till the pressure of the air inside is the same as the atmospheric pressure. The rubber tubing is pinched while this condition is satisfied and the flask is lifted out of the water, turned right way up, dried on the outside and weighed. Let the weight be W_2.

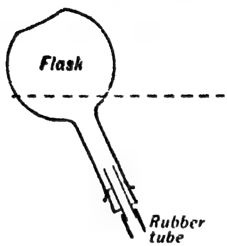


FIG. 173.—Flask in Cold Water.

Then the flask is filled completely with cold water, the stopper is inserted so that the water fills the glass tube, and the weight W_3 is found.

The weight of water filling the whole flask is $W_3 - W_1$ gm. But 1 gm. of water occupies 1 c.c. So the volume of the flask is $W_3 - W_1$ c.c. Now when the flask was in the boiling water, the air inside it occupied the whole volume and the pressure was atmospheric.

Let this volume be V_{100}, then

$$V_{100} = W_3 - W_1 \text{ c.c.}$$

When the flask was placed in the cold water at $t°$ C., the volume of the air diminished till it became V_t; and a little consideration will show that

$$V_t = W_3 - W_2 \text{ c.c.}$$

This is the case because the air at this lower temperature occupied the volume *not* occupied by the water sucked into the flask. Thus we find both V_{100} and V_t.

But it is necessary to refer the volumes to the volume at

0° C. in order to calculate the coefficient of expansion. That is, we have two equations

$$V_{100} = V_0 (1 + 100a),$$
$$V_t = V_0 (1 + at),$$

with two unknown quantities.

Divide the first by the second, then

$$\frac{V_{100}}{V_t} = \frac{1 + 100a}{1 + at},$$

which gives

$$a = \frac{V_{100} - V_t}{100V_t - tV_{100}},$$

from which the value of a may be calculated.

THE CONSTANT VOLUME AIR THERMOMETER

When a definite mass of gas is enclosed in a vessel the volume of which remains unchanged, the pressure exerted by the gas on the walls of the vessel increases as the temperature is raised. The relation between the pressure and the temperature of the gas may be examined by means of the apparatus known as the **constant volume gas thermometer** due to Jolly (1874).

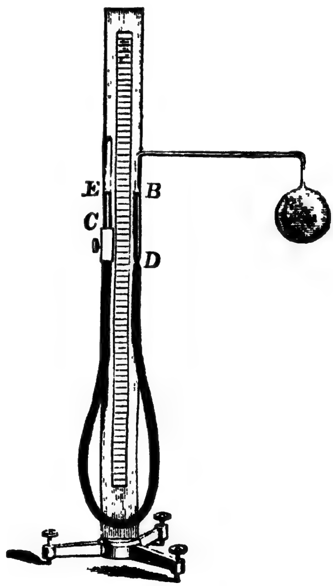


FIG. 174.—Jolly's Constant Volume Gas Thermometer.

The gas is contained in a glass globe A (Fig. 174) which may be heated to any desired temperature by the bath of water, or oil, in which it is placed. The globe is connected by a glass tube of small bore with a mercury manometer for measuring the pressure. The manometer consists of two fairly wide glass tubes BD and EC connected by a length of rubber tubing. It contains sufficient mercury to fill the rubber tube and some part of the wide glass

tubes. The level of the mercury in BD can be adjusted by raising or lowering the glass tube EC until the meniscus just touches the tip of a little glass index fixed inside B near the junction of the wide and narrow tubes.

In using the apparatus, the mercury meniscus in BD must occupy this definite position, in order that the volume of the gas enclosed in the globe A and the narrow tube may remain constant. The pressure exerted by the gas is equal to that at the level of the mercury surface at B. This pressure is found by measuring the difference in level between the surface of the mercury at B and that of the mercury in EC, and taking into account the pressure of the atmosphere on the mercury at E. The atmospheric pressure at the time when the observations are made, must be found by reading the height of the barometer.

Three points in connection with the use of the apparatus require to be emphasised :—

I. In order to determine the difference in level of the mercury surfaces accurately, the apparatus must be so arranged that the tubes C and D are quite close to the scale used in measuring the level.

II. The determination of the pressure must be made while the temperature of the gas is constant. *Great care must be taken to keep the bath in which A is immersed at a steady temperature* while EC is being adjusted, and the reading of the difference of level of the mercury surfaces is being made. This can be done more easily when the temperature is falling than when it is rising, so that it is advisable to heat the bath to the highest temperature to be used in the experiments and then allow the bath to cool slowly. As, however, this takes a considerable time, the bath may be heated to one or two degrees above any desired temperature and then the heater removed. The water is now stirred thoroughly until it has cooled to the temperature desired. The adjustments are made *approximately* while the water is cooling, are brought rapidly to their exact value, and the reading is taken when this temperature has been reached. The whole bath is then heated rapidly to a little above the next temperature desired, when the same process is repeated.

The success of the experiment depends on the temperature of the gas inside the bulb being exactly the same as the temperature of the bath outside, and upon this temperature being determined accurately.

III. When the bath is allowed to cool, *great care must be taken*

that the mercury in BD *is not drawn over into the bulb* A in consequence of the reduction of the pressure of the gas. To prevent this, lower the tube EC so that the mercury in BD may be well below the top of the tube. *When the experiment is completed the tube* EC *must always be lowered in this way.*

EXPT. 161. **Variation of the Pressure of a fixed Mass of Air with the Temperature as shown by a Mercury Thermometer when the Volume is kept constant.**—Use a water bath to heat the bulb of the air thermometer, and a mercury thermometer to take the temperature of the water bath. Heat the water to the boiling point, and when the temperature has become steady, read the thermometer, adjust the mercury in the manometer, and read the level of B and E. Then allow the temperature to fall about 20°, and again take readings of temperature and pressure. Take a series of readings in this way, allowing the temperature to fall about 20° between consecutive readings. Or the adjustments to the different temperatures may be made while raising the temperature, if the precautions mentioned in II. are taken, the final temperature being 100° C.

Record the results as follows:—

Height of Barometer = . .
Level of Index Mark at B = .

Temperature.	Level of E.	Difference of Level, E – B.	Pressure in A.

The relation between the pressure of the air and its temperature must now be represented graphically, taking the pressure as the ordinate and the temperature as the abscissa. The points so obtained should fall nearly on a straight line. Draw a straight line passing through the points so that there are about as many points above the line as below it. Find from this line, which may be taken to represent an average of the experimental results, the pressures corresponding to two chosen temperatures t_1 and t_2. Let these pressures be p_1 and p_2.

Then, if a denote the coefficient of increase of pressure of a gas with temperature, we may write

$$p_1 = p_0(1 + at_1),$$
$$p_2 = p_0(1 + at_2).$$

We can eliminate p_0 by dividing one equation by the other, and so obtain

$$\frac{p_1}{p_2} = \frac{1 + at_1}{1 + at_2}.$$

Solving for a we get

$$a = \frac{p_2 - p_1}{p_1 t_2 - p_2 t_1}.$$

Calculate the value of a by means of this equation.

We may, if we please, select as the chosen temperatures $t_1 = 0°$ C. and $t_2 = 100°$ C. Find from the graph the corresponding pressures p_0 and p_{100}, and calculate a from the equation

$$p_{100} = p_0(1 + a100).$$

To do this the graph must be extended beyond the lowest temperature used, the pressure p_0 being obtained by *extrapolation*.

EXPT. 162. **Determination of the Temperature of the Melting Point of a Substance by means of the Constant Volume Air Thermometer.**—In this experiment no mercury thermometer is to be used, but the scale of temperature defined by the constant volume air thermometer is to be employed. First determine the 'fixed points' of the thermometer. Determine the lower fixed point by observing the pressure of the air in the bulb when the surrounding bath contains melting ice. Let this pressure be p_0. Determine the upper fixed point by observing the pressure of the air in the bulb when at the boiling point. Strictly speaking, it would be necessary to surround the bulb with steam from pure water boiling under standard pressure to obtain this point accurately. For the present purpose it will suffice to surround the bulb with boiling water in the water bath. Let the corresponding pressure be p_{100}.

Then $$p_{100} = p_0(1 + a100).$$

The value of a can thus be found by direct experiment.

Now adjust the temperature of the water in the water bath till it is equal to that of the melting point of the solid. For this purpose a small quantity of the solid may be placed in a thin-walled test-tube, which may be immersed in the water bath. Read the pressure p corresponding with this temperature.

Then on the scale of the constant volume air thermometer we have

$$p = p_0(1 + at),$$

where t is the temperature to be determined, and a has the value already found experimentally.

Calculate the temperature t from this equation.

The results of experiments on gases expressed in the laws of Boyle and of Charles may be combined in the single expression

$$\mathbf{PV = RT},$$

where P denotes the pressure, V the volume of a given mass of gas, and T is the absolute temperature, *i.e.* the temperature reckoned from 273° C. below the freezing point on the centigrade scale.

R is a constant generally known as the **gas constant**.

This expression should be employed in dealing with calculations with regard to gases, except when the gas coefficient a has to be determined from experimental observations.

If unit mass of gas be considered, $V = 1/\rho$, where ρ is the density of the gas, and the gas equation may be written

$$\mathbf{P/\rho = RT}.$$

In this equation R is the gas constant reckoned for 1 gm. of gas.

CHAPTER III

CALORIMETRY

§ 1. MEASUREMENT OF QUANTITIES OF HEAT

THE subject of calorimetry deals with the measurement of quantities of heat. The **unit quantity of heat** is that quantity which is required to raise the temperature of unit mass of water one degree. The unit generally employed in scientific work is the **calorie**, which may be defined as **the quantity of heat required to raise the temperature of 1 gm. of water 1° C.** at some specified temperature. This quantity is nearly, but not exactly, the same at different temperatures, between 0° C. and 100° C., *e.g.* the 15° calorie is about 1 part in 1000 greater than the 20° calorie. In what follows these small variations will be ignored. The number of calories required to raise the temperature of m gm. of water from t_1° C. to t_2° C. will then be

$$H = m(t_2 - t_1).$$

A certain quantity of heat is required to raise the temperature of a body 1° C.—this quantity is called the **thermal capacity of the body.** The quantity of water which requires the same amount of heat as a certain body to raise its temperature 1° C. is called the **water equivalent of the body.** The water equivalent in *grams* is numerically equal to the thermal capacity in *calories per ° C.*

If the water equivalent of a body be w gm., then the heat required to raise the temperature of the body from t_1° C. to t_2° C. is

$$H = w(t_2 - t_1).$$

The thermal capacity of unit mass, or the specific heat of a substance is the number of calories required to raise the temperature of 1 gm. of the substance 1° C. If the specific heat of a substance be denoted by s calories per gram per °C., the heat required to raise the temperature of m gm. from t_1° C. to t_2° C. is

$$H = ms(t_2 - t_1).$$

This is the fundamental equation in connection with the measurement of quantities of heat.

Comparing this with the preceding equation, we see that the water equivalent $w = ms$. Hence the water equivalent of a body can be calculated as the product of the mass of the body and the specific heat of the substance.

CALORIMETERS

A vessel adapted for the measurement of quantities of heat is termed a **calorimeter** It should be arranged so as to avoid, as far as possible, transference of heat to or from external bodies. Such transference can take place by conduction, convection, or radiation To avoid conduction of heat, the calorimeter is supported by some bad conductor of heat, such as felt, cotton wool, cork, or ebonite. To avoid convection currents, the vessel is sometimes packed in cotton wool or surrounded by a vacuum jacket. To prevent transference of heat by radiation, usually the calorimeter is supported in an outer vessel, the outside of the inner vessel being brightly polished to diminish the emissivity, and the inside of the outer vessel being polished brightly to increase the reflecting power.

A Dewar's vacuum vessel (thermos flask) is a convenient calorimeter for some experiments, but as the glass does not all acquire the same temperature, there is some difficulty in deciding what value to take as its thermal capacity.

§ 2. Determination of the Specific Heat of a Solid

Expt. 163. **Simple Methods of determining the Specific Heat of a Solid.** A known mass of the solid is heated to a definite temperature and then placed in a known mass of water at the temperature of the room. The solid and the water finally attain a common temperature, which is observed. The specific heat of the solid can then be calculated.

The solid should be weighed first of all, so that the other weighings can be carried out while the solid is being heated. If the solid is a lump of metal, attach to it a fine thread or wire, and lower it into a can of water which can be heated to the boiling point. If the solid is in fragments (*e.g.* lead shot or brass filings) place the fragments in a test-tube of glass or thin metal and heat this in the boiling water. The solid must be left in the boiling water for a sufficient time for the whole to reach a steady temperature.

While the solid is being heated, weigh a calorimeter (with its stirrer) and weigh it again when about two-thirds full of water. Observe and record the temperature of the water.

When the temperature of the solid has reached that of the boiling water, transfer it as quickly as possible to the calorimeter. In the case of the fragments, the test-tube is lifted by a suitable handle and tilted so that the fragments fall into the calorimeter. Stir the water in the calorimeter and observe carefully the highest temperature recorded by the thermometer. In the case of the solid lump, a small quantity of water is unavoidably transferred to the calorimeter when the lump is lifted by the thread and placed in the latter vessel; and this introduces a serious error.

The following example illustrates the method of entering the observations and calculating the result.

Example.—Determination of the specific heat of lead shot.

Mass of lead shot	= 200 gm.
Mass of calorimeter and stirrer	= 40·0 gm.
Mass of calorimeter, stirrer, and water	252·2 gm.
Mass of water	= 212·2 gm.
Initial temperature of shot	100° C.
Initial temperature of water, t_1	15·0° C.
Final temperature of water and shot, t_2	= 17·3° C.

We assume that the heat given out by the solid in cooling from 100° C. to the final temperature t_2 is exactly equal to the heat taken in by the water and the calorimeter when their temperature rises from t_1 to t_2.

If s is the specific heat of the solid, the heat it gives out is

$$200 \times s \times (100 - t_2) \text{ calories.}$$

The 'water equivalent' of the calorimeter is equal to its mass multiplied by the specific heat of the material (0·095 say)

$$= 40 \times 0{\cdot}095 = 3{\cdot}8 \text{ gm.}$$

The total water equivalent (including both calorimeter and water)

$$= 212{\cdot}2 + 3{\cdot}8 \text{ gm.}$$
$$= 216 \text{ gm.}$$

The heat taken in by the water and the calorimeter

$$= 216 \times (17 \cdot 3 - 15)$$
$$= 496 \cdot 8 \text{ calories.}$$

We next write down an equation expressing the fact that the heat given out by the solid is equal to the heat taken in by the water and the calorimeter.

$$200 \times s \times (100 - 17 \cdot 3) = 496 \cdot 8$$
$$s = 0 \cdot 03 \text{ calories per gm.}$$

REGNAULT'S APPARATUS

For the accurate determination of the specific heat of a solid an apparatus of the type designed by Regnault may be employed. The chief points kept in view in the design of this apparatus were the heating of the solid to a fixed temperature without contact with moisture, the rapidity of transference from the heating

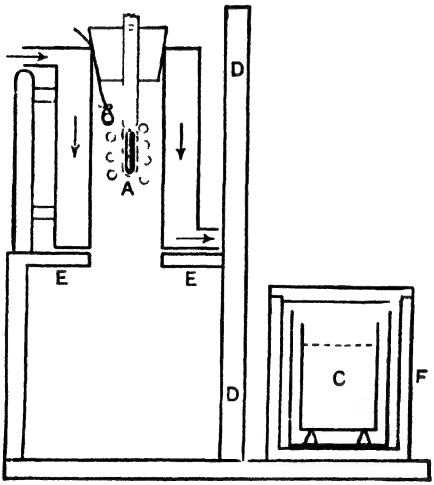


Fig. 175. Regnault's Apparatus.

chamber to the calorimeter, and the protection of the calorimeter from the heating chamber during the other parts of the experiment.

The solid at A (Fig. 175) is heated inside a double-walled steam jacket, through which a current of steam is passed from a boiler. The boiler and the outlet pipe must be arranged so as not to radiate heat to the calorimeter C, which is shielded from the steam

heater by a sliding wooden shutter D. While the solid is being heated the upper end of the chamber is closed by a cork through which passes a thermometer reading to 100° C., and the lower end is closed by part of the wooden platform E. The solid, which should be placed in contact with the bulb of the thermometer, is suspended by a fine thread held in position by the cork. In the case of a metal it is convenient to use a wire bent into the form of a helix.

EXPT. 164. **Regnault's Apparatus for the Specific Heat of a Solid.**—As it requires a long time for the solid to attain a steady temperature, the first operation, after arranging for a supply of steam, should be to weigh the solid and to suspend it in the heating chamber. Then the inner vessel of the calorimeter should be weighed, and filled about three parts full of water, after which it is weighed again to determine the mass of the water. It is then placed in position in the outer metal vessel, which is further protected by the wooden box. A sensitive thermometer is used to determine the temperature of the water in the calorimeter as accurately as possible. The solid should be allowed to remain in the heating chamber for at least five minutes *after the temperature shown by the thermometer in the chamber has become steady.* The whole process of heating the solid will take probably from twenty to thirty minutes after steam has commenced to pass.

When this steady temperature has been recorded, the heating chamber is swung round till it comes above the hole in the platform E. The shutter D is raised, and the box F containing the calorimeter is pushed into position so that the inner vessel of the calorimeter is exactly underneath the hole in the platform. The solid is lowered quickly into the calorimeter without splashing, the box F is withdrawn, and the shutter is lowered. The temperature of the calorimeter is observed carefully and the highest temperature reached recorded. In an accurate determination a cooling curve would be plotted in order to determine the correction necessary to allow for loss of heat by radiation (p. 348).

From the observations the specific heat can be calculated exactly as in the simpler experiment on p. 344.

Let m = the mass of the solid.
s = the unknown specific heat.
c = the mass of the calorimeter.
M = the mass of water in the calorimeter.

s_1 = the specific heat of the material of the calorimeter.
t = the temperature of the hot solid.
t_1 = the initial temperature of the calorimeter.
t_2 = the final temperature of the calorimeter.

Then the heat given out by the solid in cooling from t to t_2

$$= ms(t - t_2).$$

The heat taken in by the water and the calorimeter in changing in temperature from t_1 to t_2

$$= (M + cs_1)(t_2 - t_1).$$

Assuming these quantities of heat to be equal,

$$ms(t - t_2) = (M + cs_1)(t_2 - t_1),$$

an equation from which the value of s can be determined.

The student should not attempt to remember an equation of this kind, but should obtain the result in any particular case from first principles.

§ 3. Determination of the Specific Heat of Liquids

The specific heat of a liquid may be determined by the method of mixtures in several ways.

Expt. 165. **Determination of the Specific Heat of a Liquid, using a Solid of known Specific Heat.**—The solid must have no chemical action on the liquid.

This determination is carried out in exactly the same manner as that of the specific heat of a solid (Expts. 163 and 164), using the given liquid in place of water in the calorimeter.

Let s_2 denote the specific heat of the liquid, M its mass, then

$$ms(t - t_2) = (Ms_2 + cs_1)(t_2 - t_1).$$

where the other symbols have the meaning previously assigned to them.

Expt. 166. **Determination of the Specific Heat of a Liquid by Regnault's Method.**—The specific heat is determined by running the hot liquid into a thin-walled metal vessel placed in the water in the calorimeter, or conversely by running hot water into a thin-walled metal vessel placed in the given liquid in the calorimeter. Since the mixture of two liquids at the same temperature often results in the evolution of heat through chemical action, the two liquids should not, as a rule, be brought into direct contact.

Expt. 167. **Determination of the Specific Heat of a Liquid by the Method of Mixtures.**—A more convenient method is to heat the liquid in a small thin-walled glass bottle, or metal cylinder,[1] closed by a cork through which passes a thermometer. The heated bottle, after its temperature has been determined, is transferred to the calorimeter. The bottle, held by the stem of the thermometer, may be used as a stirrer. A second thermometer is used to determine the temperature of the water in the calorimeter. The final temperature is taken as the mean of the readings of the two thermometers when they differ by only one degree or less. The water equivalent of the vessel containing the liquid must of course be taken into account, as well as the water equivalent of the calorimeter.

Expt. 168. **Determination of the Specific Heat of a Liquid by means of the Calorifer.** The **calorifer** resembles a mercury thermometer with a large bulb. There are, however, only two marks on the stem. By heating the calorifer in boiling water the mercury rises above the upper mark. The calorifer is removed from the boiling water, dried, and placed in a weighed quantity of liquid in the calorimeter at the instant when the mercury reaches the upper mark. It is left in the calorimeter till the mercury sinks to the lower mark, when it is at once removed. The rise of temperature of the liquid in the calorimeter is measured by a sensitive thermometer. The same operation is then repeated, using a known weight of water in the calorimeter. Since exactly the same quantity of heat is transferred by the calorifer to the calorimeter in the two experiments, it is easy to calculate the specific heat of the liquid. The method of calculation is left as an exercise for the student.

A description of the determination of the specific heat of a liquid by the method of cooling will be found on p. 360.

§ 4. Method of correcting Calorimetric Observations for Radiation

For accurate calorimetry, the calorimeter should be enclosed in a double-walled metal vessel, between the walls of which water is placed. By this means, the calorimeter is enclosed in constant temperature surroundings, and the radiation may be assumed to be

[1] Thin-walled aluminium cylinders are obtainable which are suitable for this purpose.

proportional to the difference between the temperature of the calorimeter, and the temperature of the surrounding enclosure.

The temperature of the calorimeter is taken every 30 seconds before and after the experiment as well as during the experiment itself, and a curve showing the temperature variation with time is plotted.

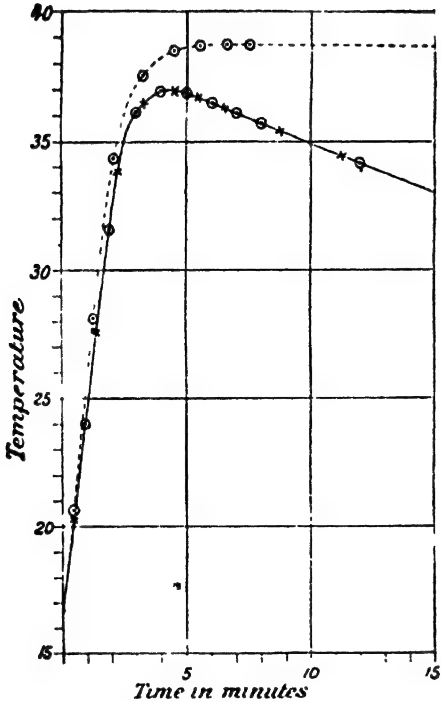


FIG. 176.—Correction Curve.

At the highest temperature reached, the rate of fall of temperature is determined from the curve; let this be x degrees per minute, the highest temperature being t degrees above the surroundings.

Divide the curve into one minute intervals, starting from the instant when the hot body was dropped into the calorimeter, and take the temperature at the middle of each of these intervals as the mean temperature during that minute. Let these temperatures be respectively t_1, t_2, t_3 degrees above the temperature of the enclosing vessel.

During the first minute, heat was lost corresponding with a mean temperature excess of t_1 degrees. If this heat had been retained, the temperature at the end of the first minute would have been higher than the temperature actually reached by an amount $x_1 = \frac{x}{t} t_1$.

During the second minute the heat lost would correspond with a loss of temperature $x_2 = \frac{x}{t} t_2$, and so on.

The temperature at the end of the first minute is x_1 degrees too low, during the second minute there is a further loss of x_2 degrees, so that the temperature at the end of the second minute is $x_1 + x_2$ degrees too low, due to the cooling in the *two* minutes which have elapsed since the hot body was introduced.

Similarly, the correction to be added at the end of the third minute is $x_1 + x_2 + x_3$, and so on. By adding on these corrections to the curve plotted, a new curve will be obtained, giving the temperatures that would have been reached if there had been no radiation losses; this curve will be horizontal at the end of the experiment, the ordinate of the horizontal portion being the corrected temperature.

If desired, *half*-minute intervals may be taken, the correction being added on every half-minute instead of every minute as described.

§ 5. Latent Heats

DETERMINATION OF THE LATENT HEAT OF WATER

Latent Heat of Fusion of Ice.—**The quantity of heat required to change one gram of ice from the solid to the liquid form without change of temperature is called the latent heat of water, or the latent heat of fusion of ice.**

When small lumps of dry ice are added to a known mass of water in a calorimeter the ice is melted, becoming water at 0° C., and the ice-cold water abstracts heat from the warm water and calorimeter, until an equilibrium temperature is reached.

If the calorimeter at the commencement of the experiment is at the temperature of the room, it will be cooled below that temperature by the addition of ice, and so will be gaining heat by radiation throughout the experiment.

To avoid error due to this cause, it is advisable to warm the calorimeter and the contained water to about 5° above the temperature

of the room, and to add sufficient ice to cool it through the same number of degrees below the temperature of room. The loss of heat during the first half of the experiment will then about balance the gain during the second half, provided the second half of the experiment does not require much longer time than the first half.[1]

Expt. 169. Determination of the Latent Heat of Fusion of Ice.—First weigh the calorimeter with the stirrer. Introduce from 100 c.c. to 200 c.c. of water and weigh again.

The difference between these two weighings gives the mass of water in the calorimeter.

Warm the calorimeter to about 5° above the temperature of the room, by placing it in a vessel of hot water.

Dry the outside of the calorimeter, and place it in a larger copper vessel, supporting it on felt, cotton wool, or cork so as to avoid transference of heat by conduction. Observe the temperature of the water with a sensitive thermometer

Dry some small pieces of ice by means of a cloth or blotting paper and add them gradually, keeping the water in the calorimeter stirred. Continue adding the ice until the temperature has fallen about 5° below the temperature of the room. Observe the lowest temperature reached after all the ice has melted.

Weigh the calorimeter again, and so find the mass of ice added.

The following example illustrates the method of entering the observations and calculating the result:—

Example.—Mass of calorimeter with stirrer		= 10·0 gm.
,, ,, ,,	and water	= 240·0 gm
,, water		= 200·0 gm.
,, calorimeter and water and ice		= 262·9 gm.
,, ice		= 22·9 gm.
Temperature of room		= 15·0 C.
Initial temperature of water, t_1		= 20·0 C.
Final ,, ,, t_2		= 10·0° C.

We have to express the fact that the heat given out by the water and calorimeter in cooling from t_1° to t_2° is equal to the heat required to melt the ice and raise the temperature of the water produced from 0° to t_2° C.

[1] This method of avoiding error cannot be applied satisfactorily if the temperature of the room is very low, or if, as sometimes happens in tropical countries, the cooling would result in the temperature being reduced much below the dew point. In such cases it would be preferable to start with the temperature above that of the room, and finish when the temperature is about the same as that of the room, applying a correction for the heat lost by radiation by the method on p. 348.

The 'water equivalent' of the calorimeter (including the stirrer) is equal to the mass of the calorimeter multiplied by the specific heat of copper (0·095)

$= 40 \times 0{\cdot}095 = 3{\cdot}8$ gm.

Heat given out by water

$=$ mass of water $\times$ fall in temperature

$= 200 \times (20^\circ - 10^\circ)$

$= 2000$ units of heat, calories.

Heat given out by calorimeter

$=$ water equivalent of calorimeter $\times$ fall in temperature

$= 3{\cdot}8 \times (20^\circ - 10^\circ)$

$= 38$ calories.

Total amount of heat given out

$= 2038$ calories.

Heat required to melt 22·9 grams of ice

$= 22{\cdot}9 \times L$ calories,

where L is the latent heat of water, that is, the number of calories required to melt 1 gram of ice.

Heat required to raise 22·9 grams of water from 0° to t_2

$= 22{\cdot}9 \times t_2$ calories

$= 229$ calories.

We write down an equation expressing the fact that the total amount of heat given out = total amount of heat absorbed; and from this equation find the value of L.

Thus $\quad 2038 = 22{\cdot}9 \times L + 229$

$22{\cdot}9\,L = 1809$

$L = 79$ heat units on the centigrade scale per gram of ice

$= 79$ calories per gram.

DETERMINATION OF THE LATENT HEAT OF STEAM

Latent Heat of Steam.—The latent heat of steam, or the latent heat of vaporization of water, is the amount of heat required to convert 1 gm. of water into steam without change of temperature.

When steam from a boiler is passed through a known mass of water in a calorimeter, some of the steam is condensed and the final temperature of the water is raised above its initial temperature. From the observed rise of temperature and the mass of steam condensed, the latent heat of steam can be calculated.

If the water in the calorimeter is at the temperature of the room at the beginning of the experiment, it will lose heat by radia-

tion as soon as its temperature is raised above that of surrounding objects. This would tend to give too small a value for the latent heat.

The error due to this cause may be reduced by commencing the experiment with the temperature of the water as much below the temperature of the room as it is above the temperature of the room at the end of the experiment. The error may also be diminished by making the duration of the heating as short as possible. For this purpose the steam should be made to issue from the nozzle in a rapid stream. A screen may be placed between the boiler and the calorimeter to prevent radiation of heat (Fig. 177).

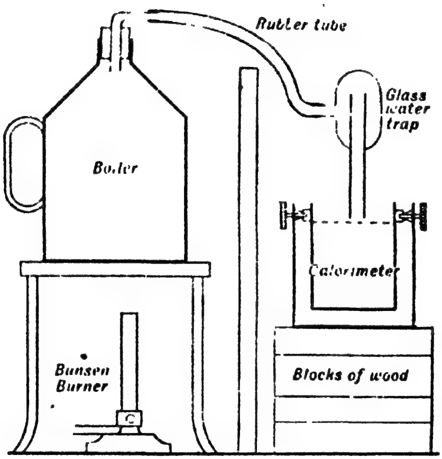


FIG. 177.—Latent Heat of Steam.

To obtain an accurate result the steam must be dry, that is, free from condensed water. A water trap of glass is provided to remove as much of the condensed water as possible. The rubber tube connecting the boiler to the water trap should be short: it may be lagged with cotton wool.

The importance of having the steam dry will be realised from the fact that carrying over 1 gm. of water with the steam introduces an error equivalent to about 500 calories.

EXPT. 170. **Determination of the Latent Heat of Steam.**—First see that the boiler contains a sufficient quantity of water, then begin to heat it by means of a gas-burner.

The calorimeter is made in two parts, an inner and an

outer vessel. Weigh the inner vessel, and fill it about two-thirds full with water.

Add small pieces of ice till the temperature of the water has fallen to about 5° C. Then weigh the inner vessel with its contents accurately.

Note the temperature of the room and again read the thermometer in the calorimeter. Estimate the temperature at which the experiment ought to be finished. For example, if the temperature of the room is 15° C., the final temperature should be about 25° C.

Remove any water drops adhering to the nozzle of the delivery tube. Pass a rapid current of steam[1] into the water in the calorimeter, at the same time thoroughly stirring the water so as to ensure uniformity of temperature. When the temperature has risen to the desired point, remove the calorimeter as quickly as possible from the neighbourhood of the boiler, and observe the highest temperature recorded by the thermometer.

Weigh the calorimeter again in order to determine the amount of steam condensed.

Read the height of the barometer.

For the standard pressure 760 mm. the temperature of steam is 100° C. Near this pressure an increase of pressure corresponding to 26·8 mm. of mercury raises the boiling point one degree C., and for small changes the rise in the boiling point is proportional to the change in pressure. Hence the temperature of steam corresponding to the observed pressure may be calculated. As, however, the temperature thus found differs but little from 100° C., the error introduced by taking the temperature of the steam as 100° C. is inappreciable compared with other unavoidable experimental errors.

Enter the results as follows:—

Mass of calorimeter	= 160·0 gm.
,, calorimeter and water	= 681·2 gm.
,, water	= 521·2 gm.

[1] In this experiment the nozzle must not be immersed in the water: it should be placed with its end only the slightest amount below the surface, so that the water surface is blown away from the end of the nozzle and the steam merely plays on the water surface. If the nozzle is put right down into the water the steam may condense too rapidly, when the water will be sucked up into the steam trap and the whole experiment spoiled. No error is introduced by escaping steam; the steam which escapes does not condense in the water, so that its latent heat is not given up, nor is its mass included in the mass of steam condensed.

Mass of calorimeter and water and steam	**=704·4 gm.**
,, **steam condensed**	**= 20·2 gm.**
Initial temperature of water $t_1°$ C.	**= 6·2° C.**
Final temperature of water $t_2°$ C.	**= 28·6° C.**
Barometric height	**=758 mm.**
Temperature of steam T° C.	**=100° C.**

Calculate the 'water equivalent' of the calorimeter. The total water equivalent is obtained by adding the mass of water in the calorimeter to the water equivalent of the calorimeter. If this is multiplied by the rise of temperature, $t_2° - t_1°$, we obtain the amount of heat absorbed by the calorimeter and the water in it. This is expressed in calories.

Now consider the heat given out in condensing the steam and in lowering the temperature of the resulting water.

Heat given out in condensing the steam

= mass of steam condensed × L
= mass of condensed water × L
= 20·2 gm. × L.

Heat given out in lowering the temperature of the resulting water from T° to $t_2°$

= mass of steam × (T − t_2)
= 20·2(100 − 28·6) calories.

Assuming that there is no loss or gain of heat by radiation, the sum of these two quantities must equal the heat absorbed by the calorimeter and the water in it.

This gives a simple equation from which L may be determined.

CHAPTER IV

COOLING

§ 1. The Law of Cooling

When a hot body is placed in an enclosure kept at a constant temperature, the temperature of the hot body will fall till at length it becomes equal to that of the enclosure. If the body be supported in such a way that the transference of heat by conduction can be neglected, the cooling process will be due partly to radiation and partly to convection. If the effect of convection currents be eliminated, as by carrying out experiments in a vacuum, **the radiation is found to be proportional to the fourth power of the absolute temperature.** This is known as **Stefan's Law.**

In the ordinary case in which a hot body cools in air at atmospheric pressure, **the rate of cooling is found to be proportional to the difference between the temperature of the body and that of its surroundings.** This is known as **Newton's Law of Cooling.** Recent experiments have shown that the law is approximately true within fairly wide limits of temperature, when the cooling takes place both by convection and by radiation.

Expt. 171. **Determination of the Rate of Cooling at different Temperatures.** — To illustrate Newton's law of cooling, support a small thin-walled metal vessel inside a larger one, in such a way as to reduce as far as possible transference of heat by conduction. Nearly fill the small vessel with hot water at a temperature of about 80° C. Take readings of the temperature of the water at intervals of half a minute until the temperature has fallen to within about ten degrees of

the temperature of the room. Plot a graph with the temperature as ordinate and the time as abscissa, taking care that the curve is drawn *smoothly* through the observed points (Fig. 178). This cooling curve will be steep at first, but will become less steep as the temperature approaches that of the room. Draw a horizontal line on the squared paper to represent the temperature of the room.

To find the rate of cooling, or the rate of change of temperature, at any particular temperature corresponding to a point P on the graph, draw a tangent to the curve at this point. Care must be taken in drawing this line so that its direction may represent as accurately as possible the direction of the curve at the point.

Let the tangent meet the vertical axis at A and the horizontal line representing the temperature of the room at B. Then the rate of change of temperature is given by the slope of this line, that is, the tangent of the angle ABC or θ. Measure the lengths AC and CB, and calculate $\tan\theta$, which is equal to AC/BC. The difference between the temperature of the body and that of the room is represented by PN. Determine this difference of temperature.

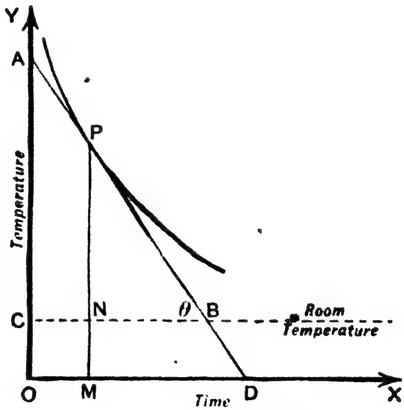


FIG. 178.—Rate of Cooling.

Then according to Newton's law of cooling, $\tan\theta$, the rate of cooling, should be proportional to PN, the difference of temperature. That is, $\tan\theta \propto PN$, $\tan\theta = kPN$, or $\tan\theta/PN = k$, a constant.

Determine this quantity for at least three points on the graph, selecting the points so as to represent fairly different portions of the complete curve, and notice whether the result is approximately constant.[1]

[1] Since $\tan\theta = PN/NB$, $\tan\theta/PN = 1/NB$.

Hence if $\tan\theta/PN$ is a constant, NB must be a constant.

This gives a simple graphical method of testing the truth of the law. Measure the length of the line NB for the different points considered, and notice whether this length is approximately constant.

Find the difference between the greatest and the least value of the results obtained, and calculate the percentage difference.

When one quantity varies with another in such a way that the rate of change of the first quantity with regard to the second is proportional to the first quantity, then the variation is said to obey the logarithmic, or exponential, law.

This is the case when a sum of money accumulates at compound interest, but then the quantity is continually increasing while in the question now under consideration the temperature of the hot body is continually diminishing. If we plot the logarithm of the excess of temperature (i.e. over the room temperature) against the time, the result should be a straight line. Students who have an elementary knowledge of the differential calculus may then verify Newton's law as follows. If the graph in question is a straight line

$$\log E - \log e = at,$$

where e is the excess at time t, E is the initial excess when $t = 0$, and a is a constant.

Differentiating,

$$-\frac{1}{e}\frac{de}{dt} = a$$

or

$$-\frac{de}{dt} = ae,$$

that is, the rate of fall at time t is proportional to the excess at this time t.

§ 2. Curve of Cooling when a Liquid solidifies

In the last experiment, Newton's Law of Cooling was illustrated by the cooling ot a thin-walled metal vessel containing hot water. The results obtained in such an experiment are modified in a characteristic way when the liquid in the vessel passes through the temperature at which solidification occurs.

In the present experiment, a substance may be used having a melting point above the ordinary atmospheric temperature, but below 100° C. For instance, naphthalene, stearin, or paraffin wax of good quality may be employed. If the substance at the beginning of the experiment is in the liquid state, and the vessel is allowed to cool, the temperature falls regularly until the point of solidification of the liquid is reached. Then, as each successive portion solidifies, it gives up its latent heat, and so prevents the temperature from falling. The temperature therefore

remains approximately constant till the whole of the substance has solidified. From this time the temperature again diminishes regularly, till at length the solid reaches the temperature of the room (Fig. 179).

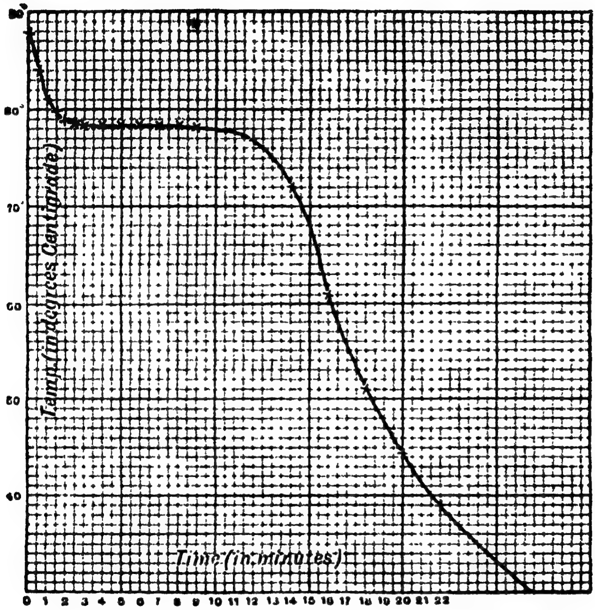


FIG. 179.—Cooling Curve showing Melting Point of Naphthalene.

EXPT. 172. **Determination of the Melting Point by a Cooling Curve.**—The small metal vessel containing the substance to be examined is heated carefully by plunging it in a vessel of hot water, till the whole of the wax has melted and reached a temperature of not more than 80° C. or 90° C.

The small vessel is then supported inside a larger, and readings of the temperature are taken at half-minute intervals. When the substance begins to solidify, it becomes necessary to leave the thermometer at rest in the centre of the solidifying mass, so that it is better *not* to stir in this experiment. Continue reading the temperature till it has fallen 10° or 15° below the melting point of the substance.

Plot a curve with the times as abscissae and the temperatures

as ordinates. Be careful so to choose the scale that the curve covers practically the whole of the sheet of paper. From the curve determine the melting point of the substance, *i.e.* the temperature at which the curve first shows a horizontal portion.

When the substance is a mixture, there may be several different melting points indicated, or no sharp change may be observed. The cheaper kinds of paraffin wax are mixtures of various members of the paraffin series which melt at different temperatures: the various constituents dissolve each other to a slight extent, and there may be no definite melting point.

Super-cooling.—An interesting case of cooling is offered by ordinary photographic 'hypo.' If this is melted and a cooling curve taken in the usual way, the temperature will fall quite steadily for a considerable time, obeying Newton's law of cooling. Suddenly solidification will commence and immediately a considerable *rise* of temperature will take place, the temperature rising to, and remaining steady at, the true melting point, until all the 'hypo' has solidified. It will then commence to fall again according to the ordinary law of cooling. The student should endeavour to formulate some theory as to the cause of the *rise* of temperature when solidification begins.

It may sometimes be observed that the temperature has fallen to only a few degrees above atmospheric temperature and yet solidification has not taken place. If the temperature falls to below 25° C. without solidification occurring, a crystal of solid 'hypo' should be dropped into the molten mass, the temperature being carefully observed the while.

§ 3. Specific Heat of a Liquid by Method of Cooling

The **quantity of heat lost per second** by a substance in given surroundings depends on the temperature of the cooling body, on the area it exposes, and on the nature of the surface exposed. **The fall of temperature per second** is equal to the heat lost per second divided by the thermal capacity of the body.

When a quantity of liquid heated to some fairly high temperature is allowed to cool in a calorimeter placed in a constant temperature enclosure, a cooling curve can be plotted from observations of the temperature of the liquid taken at half-

minute or minute intervals. The liquid can then be replaced by water, and a similar experiment carried out for the water.

If we take *identical ranges of temperature* in the two cases, the average rates of loss of *heat* will be identical, though the average rates of fall of temperature will not. If the rates of fall of temperature are found from the curves in the two cases, then we can find expressions for the rates of loss of heat, and by equating these we can determine the specific heat of the liquid.

Let M = mass of liquid used and S its specific heat,
W = mass of water,
m = mass of calorimeter, s its specific heat.

Then suppose that the temperature falls from θ_1 to θ_2 in each case, the times taken for this to take place being t_1, when the liquid is used, and t_2 with water.

The average rate of loss of heat in the first case is

$$\frac{(MS + ms)(\theta_1 - \theta_2)}{t_1}$$

and in the second case is

$$\frac{(W + ms)(\theta_1 - \theta_2)}{t_2}.$$

These rates are equal, hence

$$\frac{MS + ms}{t_1} = \frac{W + ms}{t_2}$$

From this equation S can be calculated.

EXPT. 173. **Determination of the Specific Heat of a Liquid by the Method of Cooling. I.**—A double-walled vessel with water *between* the walls is useful as a constant temperature enclosure. Paraffin oil is a suitable liquid to employ in this determination. A small calorimeter of metal, with a lid pierced for a thermometer and a stirrer, is used to contain the liquid. It is important that the outer surface of the calorimeter should be in the same condition in the two parts of the experiment. It may either be highly polished or coated with dead-black varnish.

Weigh the calorimeter. Heat some paraffin in another vessel to about 75° C. by dipping the vessel in hot water, and pour

the hot paraffin into the calorimeter. Cover the calorimeter and place it in position in the enclosure, supporting it by a non-conductor of heat so that it does not come into contact with the surrounding vessel (Fig. 180). Read the thermometer at intervals of 1 minute as the temperature falls from 70° to 30° C., keeping the liquid gently stirred. The calorimeter must be removed and weighed at the end of the observations to determine the mass of paraffin. The same procedure must then be followed, using water instead of paraffin, taking care not to alter the radiating surface in any way.

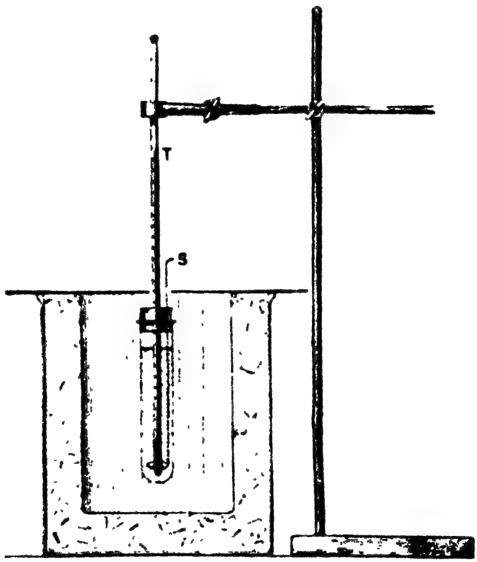


FIG. 180.—Specific Heat of a Liquid.

Plot the two cooling curves on squared paper, taking temperatures as ordinates, times as abscissae. From the curves determine the number of seconds required for the paraffin and for the water to cool through *the same range* of temperature (say from 65° to 30° C.). Calculate the specific heat of the paraffin from the formula given on p. 361.

Sometimes two calorimeters are used, one containing water and the other the liquid (paraffin). If this is done, they must be of the same metal (aluminium), have identical dimensions and be polished carefully; they are suspended at a little distance apart in

the same enclosure. Except for the saving of time in taking the cooling curves simultaneously, this method is not to be recommended, as it is impossible to ensure that the cooling surfaces, even if equal in area, shall be identical in polish. The calorimeters used are small, and it is assumed that the temperature shown by the thermometer represents the temperature of the liquid *without stirring*.

Let m_1 denote the mass of the first, m_2 the mass of the second calorimeter.

Then $\dfrac{(MS + m_1 s)(\theta_1 - \theta_2)}{t_1}$ is equal to $\dfrac{(W + m_2 s)(\theta_1 - \theta_2)}{t_2}$

Hence
$$\frac{MS + m_1 s}{t_1} = \frac{W + m_2 s}{t_2}.$$

From this equation S can be calculated.

EXPT. 174. **Determination of the Specific Heat of a Liquid by the Method of Cooling. II.**—Weigh the empty calorimeters

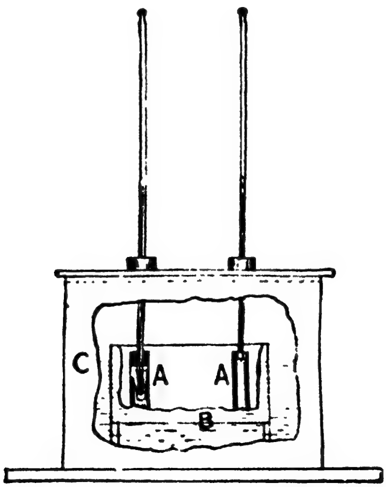


FIG. 181.—Specific Heat of a Liquid.

Fill one calorimeter about two-thirds full with paraffin, and the other with water. Support the calorimeters from the lid of the enclosure by passing thermometers through rubber stoppers as in Fig. 181. Heat the calorimeters with the contained liquid to about 75° C. by immersing them in a

vessel of hot water. Replace the lid, with the calorimeters attached, on the enclosure, taking care that the calorimeters A do not touch the metal vessel B. The space between the vessels B and C should have been filled with cold water. When the lid is in position, commence taking readings of the thermometers.

A convenient plan is to take the reading of the first thermometer when the seconds hand of a watch is at 60, and the reading of the second thermometer when the seconds hand is at 30. Readings should not be commenced unless the two thermometers indicate approximately the same temperature (between 60° and 70° C.). Continue the readings till the temperature falls below 30° C. in each case. The paraffin cools more rapidly, and therefore the paraffin will reach this temperature first. When this is the case, the readings of the paraffin thermometer can be discontinued, but the water thermometer must still be read. It is the time to cool through equal ranges of temperature which is required, not the temperature change in equal times.

At the end of these observations, remove and weigh the calorimeters to determine the mass of liquid in each. Plot two curves on a sheet of squared paper to show the cooling of the two calorimeters, taking temperatures as ordinates, times as abscissae. From the curves determine the number of seconds required in each case in cooling from θ_1 (about 60° C.) to θ_2 (about 30° C.). Calculate the specific heat of the liquid from the formula on p. 363.

CHAPTER V

THE COEFFICIENT OF THERMAL CONDUCTIVITY

§ 1. Definitions

When the temperature at one point of a body is higher than the temperature at a neighbouring point, heat tends to flow from the first point to the second. If the temperature at the two points be T_1 and T_2, and d be the distance between them, the quantity $(T_1 - T_2)/d$ is called **the temperature slope or temperature gradient.**[1] It may be expressed in degrees per centimetre.

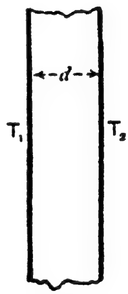


Fig. 182.—Slab with Parallel Faces.

When a slab of any material, of thickness d, with parallel faces, has one face maintained at a temperature T_1 and the other face at a temperature T_2, a steady flow of heat will eventually take place in straight lines perpendicular to the faces of the slab, and the temperature slope will be uniform and equal to $(T_1 - T_2)/d$.

The quantity of heat Q, flowing in time t through an area A measured on one face of the slab, will be proportional to the time, to the area, and to the temperature slope. It will also depend on the material of which the slab is composed. Hence we may write—

$$Q = KA\frac{T_1 - T_2}{d}t,$$

[1] In the notation of the differential calculus this may be written $\frac{dT}{dx}$, if x be taken to represent distance.

where K is a quantity depending on the material of the slab. This equation may be regarded as defining the meaning of K, **the coefficient of thermal conductivity** of the material. Solving the equation for K we find

$$K = \frac{Q/t}{A \cdot (T_1 - T_2)/d}.$$

The numerator Q/t measures the rate of flow of heat through the slab. This may be expressed in calories per second.

Hence **the coefficient of thermal conductivity** may be defined briefly as **the rate of flow of heat per unit area per unit temperature slope.**[1] The coefficient will be expressed in calories per second, per square centimetre, per unit temperature slope.

The measurement of a coefficient of thermal conductivity consequently involves the determination, after a steady state has been reached, of the three quantities: rate of flow of heat, area through which the heat flows, and temperature slope.

§ 2. Experimental Determinations

COEFFICIENT OF THERMAL CONDUCTIVITY OF A METAL BAR

The metal, of which the coefficient of thermal conductivity is to be found, is in the form of a cylindrical bar (Fig. 183). One end of the bar is heated by passing a current of steam through a steam chamber B, the other end is cooled by passing a current of water through a spiral tube encircling the bar at C. The temperature at two points, D and E, along the length of the bar is determined by thermometers T_1 and T_2. The temperature in the water is determined at the point F, where it leaves the coil, by the thermometer T_3, and at the point G, where it enters the coil, by the thermometer T_4.

Expt. 175. **Determination of the Coefficient of Thermal Conductivity of a Metal Bar.**—In carrying out the experi-

[1] In the notation of the differential calculus the equation may be written—

$$K = \frac{dQ/dt}{A\,dT/dx}.$$

ment a steady current of steam from a boiler must be passed through the steam chamber, and a steady current of water must be passed through the coil. The bar is packed round with a bad conductor of heat, such as felt, and is left until a steady state is reached. This may take from twenty minutes to half an hour. The four thermometers are read from time to time in order to see whether the temperatures recorded are steady. The final temperatures recorded will depend on the rate at which water is flowing through the coil. To secure a reasonably large difference between the temperatures T_3 and T_4 it is best to have a rather slow stream; in

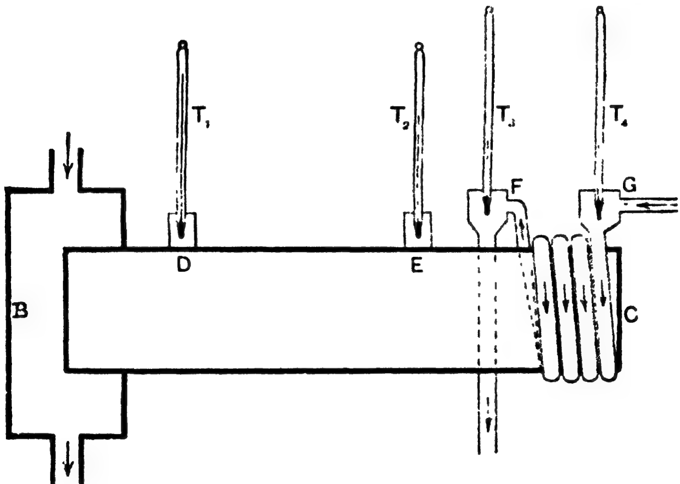


FIG. 183.—Thermal Conductivity of Metal Bar.

fact little more than a ·kle of water should issue from the coil. The quantity of water flowing through the coil per second must be determined by collecting the water that issues in a given time (two or three minutes), and either weighing it or measuring the volume in a graduated vessel. In this way is found the number of grams, m, of water passing through the coil in t seconds. The temperature of this mass of water has been raised from T_4 to T_3, so that the water must have absorbed $m(T_3 - T_4)$ units of heat from the bar. Assuming that no heat has been lost from the sides of the bar, we may write for Q/t in the definition of the coefficient of thermal conductivity $m(T_3 - T_4)/t$.

The area of cross-section of the bar can be found by

measuring the diameter with the vernier callipers, and taking $A=\pi r^2$ where r is the radius of the circular cross-section. The temperature slope can be found from the temperatures T_1 and T_2, and the distance d between the two points D and E. Thus all the quantities necessary for the determination of K can be found.

COEFFICIENT OF THERMAL CONDUCTIVITY OF A BAD CONDUCTOR IN THE FORM OF A PLATE

In the case of a bad conductor of heat, the thickness of the slab of material to be examined must be small. The thermal conductivity of a sheet of cardboard may be determined by means of an apparatus similar in principle to that used in the researches of Professor C. H. Lees.

EXPT. 176. **Determination of the Coefficient of Thermal Conductivity of a Bad Conductor such as Cardboard.** — The material is in the form of a thin circular plate. One face of the plate is heated by being placed in contact with a

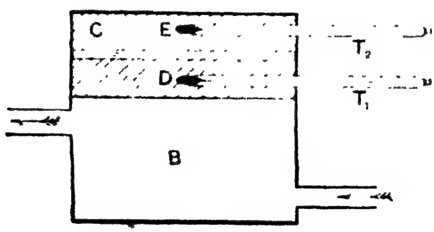


FIG. 184.—Thermal Conductivity of Cardboard.

metal chamber B (Fig. 184), through which a current of steam is passed. The other face of the plate is in contact with a circular disk C of metal. All the metal surfaces are nickel-plated. Thermometers D and E are inserted in holes in the chamber B and the disk C.

The apparatus is left until a steady state is attained, and then the readings of the thermometers are recorded.

It is assumed that the temperatures T_1 and T_2 indicated by the thermometers represent the temperatures of the two faces of the cardboard plate. The thickness of the cardboard being known, the temperature slope can then be calculated. The area through which

the heat flows is taken to be the area of the face of the card, which can be calculated from the diameter of the circle.

It now remains to determine the rate of flow of heat through the card. For this a separate experiment is necessary. When the steady state is reached in the first experiment the rate of flow of heat through the card must be exactly equal to the rate at which heat is lost from the surface of the disk C by convection and radiation. For when the temperature has become stationary there can be no accumulation of heat going on in the disk, and the heat gained by the disk must be equal to the heat lost. Consequently, if we can determine the rate at which heat is lost, we know the rate at which heat is flowing through the card.

The heating chamber B is removed, and the disk C, with the cardboard in contact with one face, is supported so that transference of heat by conduction is a minimum. This is sometimes done by suspending the disk by strings, but it may be more convenient to place the cardboard on a support, such as a block of wood, which is a bad conductor of heat.

The disk is then heated by the flame of a Bunsen burner till its temperature is 5 or 6 degrees higher than the steady temperature T_2. It is then allowed to cool until its temperature has fallen from this temperature, T_3, to a temperature T_4, the same number of degrees below T_2, and the time t taken in cooling is noted carefully.

The heat lost in cooling is $MS(T_3 - T_4)$, when M is the mass of the disk and S the specific heat of the metal. Consequently the rate at which heat is lost is $MS(T_3 - T_4)/t$. Assuming[1] that this is equal to Q/t in the formula for thermal conductivity, we have all the data necessary for the calculation of the latter quantity.

COEFFICIENT OF THERMAL CONDUCTIVITY OF A BAD CONDUCTOR IN THE FORM OF A TUBE

The Coefficient of Thermal Conductivity of a bad conductor of heat in the shape of a tube can be determined by passing a current of steam through the tube, or through a jacket surrounding the

[1] This assumption is not strictly accurate, for when the heater is removed some heat is lost from the disk by conduction through the cardboard. It would be more correct to determine the rate of loss of heat from the disk when the heater is in position, but *at the same temperature as the disk*, so that there is no slope of temperature in the cardboard. It is not difficult to realise this condition in practice, at least in an approximate fashion.

tube, and measuring the quantity of heat transmitted through the walls of the tube by the ordinary methods of calorimetry.

First Form of Apparatus.—Steam is passed *through* the tube. The tube may be immersed in a known mass of water in a calorimeter and the rise of temperature of the water in a certain time may be observed.

EXPT. 177. **Determination of the Coefficient of Thermal Conductivity of a Poor Conductor in the Form of a Tube.**—In the case of a rubber tube select a calorimeter of fairly large capacity (500 or 600 c.c.) so as to allow a considerable length of the tubing to be coiled up inside it. Weigh the inner vessel of the calorimeter and fill it about two-thirds full of water. Weigh the vessel thus filled in order to find the weight of the water. Take the temperature, T_2, of the water, which may conveniently be below that of the room to start with. Coil up the rubber tube in the water, allowing both ends to project some distance out of the calorimeter. The rubber tube must then be connected to the nozzle of a steam generator, so that a steady current of steam can be passed through it. The other end may dip into a tin can to catch any drippings from the condensed steam.

Allow the steam to pass through the tube for an observed time, until the temperature of the water has risen 15° or 20° C. Note the time during which the steam has passed, and also the final temperature, T_3, of the water.

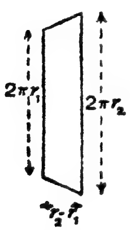


FIG. 185. Slab formed from Tube.

The length of tube immersed in the water must be measured, and for this purpose it is convenient to have two marks made on the tube at the points where it enters and leaves the water in the calorimeter. Let the length immersed be l cm. Measure also the internal and the external radius of the tube. Let these be r_1 and r_2 cm. respectively. Then the thickness of the wall of the tube is $r_2 - r_1$ cm. If we imagine the tube slit open by a cut parallel to its axis (Fig. 185), it will approximately correspond to a slab of material of thickness $r_2 - r_1$.

The area through which the heat flows may be taken approximately as the mean of the areas of the two faces of the slab, that is

$$A = \tfrac{1}{2}(2\pi r_1 l + 2\pi r_2 l) = 2\pi \bar{r} l,$$

where $\bar{r} = \frac{1}{2}(r_1 + r_2)$ is the *mean* radius of the tube.

The temperature on the outside of the tube is not constant, but we take the mean of the initial and final temperatures in calculating the temperature gradient. The temperature of the inside of the tube may be taken as $T_1 = 100°$ C. Thus the temperature gradient is

$$\frac{T_1 - \overline{T}}{r_2 - r_1},$$

where $\overline{T} = \frac{1}{2}(T_2 + T_3)$ is the *mean* temperature outside the tube.

The only other quantity required is the quantity of heat flowing through the wall of the tube in time t. As the heat goes to raise the temperature of the calorimeter and its contents from T_2 to T_3 it can be calculated easily. Thus all the quantities required for the determination of the coefficient of thermal conductivity can be obtained, and the coefficient can be calculated from the equation[1] deduced from the definition on p. 366.

$$K = \frac{Q/t}{2\pi \bar{r} l\left(\frac{T_1 - \overline{T}}{r_2 - r_1}\right)};$$

Second Form of Apparatus.—If the tube is not flexible, as in the case of a glass tube, its thermal conductivity may be determined in the following manner:—A slow stream of water is passed through the tube, the stream being maintained steady by use of a Mariotte's Bottle. The tube is inclined slightly so that it is always full of water while the experiment is in progress; it is enclosed in a steam jacket through which steam is passing (Fig. 186).

EXPT. 178. **Determination of the Coefficient of Thermal Conductivity of a Poor Conductor in the Form of a Tube.**—As the water enters the tube its *inflow* temperature T_1 is taken. As the water flows through the tube, it receives heat by conduction through the tube walls, its temperature rising to T_2 by the time it has flowed through the jacketed part of the tube. This *outflow* temperature is taken, and the emerging water is collected in a measuring cylinder. The mass of water

[1] A more accurate formula, obtained by considering the flow of heat through the wall of a hollow cylinder, gives

$$K = \frac{Q/t}{\dfrac{2\pi l(T_1 - \frac{1}{2}\{T_2 + T_3\})}{\log_e r_2/r_1}}.$$

When the tube is thin, so that $r_2 - r_1$ is small compared with r_1 or r_2, this reduces to the form above.

flowing through the tube in a certain time is noted; this interval of time should be such that at least 300 c.c. of water are collected in a measuring glass, the mass being calculated on the assumption that the density is unity. If the mass of water flowing through the tube in t seconds be M gm., the quantity of heat Q conducted through the walls of the tube in this time is $M(T_2 - T_1)$ calories.

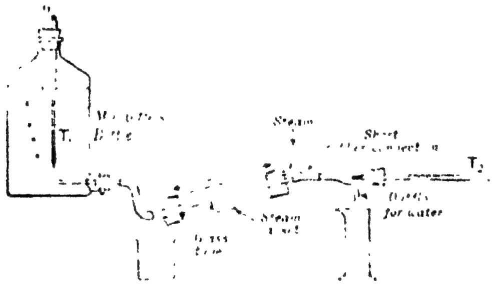


FIG. 188. Thermal Conductivity of Glass Tube.

The length of the tube between the ends of the steam jacket is measured, and also the internal and external radii of the tube. Let these be l, r_1, and r_2 respectively.

Then the average area through which the heat flows is $2\pi\bar{r}l$, where $\bar{r} = (r_1 + r_2)/2$.

The average temperature slope through the tube is

$$\frac{100 - \bar{T}}{r_2 - r_1}, \text{ where } \bar{T} = \frac{T_1 + T_2}{2}.$$

Hence[1]

$$M(T_2 - T_1) = K 2\pi\bar{r}l\left[\frac{100 - \bar{T}}{r_2 - r_1}\right]t.$$

All these quantities can be measured or observed except K, hence K can be calculated.

[1] If the more accurate equation is used

$$M(T_2 - T_1) = \frac{K 2\pi l(100 - \bar{T})t}{\log_e r_2/r_1}.$$

CHAPTER VI

THE MECHANICAL EQUIVALENT OF HEAT

§ 1. Definition and Determination of the Mechanical Equivalent of Heat

It was proved by Dr. J. P. Joule (1818–1889) that when heat was produced by the expenditure of mechanical energy, for every unit of heat produced a definite number of units of work had to be done. This number is called **the mechanical equivalent of heat.** Thus, in order to produce one calorie (gram-degree centigrade), $4{\cdot}2 \times 10^7$ ergs or $4{\cdot}2$ joules of work are required. In C.G.S. units, using the centigrade scale of temperature, the mechanical equivalent of heat is $4{\cdot}2 \times 10^7$ ergs per calorie.

Expt. 179. **Determination of the Mechanical Equivalent of Heat by the Fall of Mercury in a Tube.**—A wide glass tube about a metre in length and 3 to 4 cm. in diameter is sealed at one end and provided at the other with a well-fitting rubber cork through which passes a sensitive thermometer. About 50 c.c. of mercury is introduced into the tube and the cork is fixed securely in position. The tube is grasped firmly at its centre and held vertically so that the lower end rests level with a table. The tube is then inverted quickly so that the upper end now occupies the position formerly occupied by the lower end. This means that the tube must be rotated about a horizontal axis through the middle of its length. During the rotation the mercury remains at the end of the tube, but when the tube becomes vertical the mercury falls from one end of the tube to the other.

The work done in lifting the mercury is converted into kinetic energy during the fall, and this is converted into heat when the mercury comes to rest at the bottom of the tube. In order to secure an appreciable rise of temperature, the operation must be repeated about 50 times.

Let m = mass of mercury in the tube,
s = specific heat of mercury,
t_1 = final temperature,
t_2 = initial temperature.

Then assuming that no heat is lost, the total amount of heat produced $H = ms(t_1 - t_2)$.

Let h = the vertical distance through which the centre of gravity of the mercury falls when the tube is inverted (note that this is *not* the length of the glass tube), n = the number of times the operation is carried out.

Then the mechanical energy which disappears is $E = nmgh$.

So we have $J = \frac{E}{H} = \frac{nmgh}{ms(t_1 - t_2)} = \frac{ngh}{s(t_1 - t_2)}$.

From this result Joule's equivalent can be calculated.

Note that the value of J is independent of the mass of mercury used. In an actual experiment a very small quantity of mercury must not be used, otherwise the heat used in warming the tube would be appreciable in comparison with that used in warming the mercury. A less accurate method, attended with less risk of breaking the thermometer, is to use a solid cork, and to take the temperature of the mercury in a small beaker before and after the operations.

PRODUCTION OF HEAT BY FRICTION BETWEEN METAL CONES

The following method of determining the mechanical equivalent of heat by the friction between two metal cones is an adaptation of one employed by Joule.

Two metal cones, D and E (Fig. 187), are provided which fit closely one within the other. The outer cone is forced to rotate by attaching it to a vertical spindle driven by a flywheel turned by hand. The inner cone is prevented from rotating. Consequently friction takes place between the surfaces in contact, and the heat produced goes to heat the cones and any liquid (usually water, sometimes mercury) that may be placed in the inner cone.

The amount of heat produced is determined from a knowledge of the water equivalent and the rise of temperature.

The amount of mechanical work expended can be estimated by applying a measurable torque to the inner cone in order to prevent it from rotating, and by counting the *number of revolutions* made by the outer cone. A is a circular wooden disk which rests upon the inner cone, to which it is attached by two steady-pins. A leaden weight B is placed on the top of it to hold it in position. A string attached to the circumference of the disk passes over a pulley and is kept stretched by a known weight M (100 to 200 gm.) fastened to its other end. When the outer cone is rotated the inner cone tends to move with it, but it is prevented by the

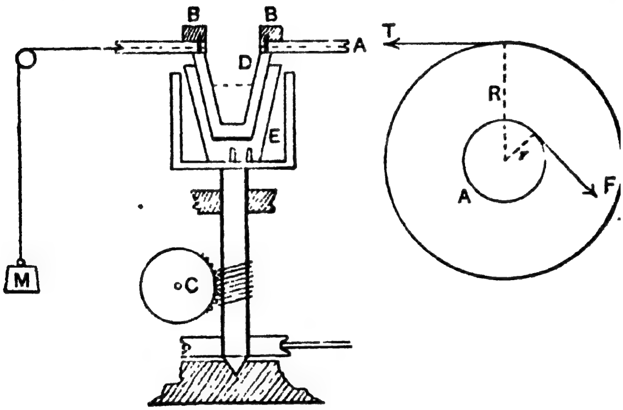


FIG. 157.—Mechanical Equivalent of Heat.

moment of the force due to the tension T in the string. *The string must always be tangential to the circumference of the wooden disk when the apparatus is in use.*

Let R be the radius of the disk, and r the mean radius of the surface of contact of the cones. Then if F represent a mean value of the friction between the cones,

$$\begin{aligned} Fr &= TR \\ &= MgR, \end{aligned}$$

where M is the mass of the suspended load.

The work done in one revolution of the outer cone, when the inner one is at rest, is $W = F2\pi r$. Consequently the work done in n revolutions $= 2\pi nFr$.

Although the values of F and r separately cannot be determined

accurately, the value of Fr can be found from the equation above. Substituting this value we find for the work done

$$W = 2\pi n M g R.$$

Consequently the mechanical work can be calculated in ergs.

EXPT. 180. **Determination of the Mechanical Equivalent of Heat by the Friction between two Metal Cones.**—In carrying out the experiment, it is essential that the friction between the cones should be suitable in amount. Otherwise it will be impossible to keep the load at a fixed level. A single drop of lubricating oil is usually sufficient to place between the inner and the outer cone. If the cones are not lubricated the surfaces in contact will 'seize.'

The adjustment of the amount of lubricant should be made before beginning the actual experiment, the apparatus being tested by turning the driving wheel so as to see whether the load can be maintained approximately at a fixed level, whilst turning at a fair speed.

Weigh the two cones together when empty, and also when the inner cone is about two-thirds full of water. Then replace the cones in the apparatus and introduce a sensitive thermometer to determine the temperature. If care be taken the thermometer itself may be used as a stirrer. Sometimes the thermometer is held in a stand and a separate stirrer is employed.

It is important to allow for any loss or gain of heat due to radiation. In order to do this, readings of the temperature should be taken at intervals of 1 minute for 5 minutes before the actual experiment commences. Then the flywheel should be set in rotation and the required number of revolutions made. The number is indicated by the counting mechanism C geared to the rotating spindle. To obtain a rise of temperature that can be measured with reasonable accuracy a large number of revolutions must be made: 500 or 1000 revolutions may be necessary. The time this takes must be noted. When this operation is completed, the temperature must be read, and readings of the thermometer taken again at intervals of 1 minute for 5 minutes. From the readings before and after the *mean rate of change of temperature* can be found, and knowing the time the experiment lasted, the actual change during this period can be calculated. This change must be taken into account in estimating the rise of temperature produced by the friction between the cones.

Calculate the number of calories of heat produced, from the water equivalent and the rise of temperature. Measure the diameter of the wooden disk with a large pair of callipers, and calculate the work done from the expression $W = 2\pi n M g R$. The mechanical equivalent should be expressed both in ergs per calorie and also in joules per calorie.

CALLENDAR'S APPARATUS FOR THE MECHANICAL EQUIVALENT OF HEAT

In this form of apparatus (Fig. 188), the water is contained in a hollow drum which is rotated by an electro-motor or by

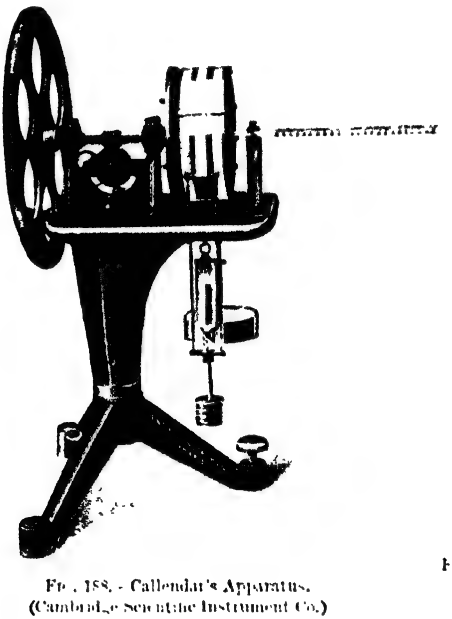


FIG. 188.—Callendar's Apparatus. (Cambridge Scientific Instrument Co.)

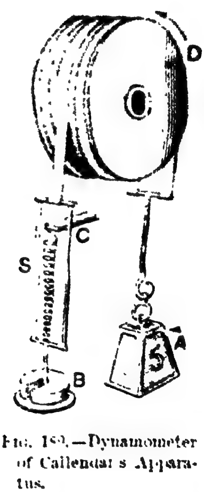


FIG. 189.—Dynamometer of Callendar's Apparatus.

hand. Over the drum passes a brake-band of silk,[1] consisting of three ribbons. The two outer ribbons are wound once round the drum and at one end carry a mass A (Fig. 189), of several (3 to 5) kgm. The other ends of these ribbons are fastened to

[1] The silk belt must be kept clean and dry, and be put away in a paper wrapper when the apparatus is not in use.

an ivory or vulcanite bar, to which the middle ribbon is also attached. This ribbon also passes over the drum, lying between the other ribbons and in continuation of them. It carries a yoke S at its other end, and from this yoke is suspended a small mass B, about 200 to 400 gm. From the lower end of the yoke passes a spring-balance which hangs from the frame of the apparatus at C; this spring acts upwards on the mass B and supports B to some extent during the experiment: its action is to steady the working of the apparatus.

Suppose the drum to be rotating in the direction of the arrow: A is raised by the friction of the band, and B is depressed. The difference in tension between the ends of the band is equal to the force of friction between the band and the drum. Now the force of friction round a drum or fixed pulley depends on the tension T_0 at the free end (see p. 94); if the weight of B were adjusted carefully, it would be possible for B to keep A balanced exactly when the drum was rotating at a certain speed. If, however, B were not adjusted exactly to this value, the band would move slowly either in the same direction as the drum or against the motion of the drum, according as B were greater or less than this special value. The slightest alteration of the coefficient of friction, due to rise of temperature, alteration of speed,[1] or any other cause, would require a readjustment of B, or if the angle of contact between the brake-band and the drum were altered through slight oscillation of either end, the band would begin to move in one direction or the other.

The adjustment of B would be troublesome if the spring-balance were not used, and readjustment would be necessary at frequent intervals in the experiment. The spring avoids all this troublesome adjustment in the following way: If, at any moment the force of friction is too large, B begins to move downwards, its weight is thus thrown on the spring to some extent, and the cord, being released from this part of the weight of B, exerts a smaller frictional force on the drum and the

[1] The force of friction between solid surfaces is nearly, but not quite, independent of their relative velocity.

downward motion of B is arrested. Diminution in the force of friction causes B to rise; its weight is then taken by the cord more completely and the friction increases accordingly, the motion of the cord being again stopped. The arrangement of a brake-band round a drum is called a **Dynamometer.**

The force of friction is equal to $T - T_0$, where T is the weight of A, and T_0 is the difference between the weight of B and the force exerted by the spring: all these forces are measured in dynes.

The **work done** is equal to the product of the frictional couple exerted on the drum, and the angle in radians through which the drum revolves: thus in *n* revolutions the work done is $2\pi n(T - T_0)R$, where R is the radius of the drum.

The number of revolutions is determined by means of a revolution counter mounted on the axle of the drum.

The **heat generated** in any given number of revolutions is determined by the rise of temperature of the water, multiplied by the thermal equivalent of the drum and contents. To avoid loss of heat from the drum by conduction, and also to give a definite value to the thermal equivalent of the drum, it is mounted at six points on its circumference on ivory or vulcanite studs, by means of which it is attached to the driving disk and spindle. The drum has a hole in the centre of its end plate, into which the thermometer is inserted and through which the water is introduced. The drum is half-filled with water before the experiment, the mass of water used, *w* gm., being determined before introducing it into the drum. As the drum revolves, the water gradually acquires a motion of rotation and rotates with the drum, being prevented from escaping from the drum by centripetal force, which keeps it in contact with the rim.

The thermometer is of a special design, being bent so that its bulb lies near the rim of the drum and inside the drum, while the graduated stem projects from the hole at the centre, and is clamped as shown in Fig. 188. The water swirls past the bulb, and its temperature is registered on the thermometer, the rotary motion ensuring thorough mixing and consequent uni-

formity of temperature throughout the liquid. The thermometer, being fixed, enables temperatures to be taken at any instant during the experiment, and it is possible therefore to plot a temperature-time curve if desired, and to correct the final temperature for radiation losses by means of this curve (p. 349). The water equivalent of the drum can be determined from its mass m and its specific heat s.

From the work done and the heat generated, the mechanical equivalent of heat J is determined by the equation

$$W = JH.$$

W is the work done and is given by $2\pi n\ (T - T_0)\ R$.

H is the heat generated and is equal to $(w + ms)\ (\theta_2 - \theta_1)$, θ_1 being the initial and θ_2 the final temperature of the water in the drum.

EXPT. 181. **Determination of the Mechanical Equivalent of Heat by Callendar's Apparatus.** The apparatus is set up to be driven either by hand or by motor. Lightly polish the drum with a clean duster and a little French chalk. Adjust the brake-bands over the drum as shown diagrammatically in Fig. 189, placing a five-kilogramme weight on the end A and a mass of 100 gm. on B, the spring-balance being fixed to the frame of the apparatus at C.

Measure an amount of water sufficient to fill the drum nearly up to the hole at the centre—between 300 and 500 c.c. will be required; let the mass of this be w gm. Introduce the water into the drum.

Place the thermometer bulb in the interior of the drum, clamping the thermometer in the clamp provided on the apparatus so that the stem projects along the axis of the drum. Care must be used in inserting the bulb, as the thermometer is fractured easily at the bends.

Start the motor and adjust its speed or the masses of A or B until the band is stationary when the drum rotates. Care must be taken that the yoke is not touching the frame of the instrument, and that the index of the spring-balance is well away from both ends of the scale. When these adjustments have been made the motor is stopped and the water allowed to come to rest. Take readings of the water temperature θ_1, and of the revolution counter.

To carry out the experiment proper, start the motor and read the temperature of the water every fifty or one hundred revolutions of the drum. Read the tension of the spring-balance between every pair of temperature readings so as to get the mean force exerted by the spring during that part of the rise of temperature. Note also the time occupied by the experiment.

After 1000 revolutions (or some other convenient number)

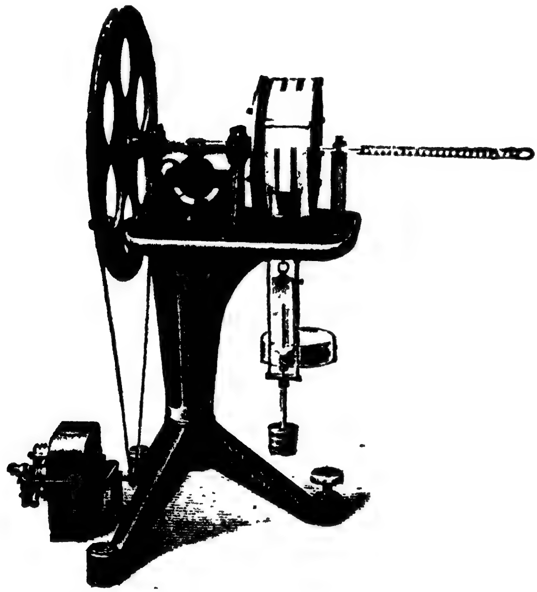


FIG. 190. Callendar's Apparatus driven by Motor. (Cambridge Scientific Instrument Co.)

stop the motor and take the temperature θ_2 when the water has come to rest. The apparatus is then left for the same length of time as that taken for the experiment, and the fall of temperature during this period is noted; let this be $\delta\theta$.

The mean excess of temperature over the surroundings during the experiment is half the final excess, and therefore the mean rate of loss of heat during the experiment would be half the rate at the end of the experiment. The correction for radiation during the experiment is therefore made by adding $\delta\theta/2$ to the observed rise of temperature, and we have

$$H = (w + ms)\left(\theta_2 - \theta_1 + \frac{\delta\theta}{2}\right),$$

where H is the **Heat generated** during the experiment.

In this expression m is the mass of the drum, and s the specific heat of the material of the drum (usually brass): m is usually stamped on the end of the drum by the maker.

To calculate the work done it is necessary to measure the radius of the drum; let this be R. Then the force of friction is given by the difference in tension between the ends of the cord. Let the mean reading of the spring-balance be C gm., then the tension T_0 at the end of the band carrying the weight B is $(B - C)$ gm. weight.

At the other end the tension T is equal to A gm. weight, and the friction force is given by

$$F = (T - T_0) \text{ dynes} = \{A - (B - C)\} g \text{ dynes}.$$

This force is exerted round the periphery of the drum, and the couple due to friction is

$$FR \text{ dyne-cm.},$$

the work done in n revolutions being equal to

$$W = 2\pi n \, FR \text{ ergs}.$$

Calculate the heat generated H and the work done W in the number of revolutions taken, and calculate the mechanical equivalent of Heat J from the equation

$$W = JH.$$

CHAPTER VII

HYGROMETRY

§ 1. DEFINITIONS

THE **Hygrometric State** or **Relative Humidity** of the air can be defined as **the percentage or fractional saturation of the air.** At any temperature t a certain maximum amount of aqueous vapour can exist in the air; this corresponds with the **Saturation Vapour Pressure,** F, of water vapour at that temperature.

The actual quantity of vapour present is rarely equal to this maximum, the aqueous vapour actually present corresponding with a saturation pressure f which is usually considerably less than F. The mass of aqueous vapour present is proportional to f, and consequently the fractional saturation can be expressed as f/F or as a percentage by $f/F \times 100$.

The aqueous vapour present in the air would be sufficient to saturate the air at a certain temperature t_1. If the air is cooled down *locally* to this temperature t_1, dew will be deposited on any flat surface exposed to this cooled air: the temperature t_1 is called the **dew point.**

To a very close degree of approximation, the saturation vapour pressure (S.V.P.) at the dew point may be taken as equal to the pressure of the vapour actually present in the air. Thus if we can determine the dew point, we can obtain the hygrometric state of the air, for the saturation vapour pressure of water vapour at any temperature can be obtained from tables (Appendix, p. 598), and the

$$\text{Hygrometric state} = \frac{f}{F} = \frac{\text{S.V.P. at the dew point}}{\text{S.V.P. at the air temperature}}.$$

§ 2. Methods of determining the Dew Point. Hygrometers

The method of cooling the air locally is to cool down a bright metallic surface. When dew is deposited on this, its bright surface assumes a dull appearance, and with practice a very slight trace of dew can be detected. If the temperature of the surface is then taken, this temperature will be the dew point. Any apparatus designed for this purpose is called a **Hygrometer.**

DANIELL'S HYGROMETER

Daniell's Hygrometer is shown in Fig. 191. In this form the metallic surface is a gold band fused round the lower glass bulb A. Inside this bulb is a thermometer the stem of which passes up the tube connecting this bulb with the second bulb B on the other side of the stand. The two bulbs and the connecting tube contain ether and ether vapour only.

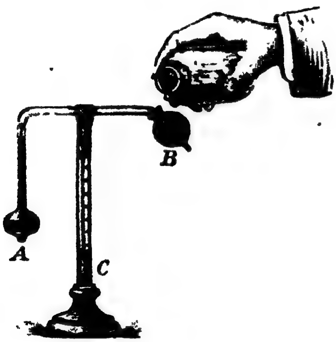


Fig. 191.—Daniell's Hygrometer.

By pouring ether on the cloth surrounding the upper bulb, and causing it to evaporate rapidly, this bulb is cooled. The ether vapour inside the bulb is condensed, and its place is taken by more vapour which passes over from the other bulb. Condensation is continued in the upper bulb, and vapour passes over from the lower bulb to replace the condensed vapour, so that continuous evaporation goes on in the lower bulb so long as the upper is being cooled.

The evaporation inside the lower bulb causes a steady fall of temperature in it, and the gold band finally cools to the dew point. When the first trace of dew is noticed, the temperature of the thermometer inside the apparatus is taken and also the temperature of the air of the room. Usually a second thermometer is mounted on the stand of the instrument for this latter purpose.

The first of these temperatures is taken as the dew point, and the hygrometric state is calculated from these observations.

The apparatus is not a good one. The thermometer inside the apparatus is separated from the gold band by a mass of liquid at least 1 cm. thick, and then by a layer of glass of from 1 to 2 mm. thickness. The liquid is practically *still*, and there may be considerable variations of temperature in the liquid itself; the glass too is a bad conductor of heat, and consequently the temperature of the thermometer may be from 1° C. to 2° C. below the temperature of the gold band, the value obtained for the dew point being wrong to the same extent.

There are other objections to this form of instrument; the air all round it is charged with ether vapour and also the rate of cooling cannot be regulated, being determined by the rate of evaporation of the ether on the cloth.

EXPT. 182. **Determination of the Dew Point with Daniell's Hygrometer.**—Read the temperature of the air in the room by means of the thermometer mounted on the stand of the instrument. Pour some ether on the muslin surrounding the upper bulb, and watch the gold band for the first trace of a deposit of dew. If the surface is touched from time to time with the end of a long paper spill or a feather, the presence of dew may be detected more easily. Immediately the deposit is noticed read the temperature of the thermometer inside the apparatus.

Find from the table (p. 598) the saturation vapour pressure corresponding with these two temperatures, and calculate the relative humidity.

REGNAULT'S HYGROMETER

The form of hygrometer designed by Regnault, when constructed and used properly, avoids the disadvantages of Daniell's Hygrometer. An *open* glass tube A is fitted at its lower end with a silver cap B (Fig. 192). In this is placed sufficient ether to fill the silvered cap, and a thermometer dips into the ether. By means of two tubes CD and EF fitted as shown, a current of air is aspirated through the apparatus, the air bubbling through the liquid and passing out through the side tube G. The air becomes charged with ether vapour as it bubbles through, and the rapid evaporation lowers the temperature of the liquid. This is in immediate contact with the silver cap and with the thermometer, and is well stirred by the bubbling air, consequently the thermometer, the liquid, and the silver cap will all be at the same temperature.

Dew forms on the silver cap as soon as the temperature of the dew point is reached, and the dew point can therefore be

determined very accurately by taking the temperature of the thermometer when dew is first observed on the cap.

EXPT. 183. **Determination of the Dew Point with Regnault's Hygrometer.**—Connect the short glass tube to the aspirator. The dew point should first be determined approximately by using a rapid current of air. This causes rapid cooling, and the dew will not be noticed until the temperature is slightly below the true dew point. If now the current of air is stopped, the whole apparatus will warm up *slowly* and the dew will disappear. The temperature at which the dew disappears should be noted; this will be much nearer the true dew point than the value previously taken, but will be somewhat too high.

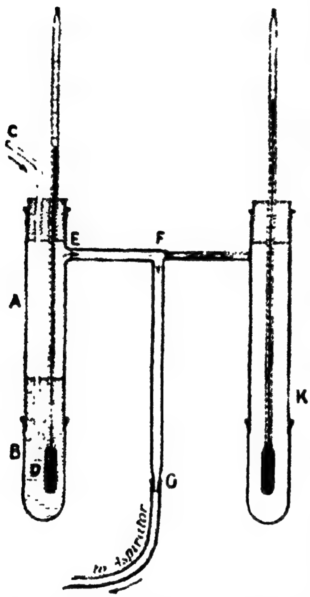


FIG. 192.—Regnault's Hygrometer.

The aspirator is now set working again so as to aspirate a very slow current of air through the apparatus. By this means the temperature will be lowered again but very slowly, and the appearance of dew will be noticed very soon after the dew point is reached; a more accurate value of the dew point will thus be obtained.

By alternately cooling the bulb and allowing it to warm up again in the manner described, temperatures will finally be obtained for the appearance and disappearance, which will not differ by more than 0·2 of a degree. When this is the case the mean of these may be taken as the dew point. The thermometer in the tube K gives the temperature of the room.

Find from the table (p. 598) the saturation vapour pressure corresponding with the dew point, and also with the temperature of the room, and deduce the relative humidity.

NOTE.—In order to detect the smallest trace of dew it is convenient to use a long dry quill, or a spill of paper formed by rolling up a half sheet of note-paper. This should be held at one end, and the silver cap gently stroked at one point with the other end of the paper or quill. Any deposition of dew

can be detected in this way, as the moistened surface of the silver shows a higher polish where the paper has been stroked along it. The hand must not approach within 20 cm. of the silver cap, and the apparatus should be watched through a large pane of glass; the experiment should not be carried out near any place where a large surface of water is exposed.

Many instrument makers supply a complete glass test-tube with a silver cap slipped on the end as a Regnault's Hygrometer. The use of such an apparatus reintroduces one of the main errors which Regnault's apparatus was designed to avoid, by placing a badly conducting medium between the silver cap and the thermometer.

The end of the test-tube must be cut off with a file and the silver cap cemented to the tube so as to have the tube closed with a silver cap in immediate contact with the ether.

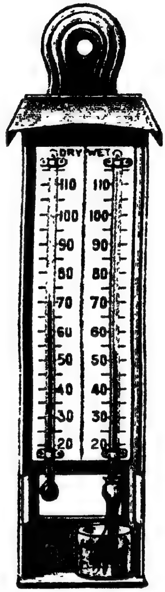


FIG. 193.—Dry and Wet Bulb Thermometers.

WET AND DRY BULB HYGROMETER

Two thermometers are arranged on a stand; one is exposed to the air, and the bulb of the other is wrapped round with a cloth that is kept moist by dipping at its lower end into a small vessel of water (Fig. 193). The drier the air, the more rapidly will evaporation take place from the wet bulb and the lower will be its temperature. By reading the two temperatures an estimate may be made of the hygrometric state, tables for the reduction of the readings having been found by trial, using a Regnault's hygrometer for comparison. Such a table is given on p. 388.

The instrument, though used extensively by meteorologists, is not of direct scientific value.

CALCULATION OF THE MASS OF AQUEOUS VAPOUR PER LITRE OF THE ATMOSPHERE

A litre of hydrogen at N.T.P. would weigh 0·09 gm.

At a pressure f mm. and a temperature t_1, its mass would be

$$0{\cdot}09 \times \frac{f}{760} \times \frac{273}{273+t_1} \text{ gm.}$$

Now we have aqueous vapour at a pressure f mm. when measured at a temperature t_1 (the dew point). Aqueous vapour is nine times as dense as hydrogen under similar conditions. Hence the mass of aqueous vapour present per litre is

$$0{\cdot}81 \times \frac{f}{760} \times \frac{273}{273+t_1}.$$

It is contended sometimes that the aqueous vapour is present at a pressure f mm. at the temperature of the air, in which case t_1 in the above expression should be replaced by t the air temperature. Either method may be adopted for the calculation, the percentage error, if any, due to using either expression being much less than the percentage error of experiment in determining f.

The mass of aqueous vapour per litre can be determined also by chemical means by aspirating a known volume of air through weighed drying tubes and finding the mass of aqueous vapour absorbed by these.

Wet and Dry Bulb Hygrometer

The first vertical column gives the temperature of the dry bulb thermometer. The first horizontal row gives the difference between the two thermometers. The remaining figures give the actual vapour pressure in mm. at the time of observation. When the air is saturated the difference between the thermometers is zero, and the second vertical column gives the saturated vapour pressure.

t° C	0	1	2	3	4	5	6	7	8	9	10
0	4 6	3 7	[illegible]	2 1	1 3						
1	4 9	4 0	3 2	2 4	1 6	0 8					
2	5 3	4 4	3 4	2 7	[illegible]	1 0					
3	5 7	4 7	3 7	2 9	2 2	1 3					
4	6 1	5 1	4 1	3 2	2 4	1 6	0 8				
5	6 5	5 5	4 5	3 5	2 6	1 8	1 0				
6	7 0	5 9	4 9	3 9	2 9	2 0	1 1				
7	7 5	6 4	5 3	4 3	3 3	2 3	1 4	0 4			
8	8·0	6·9	5 8	4 7	3 7	2 7	1 7	0 8			
9	8 6	7 4	6 3	5 2	4 1	3 1	2 1	1 1	0 2		
10	9 2	8 0	6 8	5 7	4 6	3 5	2 5	1 5	0 5		
11	9 8	8 6	7 4	6 2	5 1	4 0	2 9	1 9	0 9		
12	10 5	9 2	8 0	6 8	5 6	4 5	3 4	2 3	1 3		
13	11 2	9 8	8 6	7 3	6 2	5 0	3 9	2 8	1 7		
14	12 0	10 6	9 2	8 0	6 7	5 6	4 4	3 3	2 2	1 1	
15	12 8	11 3	9 9	8 6	7 4	6 1	5 0	3 8	2 7	1 6	0 5
16	13 6	12 1	10 7	9 3	8 0	6 8	5 5	4 3	3 2	2 1	1 0
17	14 5	13 0	11 5	10 1	8 7	7 4	6 2	4 9	3 7	2 6	1 5
18	15 5	13 8	12 3	10 9	9 5	8 1	6 8	5 5	4 3	3 1	2 0
19	16 5	14 7	13 2	11 7	10 3	8 9	7 5	6 2	4 9	3 7	2 5
20	17 5	15 7	14 1	12 6	11 1	9 7	8 3	6 9	5 6	4 3	3 1

PART IV

ADDITIONAL EXERCISES IN HEAT

1. Determine the fixed points of the thermometer provided, and use it to find the melting point of the given solid.

2. Determine the temperature of the room by means of an ungraduated thermometer, ice and steam.

3. Standardise the given thermometer and use it to find at what temperature the given substance coagulates when heated.

4. Find the mean coefficient of expansion of the given liquid between 20° C. and 30° C., and also between 30° C. and 40° C.

5. Find the boiling point of a liquid, and determine the change in the boiling point produced by adding 10 per cent by weight of a solid.

6. Find the density of water at 20° C., 40° C., and 60° C., being given the coefficient of cubical expansion of glass.

7. Find the density of the given liquid at 20° C., 40° C., and 60° C. by means of a hydrostatic balance. Determine whether the coefficient of expansion between 20° C. and 40° C. is the same as that between 40° C. and 60° C.

8. Find the coefficient of apparent expansion of the given liquid, using a bulb of known volume to which is attached a tube of known diameter.

9. Find the temperature coefficient of increase of pressure of air at constant volume between the melting point and the boiling point of water.

10. Plot a graph showing how the pressure of the given volume of air varies with the temperature as indicated by a mercury thermometer.

11. Find the water equivalent of the given calorimeter.

12. Find the thermal capacity of the given mass of metal.

13. Find what would be the water equivalent of a calorimeter, weighing 150 grams, made of the given metal.

14. Find the specific heat of a liquid, being given that of a solid which has no chemical action upon it.

15. Find the specific heat of paraffin oil by adding ice.

16. Find the specific heat of the given liquid by condensing steam in it. Latent heat of steam = 540 calories per gram. Neglect heat of dilution.

17. Having been given a known weight of water in a calorimeter of known weight, condense steam in it, and find from thermometric observations the weight of steam condensed, assuming the latent heat of steam to be 537 calories per gram.

18. Heat the given vessel of water over a Bunsen burner and find the time taken for the temperature to rise from 40° C. to 80° C. Now boil the water for a measured time, and deduce an approximate value for the latent heat of steam.

19. Compare the rates of cooling at a given temperature of the blackened and silvered test-tubes when filled with warm water.

20. Plot a cooling curve for a calorimeter containing a known amount of water. Calculate the number of calories lost per second when the temperature of the calorimeter is 20° above that of its surroundings.

21. Find the mass of aqueous vapour in 1 litre of air in the room.

22. Determine the dew point by two different methods.

23. Being given the coefficient of thermal conductivity of the metal bar, determine the temperature at the marked point without using a thermometer at that point.

24. Determine the coefficient of thermal conductivity of a sheet of ebonite.

25. Compare the coefficients of thermal conductivity of ebonite and cardboard.

26. Find the coefficient of thermal conductivity of porcelain in the form of a tube.

PART V

MAGNETISM

CHAPTER I

FUNDAMENTAL PROPERTIES AND LAWS

§ 1. FUNDAMENTAL PROPERTIES AND DEFINITIONS

MAGNETS are characterised by their power of attracting small particles of iron, and by the property of setting in a definite direction when suspended so as to be able to turn freely. When a magnet can turn about a vertical axis, a certain direction fixed with regard to it becomes parallel to a direction fixed with regard to the earth. The first direction is that of the **Magnetic Axis** of the magnet, the second that of the **Magnetic Meridian.** A magnet of any shape usually behaves as though forces of attraction or repulsion originated from two points or regions in its substance, which may be termed its **poles.** The pole which points towards the north is called the **North** (or north-seeking) **Pole,** the other the **South** (or south-seeking) **Pole** of the magnet. **North** polarity is usually taken as **positive, South** polarity as **negative.** Unlike poles (poles of opposite sign) attract, like poles (poles of the same sign) repel one another.

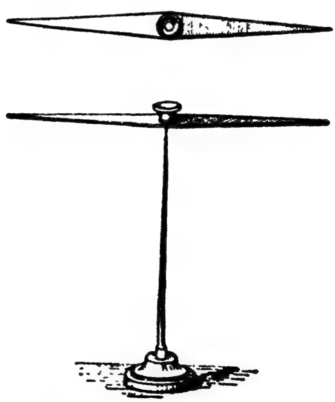


FIG. 194.—Magnetic Needle.

Definition of Unit Pole.—The unit magnetic pole is that pole

which repels an equal and similar pole at a distance of 1 cm. from it, in air, with a force of 1 dyne.

The strength of the magnetic field at any point is determined by the force in dynes acting on a unit north pole placed at that point. This is sometimes called the **magnetic force**[1] or the **magnetic intensity** at the point. The latter expression is not to be recommended as it is liable to be confused with intensity of magnetisation.

Note that **magnetic force** is a quantity of a different kind from a mechanical force; it is measured, not in dynes, but in **dynes per unit pole** or **gausses**.

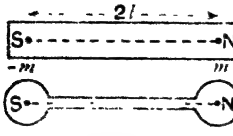


FIG. 195.—Bar Magnet and Ball-ended Magnet.

The **magnetic moment** M of a magnet is the moment of the couple required to hold the magnet with its axis at right angles to a magnetic field of unit intensity. It is numerically equal to the product of the pole strength m and the distance between the poles.

$$M = m \times NS.$$

§ 2. Plotting Magnetic Fields

A line of magnetic force is a line drawn in a magnetic field, such that its direction at any point of its length is the direction of the resultant magnetic force at that point, *i.e.* the tangent to the curve points in the direction in which a *small* magnet would set itself if placed at that point. The *positive* direction of a line of force is the direction in which a *north* pole would be urged. Lines of magnetic force may be regarded as starting from north magnetic poles and as ending at south magnetic poles. They are, however, regarded generally as passing completely through the substance of the magnet so as to form closed curves.

[1] Some writers object to the term 'magnetic force,' as there is a risk of its being confused with ordinary mechanical force. It has, however, the sanction of Maxwell's classical treatise, and is less cumbrous than 'strength of magnetic field.' If regarded as a compound expression, magnetic-force, it is easy to distinguish it from a mechanical force.

Lines of magnetic force in air may be traced experimentally in two different ways, by means of iron filings or by means of a compass needle.

EXPT. 184. **Plotting Magnetic Fields with Iron Filings.**—A sheet of glass is supported in a horizontal position on two strips of wood placed on the table, and one or more magnets are arranged below it. A sheet of paper is then placed on the glass, and fine iron filings are dusted on to the paper through a piece of muslin. If the glass is tapped gently, the filings will arrange themselves in the direction of the lines of force.

If a permanent record of the lines is desired, it can be secured by using a piece of paper which has been soaked in paraffin wax. On gently warming the glass plate the filings adhere to the paraffin. Another method is to obtain a photograph of the filings, using a camera pointing vertically downwards; or a print on 'blue' paper can be obtained easily by using a sensitised paper to receive the filings and afterwards exposing and developing in the usual way.

PLOTTING LINES OF MAGNETIC FORCE WITH A COMPASS NEEDLE

It is most instructive to plot the lines of magnetic force in various cases, using a small compass needle. The 'charm' compasses, in which both the top and bottom face are of glass, are most convenient for this purpose. Such compasses should be handled by the rim and not by the glass faces.

A large sheet of drawing paper should be fixed to a drawing board, and the latter placed so that one edge may be coincident with the edge of the table. By this means it may be replaced in the same position if it should be moved accidentally in the course of the experiment.

Place the compass on the paper, and when it has come to rest make a dot with a pencil opposite each end of the needle. Then shift the compass so that the S. pole lies just above the dot which was near the N. pole, and make another dot near the new position of the N. pole. Repeat this process, so that a row of dots is marked on the paper. Draw a freehand line passing through the row of dots. This will represent one line of mag-

netic force. Repeat the process about 2 cm. from this line and obtain a second line. In the same way draw a third line of magnetic force about 2 cm. from the second. If the magnetic field is due to the earth alone, the three lines thus obtained should be approximately straight and parallel to one another, since in the comparatively small space occupied by the drawing board the earth's field may be considered uniform.

If the field is plotted in the vicinity of any magnetic material or near to a conductor carrying a current, the field obtained is more complex in character than that just described. The lines of force obtained will represent the resultant field due to the superposition of the field of the earth and the field of the magnetic material or of the current. The lines of force will be curved in various ways, depending on the nature of the fields and on the manner in which they are superposed.

Lines of force starting close together will diverge considerably at points along their lengths, and may converge again where they re-enter the magnetic material. As the lines diverge the field is weakened, and some idea can be obtained of the relative intensity of the field at various points by noting the extent to which two lines, originally close together, have diverged by the time they have reached the points in question.

It is important to remember that **lines of magnetic force cannot cross one another**; for, if they did, the magnetic force at the point of intersection would be in two different directions at the same time. In general, there will be a line of force passing through any point chosen; there are, however, exceptional cases in which the lines of force seem to avoid certain points, and no line will be found to pass through these points. If there is no line of force passing through a point, the magnetic force at that point is zero. Such a point in a magnetic field is called a **neutral** or a **null point**.

In the neighbourhood of such a point the magnetic field is extremely weak, and it is therefore difficult to determine the direction in which the compass needle tends to point. Accordingly when a neutral point is suspected, the lines of force

should first of all be plotted at some distance from it, where the field is stronger, and afterwards the lines plotted should approach nearer and nearer to the neutral point. *It is impossible to find the neutral point by finding the place where the needle will point in any direction.*

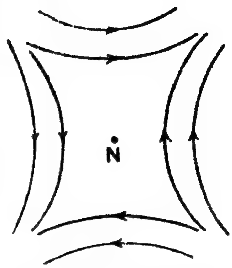


FIG. 180.—Neutral Point.

A *neutral* point is enclosed generally by four sets of lines of force, forming a curvilinear quadrilateral. By continually diminishing the size of the quadrilateral the position of the neutral point N can be found with considerable accuracy. It will be shown later how information of importance can be obtained by finding the position of such a point in certain particular cases.

EXPT. 185. **Plotting the Lines of Magnetic Force due to the Earth's Field.**—To find the character of the lines of magnetic force due to the earth's field, select a position at a distance from iron girders, pipes, or stoves, and remove all magnets, or masses of iron, from the neighbourhood. Starting from one edge of the paper, obtain a row of dots as described above. In the absence of disturbing magnets these dots should lie nearly on a straight line. Starting again about 2 cm. from this line, repeat the process and obtain a second line of force. Draw about half a dozen lines in this way, and verify the fact that the field is approximately uniform by showing that the lines are nearly straight and parallel. The direction thus obtained is the direction of the magnetic meridian at the place where the experiment is carried out.

EXPT. 186. **Plotting the Lines of Magnetic Force due to a Bar Magnet and the Earth together.**—Place a magnet in any position on a drawing board covered with paper, and plot round it the field due to the magnet and the earth's magnetism together. Before beginning to trace the lines of force, mark the position of the magnet on the paper so that it can be put back in its proper position if accidentally displaced. Some judgment is required in choosing the starting-points for the separate lines so that they may be neither too widely separated nor too closely packed together. Wherever two lines are converging at a small angle to a pair of points close together,

it is unnecessary to plot a third line between them in view of the fact that lines of force cannot cross.

In general, *two* neutral points will be found in the field near to a bar magnet, for there are two points at which the magnetic field due to the magnet exactly counterbalances the earth's field. The position of these two points relative to the magnet depends on the position of the magnet relative to the earth's field, and if possible the field of the *same* magnet should be plotted with the magnet in various positions. Cases of special interest arise when the magnet occupies a symmetrical position with regard to the magnetic meridian, *i.e.* when it is perpendicular or parallel to the lines of force due to the earth's field. It is suggested that the field should be plotted in each of these positions, and also in one or two positions where the magnet lies unsymmetrically across the earth's lines of force.

FIELD DUE TO A SINGLE POLE IN THE EARTH'S FIELD

It is sometimes desirable to be able to experiment with a single magnetic pole, and for this purpose it is convenient to make use of a very long (50 to 100 cm.) ball-ended magnet, so that the second pole may be placed so far away from the place where the test is being made that its effect may be neglected.

EXPT. 187. **Field due to a Single Pole in the Earth's Field.** —For the present experiment, support the magnet in a stand of wood with its axis vertical. Let the lower pole rest on a sheet of drawing paper fixed to a horizontal drawing board. Plot the lines of force due to the combined action of this pole and the horizontal component of the earth's magnetic field. Determine carefully the position of the neutral point, and measure the distance (r cm.) between this point and the pole.

Since the magnetic force due to a single pole varies inversely as the square of the distance from the pole, the magnetic force due to the pole of strength $m = m/r^2$. But at the neutral point this is equal to H, the horizontal component of the earth's field. Thus $m/r^2 = H$, or $m = Hr^2$. Assuming H to be known, m can be calculated. In London, H may be taken as 0·185 C.G.S. units.

FIELD DUE TO A BAR MAGNET IN THE EARTH'S FIELD

Position I.—The magnet is placed on a horizontal surface with its axis in the magnetic meridian and its *north* pole

pointing to the *north*. In this position there is a *neutral point* on either side of the magnetic axis, where the horizontal component due to the earth's magnetism is balanced exactly by the magnetic force of the magnet.

If the bar is magnetised uniformly its poles will be equidistant from the centre. The poles of a bar magnet are not at the extreme ends of the bar. Their positions must be found experimentally by drawing the lines of force near the ends of

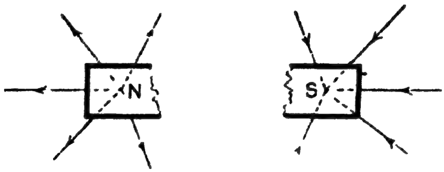


FIG. 197.—Poles of Bar Magnet.

the bar and determining the points where the directions of the lines approximately meet (Fig. 197). The neutral points will lie in a line drawn through the centre, midway between the two poles at right angles to the length.

Let P represent the position of a neutral point at a distance d cm. from either pole (Fig. 198).

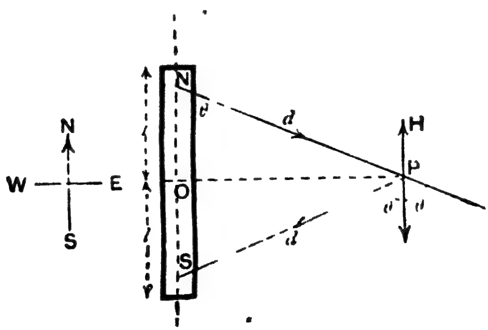


FIG. 198.—Neutral Point in the Earth's Field.

The magnetic force at P, due to a pole m at N, is equal to m/d^2 along NP. The force due to a pole $-m$ at S is equal to m/d^2

along PS. The resultant of these two forces is evidently perpendicular to OP, and its magnitude is

$$F = 2\frac{m}{d^2}\cos\theta = 2\frac{m}{d^2}\frac{l}{d} = \frac{M}{d^3},$$

since M, the magnetic moment $= 2ml$. But P is a 'neutral point'; therefore at P

$$F = H,$$

the horizontal intensity of the earth's field;

$$\therefore \frac{M}{d^3} = H,$$

or

$$M = Hd^3.$$

EXPT. 188. **Determination of the Magnetic Moment of a Bar Magnet, Neutral Point Method I.**— Plot the lines of force due to a bar magnet by means of a small compass, when the magnet is placed in the magnetic meridian with its N. pole pointing towards the north. Determine the positions of the neutral points as accurately as possible, and measure the distances from the poles. Calculate the magnetic moment of the magnet from the equation

$$M = Hd^3.$$

In London, H may be taken as 0·185 C.G.S. units; d must be in centimetres.

Measure the distance between the two poles, and deduce the pole strength m of the magnet.

Position II.—The magnet is placed in the magnetic meridian with its N. pole pointing towards the south. Two neutral points will be found somewhere on the axis of the magnet produced. If the mean distance of a neutral point from the *centre* of the magnet be r, the magnetic force due to the magnet is in this case $F = \frac{2M}{r^3}$, approximately (p. 415). Consequently at a neutral point

$$\frac{2M}{r^3} = H,$$

and therefore

$$M = \frac{Hr^3}{2}.$$

EXPT. 189. **Determination of the Magnetic Moment of a Bar Magnet, Neutral Point Method II.**—Place a bar magnet with its axis in the magnetic meridian, and its N. pole pointing towards the south. Plot the lines of magnetic force, using a small compass, and find the positions of the neutral points. Measure the distance from each neutral point to the *centre* of the magnet, and deduce the value of the magnetic moment.

Position III.—The magnet is placed in any position in the magnetic field of the earth. Two neutral points will be found symmetrically situated with regard to the centre of the magnet. The resultant magnetic field may be regarded as due to three forces: one due to the earth, which is supposed known in direction and magnitude, and two due to the magnetic poles in the direction of the lines joining the point to the poles. If the point in question is a neutral point, these three forces must be in equilibrium. By resolving the forces in a direction parallel to the earth's field we can obtain an expression for the strength of a magnetic pole in terms of distances and angles which can all be measured from the diagram.

If the magnet lies in some such position as is shown in Fig. 199 (the axis east and west is convenient), the neutral points will be found as at A and B.

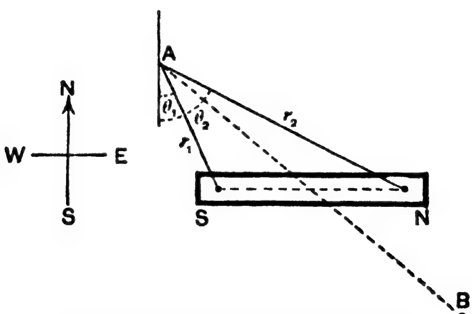


FIG. 199.—Neutral Points in the Earth's Field.

Draw a line through A pointing towards the magnetic north. Join AS(r_1) and AN(r_2). If the angles made by AS and AN with the magnetic meridian at A are θ_1 and θ_2 respectively, and m is the pole strength—

$$F = \frac{m}{r_1^2}\cos\theta_1 - \frac{m}{r_2^2}\cos\theta_2,$$

and $$F = H = 0{\cdot}185 \text{ (in London)}.$$

By measuring r_1 and r_2, θ_1 and θ_2, m can be calculated.

The proof of this expression is left as an exercise for the student.

Expt. 190. **Determination of the Magnetic Moment of a Bar Magnet, Neutral Point Method III.**—Place a bar magnet with its axis east and west. Plot the lines of magnetic force, and find the positions of the neutral points as accurately as possible. Measure r_1 and r_2, θ_1 and θ_2, for *each* neutral point, and deduce the value of m, the pole strength in each case.

§ 3. Magnetic Axis and Magnetic Meridian

When a magnet is suspended so that it can turn freely about a vertical axis, a certain direction fixed with regard to the magnet tends to become parallel to a certain direction fixed with regard to the earth. The direction fixed relatively to the magnet is the direction of the **magnetic axis of the magnet**; the direction fixed relatively to the earth is the direction of the **magnetic meridian.** In the case of a long, thin magnet, the magnetic axis may be taken to correspond with the direction of the length of the magnet, but in the case of a broader magnet—an ordinary *bar* magnet—it is not allowable to assume that the direction of the magnetic axis coincides with the direction of symmetry. The following experiment illustrates the method used in magnetic observatories for determining the magnetic axis of a magnet and the magnetic meridian.

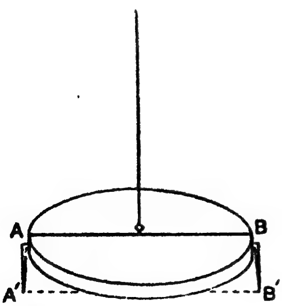


Fig. 200.—Magnetised Disk.

In order to take an extreme case, we construct a magnetised disk whose magnetic axis is quite unknown. To do this, a light bar magnet is enclosed in a flat circular wooden box, so that its position is concealed entirely. A reference line is traced on each flat face of the box, joining two points, A and B, which are at opposite ends of a diameter (Fig.

200). The problem is to find the angle between the axis of the concealed magnet (or of the whole disk) and this reference line.

Expt. 191. Determination of the Magnetic Axis of a Magnet and the Magnetic Meridian at any Place.—The magnetised disk is suspended from the centre of one face by a fine thread, which should be as free from torsion as possible. If the thread is not torsionless there will be a couple acting on the disk due to the torsion of the suspension, and the position of rest of the disk will be determined by the action of this couple in addition to the magnetic couple due to the earth's field. A sheet of paper is *fixed* horizontally, immediately below but not in contact with the disk. The position of the reference line when the disk has come to rest must be marked on the paper. It is not necessary, however, to wait till the plate comes to rest of its own accord. Notice the positions of the extremities of its swings, and then gently check the motion halfway between these two positions. In order to mark the position of rest of the reference line accurately on the paper, it is convenient to have a metal pointer fixed to the circumference of the plate at each end of the line. A line such as A′B′ is thus obtained on the paper.

The plate should then be turned upside down and suspended from the opposite face, and the new position of the reference line determined; let this be A″B″ (Fig. 201).

No lines should be drawn on the plate itself.

In this way two lines, A′B′, A″B″, are obtained on the paper, these lines being inclined to one another at a definite angle. A little consideration will show that the direction of the magnetic axis must bisect the angle between the two lines, for the magnetic axis coincides with the fixed meridian, and the reference line must be at the same angle with the meridian on one side in the first case, as it is on the other side in the second case.

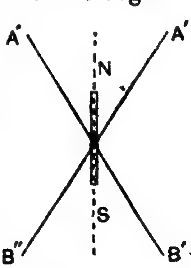


Fig. 201.—Magnetic Meridian.

As there are two bisectors between the lines AA′ and BB′ it is necessary to determine which of the two coincides with the meridian.

The points A′ and A″ were made by the same pointer A, one for each position of the plate. Similarly B′ and B″ were both made by the pointer at B on the plate. Thus in reversing the plate and allowing it to take up its second position of equilibrium, it has virtually been rotated about the diameter passing midway A′ and A″,

and B′ and B″. Hence the line marked NS in the diagram bisecting A′OA″ and B′OB″ is the magnetic axis of the plate and the magnetic meridian.

Measure with a protractor the angle between the line on the paper representing the magnetic axis (or the magnetic meridian) and the reference line A′B′.

Measure also the angle between the magnetic meridian and some line fixed in the room such as the edge of a laboratory table.

§ 4. Tractive Force for a Bar Magnet

Measurements of the tractive force at different points in the length of a magnet were made by Coulomb. He observed the weights of iron which could be supported at the points. His results may be represented by a curve of the form shown in Fig. 202, in which the ordinates are proportional to the force at different points in the length. If the distribution of magnetism is uniform the curve is symmetrical about C, the centre of the magnet.

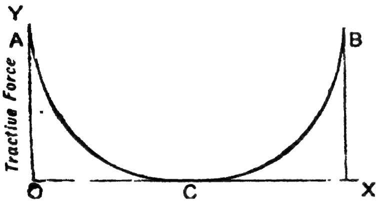


Fig. 202.—Tractive Force of Bar Magnet.

Expt. 192. **Determination of the Distribution of Tractive Force along a Bar Magnet.**—Mark off the magnet into, say, 10 equal parts, and place it on a levelling table beneath the scale-pan of a 'hydrostatic' balance. From the hook of the scale-pan suspend a small iron ball. Adjust the levelling table so that the pointer of the balance is near the middle of the scale when the ball is in contact with the magnet, and then find the weight that must be placed in the other scale-pan in order to separate the ball from the magnet. Repeat the observation with the ball at different points marked in the length of the magnet.

The attraction depends largely on the closeness of contact between the ball and the magnet; any grease or dirt between them may diminish the force by several grams-weight. The ball may be rubbed slightly across the breadth of the magnet in putting it into position, thus removing any dirt and ensuring greater consistency in the observations.

In order to prevent injury to the balance when the iron ball

leaves the magnet, the weights must be added very carefully, and wooden blocks must be placed under the scale-pan containing them, so as to limit the movement of the balance.

Determine the weight of the ball when the magnet is not near it. This weight must be subtracted from the weights

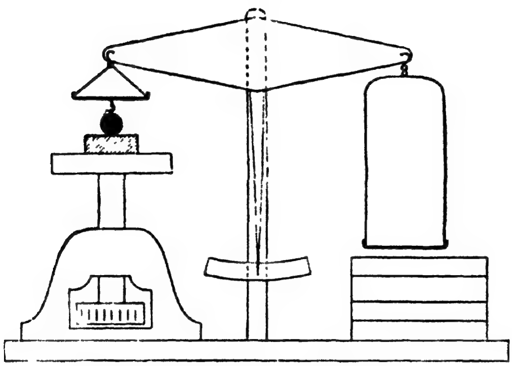


FIG. 203.—Determination of Tractive Force.

observed in the previous observations, so as to give the force due to the attraction of the magnet.

Plot a curve showing the attraction at different points in the length of the magnet.

Instead of using a hydrostatic balance in this experiment, a spring balance may be employed. In this case the levelling table may be lowered, or, keeping the magnet fixed, the spring balance may be raised gently till the ball leaves the magnet. The reading of the spring balance must be taken just as the separation occurs.

CHAPTER II

MAGNETOMETRY

§ 1. The Deflection Magnetometer

In its simplest form the **magnetometer** consists of a pivoted or suspended magnetic needle, free to turn about a vertical axis, and provided with a circular graduated scale for measuring deflection. The needle and circular scale are contained usually in a case of wood or of brass, provided with a glass top, through which the

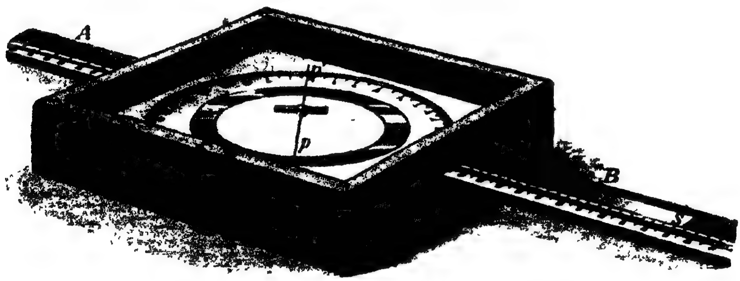


Fig. 204.—Deflection Magnetometer.

reading can be taken. A long, light pointer pp' is fitted to the needle, so that the movement of the needle may be measured over a circle of large diameter although the needle itself is small: the circle is usually large enough to give readings accurate to about 1° of angle. In order to avoid parallax in reading the position of the pointer, the base of the magnetometer is provided generally with a plane mirror. In taking a reading, the eye of the observer must be placed in such a position that the reflection

of the pointer is hidden by the pointer itself; the observer will then be looking straight down on the scale, and the true reading will be obtained.

For very accurate work a **mirror magnetometer** is used in conjunction with a lamp and scale. In this form there is a mirror fixed to the needle, a beam of light from the lamp strikes this mirror, and the motion of the reflected beam over the scale is used to measure the deflection of the needle. The beam of light serves as a long weightless pointer, the angle it turns through being *twice* the deflection of the needle (p. 232).

The magnetometer is usually set up so that the zero reading is obtained when the needle is under the influence only of the horizontal component H of the earth's magnetic field. A second field of strength F, in a direction at right angles to H, is then applied to the needle, say by means of a magnet placed in the neighbourhood of the instrument. The pointer is thus deflected through an angle θ. By measuring this angle we can determine the relation between F and H. For this purpose the following important theorem is required:—

When a magnet is placed in a magnetic field due to the superposition of **two mutually perpendicular magnetic fields** both of which are uniform, its position is determined by the relation

$$\frac{F}{H} = \tan\theta,$$

where F and H are the strengths of the magnetic fields and θ is the angle between the axis of the magnet and the direction of the field H.

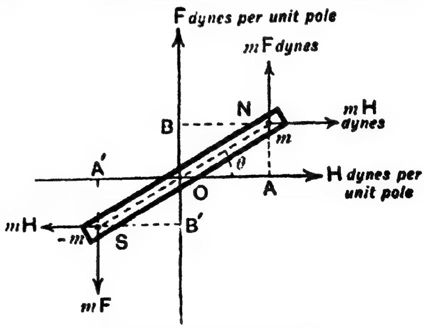


Fig. 205.—Proof that F = H tan θ.

Let NS (Fig. 205) represent the magnet, whose pole strength is *m*. The N. pole is under the action of two forces, viz. *m*H dynes parallel to H, and *m*F dynes parallel to F. The S pole is under the action

of equal forces in the contrary sense. These forces constitute couples which keep the magnet in a position of equilibrium.

Taking moments about the centre O of the magnet we get

$$mF \times OA = mH \times OB,$$

or

$$\frac{F}{H} = \frac{OB}{OA} = \frac{AN}{OA}$$

$$= \tan \theta,$$

and consequently $F = H \tan \theta$.

Thus if we know the ratio of F to H, we can calculate the angle θ; and if we know the strength of the field H and can observe the angle θ, we can determine the strength of the field F.

In most experiments with the magnetometer the field F is only approximately uniform. For this reason it is important that the needle of the magnetometer should not be too long. If the needle is short only a small error is introduced by treating the field F as a uniform field.

§ 2. Comparison of Magnetic Fields by the Magnetometer

Expt. 193. **Field due to a Single Pole.**—Determine the direction of the magnetic meridian by means of the magnetometer needle. Place a metre scale on the table at right angles to the meridian, and put the magnetometer box on the scale so that the centre of the box is directly above the central division of the scale. Carefully adjust the box and the metre scale so that the pointer is at the zero division, and the scale points exactly east and west (magnetically). In some forms of magnetometer the metre scale is fitted permanently (AB, Fig. 204).

In this experiment the long ball-ended magnet described previously (p. 398) should be used. As the intention is to examine the effect of one pole only, the second pole must be placed in such a position that it will have no effect on the reading of the magnetometer. This can be done by supporting the magnet vertically in a wooden stand.

The upper pole of the magnet is at a considerable distance (approximately 1 metre) from the magnetometer, while the lower pole is seldom used at a distance greater than 20 cm. from the magnetometer. In the extreme case, therefore, the magnetic force due to the upper pole is not greater than 4 per cent of the force due

to the lower pole. By placing the magnet vertically, the horizontal component of the force due to the upper pole at the magnetometer is reduced to about $\frac{1}{8}$th of the total force due to it when the distance of the magnetometer is greatest.

The extreme value of the horizontal force due to the upper pole is thus reduced to less than 1 per cent of the force due to the lower pole, an error which is much below the probable errors of reading.

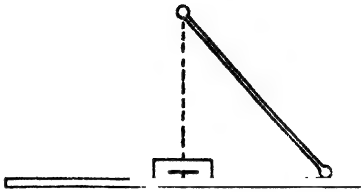


FIG. 206.—Field due to Single Pole.

If the magnet is tilted somewhat so as to bring the upper pole over the middle of the magnetometer (Fig. 206), any horizontal force due to it may be quite eliminated, though this is not absolutely necessary, as will be seen from the above.

The first pole of the magnet must be on the metre scale, so that its field at the magnetometer lies east and west. In this position it will produce a magnetic force $F = m/r^2$ at the centre of the magnetometer, if m is the strength of the pole and r its distance from that centre. Consequently a deflection θ will be produced, given by $F = H \tan \theta$, or $m/r^2 = H \tan \theta$. Hence $r^2 \tan \theta = m/H$ = a constant for the particular strength of pole under test.

Consequently, if we take a series of readings of r and θ, reading *both* ends of the pointer, we should find that $r^2 \tan \theta$ is constant.

Tabulate the results under the headings: r, θ, $\tan \theta$, $r^2 \tan \theta$.

If the numbers in the last column of the table are approximately constant, the result may be regarded as a verification of the law that **the magnetic force due to a single pole varies inversely as the square of the distance from the pole.**

EXPT. 194. **Field due to a Bar Magnet.**—Place the magnet, which should be a short bar magnet strongly magnetised, on the metre scale with its axis pointing east and west. In this 'end-on' position it will produce a magnetic field at the magnetometer pointing east and west, and of the approximate strength $F = 2M/r^3$, where M is its magnetic moment and r is the distance from the *centre* of the magnetometer to the *centre* of the magnet. *This is only true provided the length of the magnet is small in comparison with the distance r.*

The magnetometer needle will be deflected through an

angle θ given by $F = H \tan \theta$, so that $2M/r^3 = H \tan \theta$. Hence $r^3 \tan \theta = 2M/H =$ a constant for the particular magnet under test.

Note the distance r and read the deflection θ for both ends of the pointer. Now turn the magnet end for end, keeping its centre at the same point, and take two more readings. Take the mean of these readings as the true deflection. Tabulate the results under the headings: r, θ, $\tan \theta$, $r^3 \tan \theta$. The values obtained in the last column will be approximately constant.

This shows that the strength of the field along the axis of a short bar magnet varies inversely as the cube of the distance from the centre of the magnet.

§ 3. Comparison of Magnetic Moments by the Magnetometer. Elementary Treatment

In the first instance an elementary discussion of the comparison of magnetic moments is given, on the assumption that the lengths of the magnets compared are small enough to be neglected in comparison with the distance from a magnet to the magnetometer.

Expt. 195. **Comparison of Magnetic Moments, using the 'End-on' (or A) Position.**—Place the magnetometer box on a metre scale laid flat on the table, so that the centre of the magnetometer coincides with the centre of the scale, the zero line of the magnetometer pointing exactly along the length of the metre scale. Turn the scale round till it points east and west, as judged by the magnetic needle of the magnetometer.

(i.) **Method of Tangents, or Method of Equal Distances.**—Place the first magnet (magnetic moment M_1) with its centre at a definite division on the metre scale and with its axis pointing east and west. The distance from the magnetometer must be large compared with the length of the magnet, but must not be so large as to give a very small deflection of the needle. A deflection anywhere between 15° and 55° would be suitable. Take readings

of both ends of the needle, taking care to avoid errors due to parallax. Reverse the magnet, end for end, keeping its centre at the same graduation as before, and again read the position of the needle.

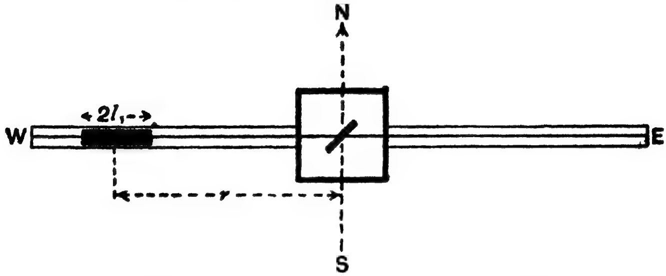


FIG. 207.—'End-on' Position. Method of Tangents.

Now repeat the observations with the magnet on the other side of the magnetometer at the same distance from it. Let the mean of all the readings be θ_1.

Take readings in exactly the same way with the second magnet (magnetic moment M_2), placing the centre of the magnet in the positions occupied by the centre of the first magnet. Let the mean of all these readings be θ_2.

Then, approximately,

$$\frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2}.$$

For $F_1 = \frac{2M_1}{r^3}$, $F_2 = \frac{2M_2}{r^3}$, and $F_1 = H\tan\theta_1$, and $F_2 = H\tan\theta_2$,

$$\therefore \frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2}.$$

(ii.) **Method of no Deflection.**—In this method the two magnets are used at the same time, one to the east, the other to the west of the magnetometer, and their distances from the magnetometer are adjusted till there is no deflection of the needle. Measure the distances r_1, r_2, from the centre of the magnetometer to the centres of the magnets.

Reverse the magnets, keeping r_1 unaltered, and readjust the second magnet till no deflection is given: r_2 may now

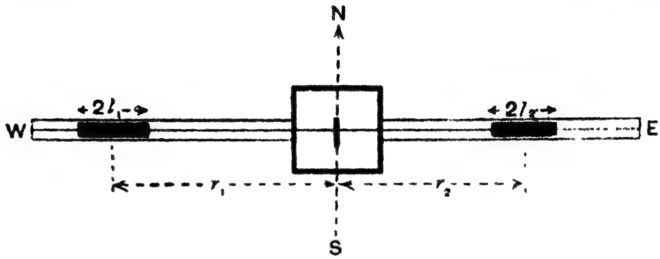


FIG. 208.—'End-on' Position. Method of no Deflection.

have a slightly different value. Take the mean of the two values of r_2.

Then, approximately,

$$\frac{M_1}{M_2} = \frac{r_1^3}{r_2^3}.$$

For $F_1 = \frac{2M_1}{r_1^3}$, and $F_2 = \frac{2M_2}{r_2^3}$, and since the deflection is zero $F_1 = F_2$,

$$\therefore \frac{M_1}{M_2} = \frac{r_1^3}{r_2^3}.$$

EXPT. 196. **Comparison of Magnetic Moments, using the 'Broadside-on' (or B) Position.**—Turn the metre scale till it lies in the magnetic meridian, the magnetometer box being still at the centre of the scale. In order that the pointer may still point to the zero of the circular scale the box also must be turned through a right angle on the metre scale.

(i.) **Method of Tangents, or Method of Equal Distances.**—Place the first magnet on the metre scale with its axis pointing east and west, and read the position of the pointer. Reverse the magnet end for end and again take readings.

Repeat the observations with the magnet on the other

side of the magnetometer at the same distance from it. Let the mean of all the readings be θ_1.

Take readings in the same way with the second magnet at the same distance from the magnetometer, and let the mean be θ_2.

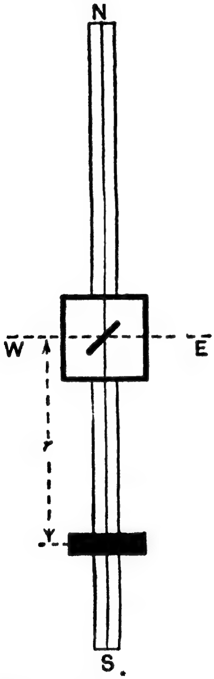


FIG. 209.—'Broadside-on' Position. Method of Tangents.

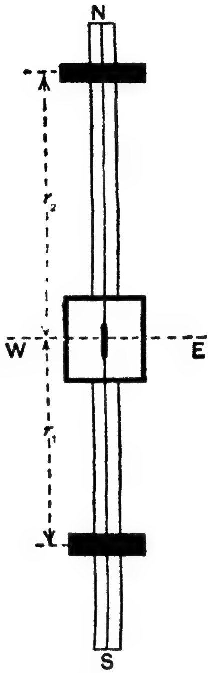


FIG. 210.—'Broadside-on' Position. Method of no Deflection.

Then, approximately,

$$\frac{M_1}{M_2} = \frac{\tan \theta_1}{\tan \theta_2}.$$

For $F_1 = \frac{M_1}{r^3}$, and $F_2 = \frac{M_2}{r^3}$, and $F_1 = H \tan \theta_1$, and $F_2 = H \tan \theta_2$,

$$\therefore \frac{M_1}{M_2} = \frac{\tan \theta_1}{\tan \theta_2}.$$

(ii.) **Method of no Deflection.**—One magnet is placed to the north, the other to the south of the magnetometer, and their distances, r_1, r_2, from the magnetometer are adjusted till there is no deflection of the needle (Fig. 210). Reverse the magnets, keeping r_1 constant, and take the mean of the values of r_2. Both magnets must be pointing east and west.

Then, approximately,

$$\frac{M_1}{M_2} = \frac{r_1^3}{r_2^3}.$$

For F_1 and F_2 are equal, and

$$F_1 = \frac{M_1}{r_1^3}, \quad F_2 = \frac{M_2}{r_2^3},$$

$$\therefore \frac{M_1}{r_1^3} = \frac{M_2}{r_2^3}.$$

Thus there are in all **four different methods** of making the comparison, **two** in the '**end-on**,' **two** in the '**broadside-on**' position. **In all cases the axes of the magnets under examination point east and west.**

§ 4. Comparison of Magnetic Moments by the Magnetometer. More Advanced Treatment

A. Magnetic Force at a Point on the Axis of the Magnet—'End-on' Position.—The magnetic moment of a magnet length

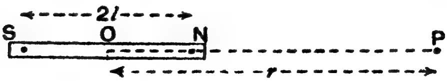


Fig. 211.—'End on' Position.

$2l$, poles $+m$ and $-m$, is $2lm = M$. The force at P on unit north magnetic pole is

$$\frac{m}{NP^2} = \frac{m}{(r-l)^2},$$

repulsion from N,

and $$\frac{m}{SP^2} = \frac{m}{(r+l)^2},$$

attraction towards S.

The resultant magnetic force at P is

$$F = \frac{m}{(r-l)^2} - \frac{m}{(r+l)^2}$$

$$= \frac{m}{(r^2-l^2)^2}\{(r+l)^2 - (r-l)^2\}$$

$$= \frac{m4rl}{(r^2-l^2)^2} = \frac{2Mr}{(r^2-l^2)^2}.$$

When r is large compared with l the term in l^2 may be neglected, and the expression becomes

$$F = \frac{2M}{r^3} \text{ (approximately).}$$

B. Magnetic Force at a Point in the Equatorial Plane of the Magnet—'Broadside-on' Position.—In this case the point P is in a line bisecting the magnet at right angles (Fig. 212). The components of the magnetic force at P are m/NP^2 along NP and m/SP^2 along PS.

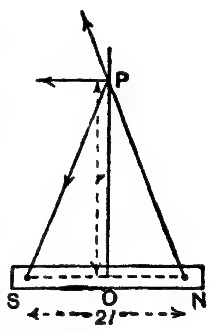


FIG. 212.—'Broadside-on' Position.

Each of these components, which are equal in magnitude, may be resolved into forces along and perpendicular to OP. The forces in the direction OP balance one another. The forces at right angles to OP give

$$F = \frac{m}{NP^2}\cos PNO + \frac{m}{SP^2}\cos PSO = \frac{2m}{NP^2}\frac{ON}{NP} = \frac{2ml}{NP^3} = \frac{M}{(r^2+l^2)^{\frac{3}{2}}}$$

or $$F = \frac{M}{r^3}$$

approximately, when r is large compared with l.

We compare the magnetic force F due to the magnet with

the horizontal force H of the earth by the expression $F = H \tan \theta$ (p. 407). We assume that the needle of the magnetometer is so small that the field in its neighbourhood may be considered uniform.

The foregoing results provide four different methods for comparing the magnetic moments of two magnets by means of the magnetometer. For further experimental details see the elementary treatment (p. 410).

EXPT. 197. **Comparison of Magnetic Moments, using the 'End-on' Position** (corresponding with Expt. 195).

(i.) **Method of Tangents, or Method of Equal Distances.**—Place each magnet in turn at the same distance r from the magnetometer, arranging the magnet so that its axis passes through the centre of the needle, and is at right angles to the magnetic meridian. The deflecting magnet must be placed with its axis pointing east and west. When the latter adjustment is secured the magnet produces the maximum deflection of the needle.

Observe the deflections θ_1 and θ_2 of the pointer in the two cases.

Then $F_1 = \dfrac{2M_1 r_1}{(r_1^2 - l_1^2)^2}$, and $F_2 = \dfrac{2M_2 r_2}{(r_2^2 - l_2^2)^2}$, and in this case $r_1 = r_2 = r$ (say).

Hence
$$\frac{F_1}{F_2} = \frac{2M_1 r/(r^2 - l_1^2)^2}{2M_2 r/(r^2 - l_2^2)^2} = \frac{H \tan \theta_1}{H \tan \theta_2},$$
$$\frac{M_1}{M_2} = \frac{(r^2 - l_1^2)^2}{(r^2 - l_2^2)^2} \times \frac{\tan \theta_1}{\tan \theta_2}.$$

If the magnets are of approximately the same length, then
$$\frac{M_1}{M_2} = \frac{\tan \theta_1}{\tan \theta_2}.$$

Note that r is the distance from the *centre* of the deflecting magnet to the centre of the magnetometer needle.

(ii.) **Method of no Deflection.**—Arrange the magnets in positions similar to the above, but one on either side of the needle. Adjust their distances till there is no deflection of the magnetometer needle.

If r_1 and r_2 are the distances, we have made $F_1 = F_2$,

$$\frac{2M_1 r_1}{(r_1^2 - l_1^2)^2} = \frac{2M_2 r_2}{(r_2^2 - l_2^2)^2}, \text{ i.e. } \frac{M_1}{M_2} = \frac{(r_1^2 - l_1^2)^2}{(r_2^2 - l_2^2)^2}\frac{r_2}{r_1}.$$

If r_1^2 and r_2^2 are both large compared with l^2 this reduces to

$$\frac{M_1}{M_2} = \frac{r_1^3}{r_2^3}.$$

EXPT. 198. **Comparison of Magnetic Moments using the 'Broadside-on' Position** (corresponding with Expt. 196).

(i.) **Method of Tangents, or Method of Equal Distances.**—Place each magnet in turn at the same distance r from the centre of the needle and observe the deflection produced in each case. Note again that the deflecting magnets must be placed with their axes pointing east and west.

Then, generally,

$$F_1 = \frac{M_1}{(r_1^2 + l_1^2)^{\frac{3}{2}}}, \text{ and } F_2 = \frac{M_2}{(r_2^2 + l_2^2)^{\frac{3}{2}}}.$$

In this case $r_1 = r_2 = r$ (say).

$$\frac{F_1}{F_2} = \frac{M_1 (r^2 + l_1^2)^{\frac{3}{2}}}{M_2 (r^2 + l_2^2)^{\frac{3}{2}}} = \frac{H \tan \theta_1}{H \tan \theta_2},$$

$$\frac{M_1}{M_2} = \frac{(r^2 + l_1^2)^{\frac{3}{2}} \tan\theta_1}{(r^2 + l_2^2)^{\frac{3}{2}} \tan\theta_2}.$$

If the magnets are of approximately the same length,

$$\frac{M_1}{M_2} = \frac{\tan \theta_1}{\tan \theta_2}.$$

(ii.) **Method of no Deflection.**—Place the magnets one on either side of the needle with their centres north and south of it, and adjust their distances until the magnetometer needle remains in the magnetic meridian.

If r_1 and r_2 are the distances of the magnets from the needle,

$$\frac{M_1}{M_2} = \frac{(r_1^2 + l_1^2)^{\frac{3}{2}}}{(r_2^2 + l_2^2)^{\frac{3}{2}}}.$$

If l_1 and l_2 are both small in comparison with r, this reduces to

$$\frac{M_1}{M_2} = \frac{r_1^3}{r_2^3}.$$

Note on making Observations of θ and of r.—If the deflecting magnet is not magnetised uniformly, its magnetic

equator is nearer to one end than the other. The true distance r is therefore not the distance from the centre of the needle to the centre of the bar. Again, if the pivot of the needle is not exactly in the centre of the magnetometer box, the value taken for r will be wrong on this account also. To avoid errors due to these two causes, the deflection should be taken first with the magnet one way round, then with the magnet reversed. After this the magnet should be placed on the other side of the magnetometer at an equal distance r, and the needle deflection determined again for both positions of the magnet on this side. Each time the two ends of the pointer must be read so that eight readings of θ are taken; the mean of these is taken as the true deflection θ.

In the method of no deflection one magnet must always be taken at the same distance r_1. After taking r_2 to correspond with r_1, both magnets must be reversed, when a new value of the second distance r_2 may be found to be necessary in order to get zero deflection. The magnets are then placed on the other sides of the magnetometer, and two more values of r_2 are found to give zero deflection when the first magnet is at a distance r_1. The mean of the four values of r_2 is used in the calculations.

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CHAPTER III

THE OSCILLATIONS OF A MAGNET IN A MAGNETIC FIELD

§ 1. Comparison of Magnetic Fields by Oscillations

When a magnet is suspended so that it can oscillate about an axis of symmetry in a uniform magnetic field the time of one complete vibration is given by the formula (deduced on the assumption that the motion is approximately Simple Harmonic Motion, p. 161),

$$T = 2\pi\sqrt{\frac{I}{MH}},$$

where T is the periodic time,

I is the moment of inertia about the chosen axis,

M is the magnetic moment,

H is the strength of the magnetic field.

If the *same* magnet be set in oscillation at various points of a magnetic field, I and M remain constant, but T and H vary. The formula shows that

$$HT^2 = \frac{4\pi^2 I}{M}$$

$$= C \text{ (where C is a constant).}$$

Thus if the value of the constant C be determined once for all by making the magnet oscillate in a magnetic field of *known* strength, the strength of any other field may be found by finding the value of T in that field.

EXPT. 199. **Determination of the Strength of the Field at any Point assuming the Earth's Field known.**—The oscillating needle used in this experiment is a small steel magnet only about 2 cm. in length mounted horizontally in a small brass stem suspended by a single silk fibre. A heavy short piece of 'rat-tail' file is suitable as the needle. A light aluminium pointer may be attached to the stem in order to make it easier to observe the oscillations. The apparatus should be protected from air currents by a glass shade. Place the apparatus on a table remote from masses of iron such as

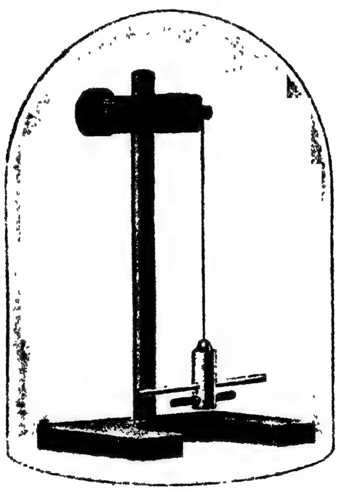

FIG. 213.—Searle's Oscillating Needle.

iron pipes, stoves, etc. Remove all other magnets, including knives and keys, from the neighbourhood. Set the needle oscillating by bringing a magnet momentarily near to it, but do not allow the amplitude of the oscillations to exceed a few degrees, as the motion approximates to Simple Harmonic Motion only when the amplitude is small. Observe the time taken for the needle to complete a number of complete oscillations (40 or 50 should be observed if possible), and calculate the time T_0 for one complete oscillation.

Assuming H_0, the horizontal component of the earth's magnetic field, to be known, find the value of the constant C by means of the equation

$$H_0 T_0^2 = C.$$

The object of the experiment is to determine H, the horizontal field at some other place. Move the apparatus to the spot where the strength of the field is to be measured, and again observe the time T of an oscillation. Using the value of C just found, determine the value of H from $HT^2 = C$. In this way a magnetic survey of the laboratory can be carried out.

ELIMINATION OF THE EFFECT OF THE EARTH'S FIELD

It is frequently necessary to compare two fields neither of which is known. This could be done as already described if the two fields could be isolated, but in general this cannot be done, and the needle would have to oscillate in a field compounded of the earth's field and one or other of the fields to be compared.

If one of the fields (F) is arranged *parallel to* the earth's field (H_0) the resultant field (H) will be either the sum or the difference of F and H_0: the needle can now be allowed to oscillate in this compound field and its period of swing (T) determined. It is important to note that greater accuracy is obtained if the composite field is made the *sum* of F and H_0. If possible, therefore, the field F should be arranged so as to assist the earth's field, so that the needle oscillates more rapidly than in the earth's field alone, *and still points in the same direction as when in the earth's field.*

If T_0, the period of swing in the earth's field, be known we have, from the fundamental equation,

$$H_0 = \frac{C}{T_0^2},$$

and

$$H = \frac{C}{T^2}.$$

But

$$H = H_0 + F,$$

or

$$F = H - H_0.$$

Therefore

$$F = \frac{C}{T^2} - \frac{C}{T_0^2},$$

or

$$F = C\left\{\frac{1}{T^2} - \frac{1}{T_0^2}\right\}.$$

When two fields F_1 and F_2 are to be compared they must be combined separately with the earth's horizontal field H_0 in such a way as to assist it. The periods of swing of the needle are determined in the two compound fields, and the ratio of F_1 to F_2 is given by

$$\frac{F_1}{F_2} = \frac{\left\{\frac{1}{T_1^2} - \frac{1}{T_0^2}\right\}}{\left\{\frac{1}{T_2^2} - \frac{1}{T_0^2}\right\}}.$$

The student is required to deduce this expression from the discussion detailed above.

EXPT. 200. **Verification of the Law of Force for a Single Magnetic Pole.**—Observe the time taken for the oscillating needle to execute fifty complete swings when all other magnets are removed from its neighbourhood. The needle should be protected from draughts, and the amplitude of the oscillations should not exceed a few degrees.

Let the time of vibration in this case be T_0, the strength of the field H_0 being due to the earth's magnetism.

Then $H_0T_0^2 = C$ (C being a constant).

If H_0 is known the value of C could be calculated from this equation, but as it cancels out in the working of the results it is unnecessary to do this.

We have then the relation that

$$H_0 = \frac{C}{T_0^2}.$$

Next take one of the very long ball-ended magnets already referred to, and support it vertically in a wooden stand. Place the lower pole somewhere along a line passing through the centre of the needle, and directed north and south (magnetic).

The needle now swings either more or less rapidly than before, or it may perhaps try to turn round end for end; this depends on the nature of the pole and the position in which it is placed.

It is important to note that the pole should be placed in such a position that the oscillating needle points in the same direction as when in the earth's field alone, and *has a shorter period of swing* than before.

Since the period of oscillation is smaller, the field H in which the magnet oscillates is stronger than before. This combined field is equal to the sum of the field F due to the pole of the magnet, and H_0 due to the earth.

Thus $$H = H_0 + F.$$

Place the lower pole of the magnet at various distances r_1, r_2, r_3, etc., from the oscillating needle, the pole of the magnet being always on the same side of the needle, and in the meridian line passing through the needle. Take the different distances r_1, r_2, etc., ranging from about 5 cm. to 20 cm. Find the times of swing T_1, T_2, T_3 of the needle for the various distances of the pole.

We wish to show by this experiment that the field of a single pole varies inversely as the square of the distance from the pole, *i.e.* we wish to show that F is proportional to $1/r^2$.

We can show this if we prove that

$$F_1 r_1^2 = F_2 r_2^2 = F_3 r_3^2, \text{ etc.}$$

Now $$F_1 = C\left\{\frac{1}{T_1^2} - \frac{1}{T_0^2}\right\} \text{ (p. 421)},$$

$$F_2 = C\left\{\frac{1}{T_2^2} - \frac{1}{T_0^2}\right\}, \text{ etc.}$$

Thus we shall show all that we wish if we prove that

$$C\left\{\frac{1}{T_1^2} - \frac{1}{T_0^2}\right\} r_1^2 = C\left\{\frac{1}{T_2^2} - \frac{1}{T_0^2}\right\} r_2^2 = C\left\{\frac{1}{T_3^2} - \frac{1}{T_0^2}\right\} r_3^2 = \text{etc.}$$

The constant C occurs in each of these, and therefore can be cancelled throughout without affecting the equality, so that if we can show

$$r_1^2\left\{\frac{1}{T_1^2} - \frac{1}{T_0^2}\right\} = r_2^2\left\{\frac{1}{T_2^2} - \frac{1}{T_0^2}\right\} = r_3^2\left\{\frac{1}{T_3^2} - \frac{1}{T_0^2}\right\} = \text{etc.},$$

we shall have proved that

$$F_1 r_1^2 = F_2 r_2^2 = F_3 r_3^2 = \text{etc.},$$

i.e. that F is proportional to $\frac{1}{r^2}$.

Arrange the observations as follows :—

Distance of Pole r (cm.).	Period of Swing T.	$\frac{1}{T^2}$	$\frac{1}{T^2}-\frac{1}{T_0^2}$	$r^2\left\{\frac{1}{T^2}-\frac{1}{T_0^2}\right\}$
5				
6				
7				
8				
10				
12				
15				
20				
Infinity.	$T_0=$			

The period when the pole is at infinity is evidently the period of swing in the earth's field alone.

The last column will prove to be approximately constant, thereby showing that the force varies inversely as the square of the distance from a single pole.

The effect of the upper end of the magnet is negligible throughout these observations, as is shown in an earlier experiment (Field of Single Pole by Magnetometer, p. 408).

§ 2. Comparison of Magnetic Moments by Oscillations

When a magnet is suspended by a fine fibre so that its axis hangs in a horizontal position, in a magnetic field of strength H, the axis will assume a certain equilibrium direction. If the magnet be disturbed slightly from this equilibrium position it will execute vibrations about it.

If the oscillations are small the time of each oscillation is the same—the vibrations are isochronous. The time depends on the form and mass of the magnet, and on the couple tending to bring it back to its equilibrium position.

The time of a complete vibration (backwards and forwards) is given by

$$T = 2\pi\sqrt{\frac{I}{MH}}.$$

Hence if I and H are kept constant, the square of the time of vibration is inversely proportional to the magnetic moment of the suspended system.

$$T^2 = \frac{4\pi^2 I}{MH} = \frac{4\pi^2 I/H}{M} = \frac{K}{M},$$

where K is some constant $= \dfrac{4\pi^2 I}{H}$.

If I is not constant, T^2 is proportional to $\dfrac{I}{M}$, when the same field is used.

EXPT. 201. **Comparison of the Magnetic Moments of Two Magnets by oscillating them separately.**—Suspend one of the magnets from a fine thread, so that it can oscillate in a horizontal plane. Note its time of oscillation by taking 50 complete oscillations when under the action of the earth's field alone. Let this time be T_1.

Remove this magnet and replace it by the second, allowing this to swing as nearly as possible in the same position as the first. Take its time of oscillation as before: let this be T_2.

In making these oscillation experiments the twist in the thread must be taken out first by allowing it to untwist under a weight equal to the weight of the magnet to be suspended. If this is not done the magnet will not oscillate about a north and south line: it will be deflected from that line by the couple due to the twist in the suspending thread. The magnets should oscillate in a closed box with glass sides so that the oscillations can be observed, and yet the motion shall not be affected by draughts (Fig. 216).

The swings should be counted and timed as the magnet swings through its middle position, and the angle of swing should not exceed 5° on either side of this position.

Then $$T_1 = 2\pi\sqrt{\frac{I_1}{M_1 H}},$$

and $$T_2 = 2\pi\sqrt{\frac{I_2}{M_2 H}}.$$

Thus $$\frac{T_1^2}{T_2^2} = \frac{I_1 M_2}{I_2 M_1},$$

or $$\frac{M_1}{M_2} = \frac{I_1 T_2^2}{I_2 T_1^2}.$$

Calculate I_1 and I_2 from the masses and dimensions of the magnets (see page 595 for the calculation of Moments of Inertia) and determine M_1/M_2.

If the magnets are similar in size and shape, and of the same density, $I_1 = I_2$.

EXPT. 202. Determination of the Ratio of the Magnetic Moments of two Magnets by allowing them to oscillate together.—Place the two magnets together in a suitable stirrup (Fig. 214) with their north poles pointing in one direction. Suspend them from a thread from which the twist has been removed (p. 425), and allow them to swing inside an oscillation box in the earth's field.

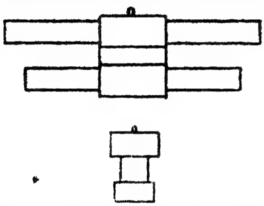

FIG. 214.—Two Magnets in Stirrup.

Observe the period of swing in the usual way. Let this be T_1.

Take out one magnet (the weaker[1]), and replace it in the stirrup with its axis reversed. Again take the period of swing T_2.

The Moment of Inertia of the suspended system is not altered by reversing one of the magnets, but the magnetic moment of the system is in the first case equal to $M_1 + M_2$, and in the second case $M_1 - M_2$, M_2 being the magnetic moment of the magnet which is reversed.

$$\therefore \frac{T_1^2}{T_2^2} = \frac{M_1 - M_2}{M_1 + M_2},$$

and hence
$$\frac{M_1}{M_2} = \frac{T_1^2 + T_2^2}{T_2^2 - T_1^2}.$$

The student should prove these results for himself.

[1] The 'weaker' magnet can be found before the experiment by any rough means such as bringing the magnets in turn near to a compass needle: that having the smaller effect at the same distance is evidently the weaker.

CHAPTER IV

THE EARTH'S MAGNETIC FIELD

§ 1. Specification of the Field

Three quantities are necessary in order to specify completely the magnetic field at any point, for any vector quantity can be determined if we know its magnitude and direction, and in three dimensions two quantities are required to fix a definite direction. The three quantities usually employed in defining the earth's magnetic field at any point are :—

(1) The **Horizontal Component** of the magnetic force.

(2) The **Declination,** that is the angle between the magnetic meridian and the geographical meridian.

(3) The **Dip,** that is the angle between the direction of the resultant magnetic force and the horizontal plane.

We consider here only the first and third, since the determination of the Declination requires astronomical observations in finding the geographical meridian. So far as the magnetic observations are concerned, the principle involved is exactly that in the experiment already described for finding the magnetic axis of a magnet and the magnetic meridian (p. 402).

§ 2. Determination of the Horizontal Component of the Earth's Field

The method to be described, which is due to Gauss, is employed usually to determine the horizontal component of the earth's field. It can be applied, however, to the determination of any magnetic field which is uniform throughout a sufficiently large volume.

The method involves two separate experiments, which of course must be done at the same place. The first consists in finding the period of swing of a freely suspended magnet of known Moment of Inertia; and the second in comparing by means of a magnetometer the field due to this magnet with the earth's field.

Before beginning the experiments remove all iron objects from the neighbourhood.

(A) **The Oscillation Experiment.**—If T is the time of one complete swing of the magnet, when oscillating freely in the earth's horizontal field

$$T = 2\pi\sqrt{\frac{I}{MH}}, \qquad \text{(p. 161)}$$

where H = Horizontal Component of the Earth's Field,
M = Magnetic Moment of the Magnet,
I = Moment of Inertia of the Magnet.

Hence $$MH = \frac{4\pi^2 I}{T^2},$$

and thus **MH** can be calculated in C.G.S. units if I is known.

The bar is a regular geometrical shape, and hence its Moment of Inertia I can be calculated from its mass and dimensions. If the bar be rectangular, as is usually the case,

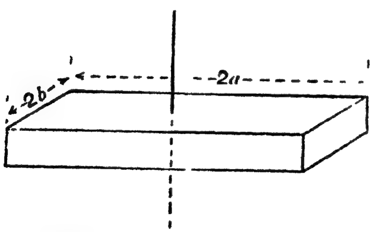

Fig. 215.—Rectangular Bar Magnet.

$$I = m\frac{a^2 + b^2}{3},$$

where m is the mass of the magnet, and a and b are the half-lengths of that face of the magnet which was horizontal during the oscillations (Fig. 215).

For a bar of any regular shape, the required expression for I can be found from the Appendix (p. 595).

Expt. 203. **Determination of MH.**—Before suspending the magnet be sure that the suspending fibre is not twisted. To ensure this, suspend in the stirrup a brass bar of the same

mass as the magnet, and wait for the fibre to untwist. The motion of the brass bar must be checked every few revolutions, otherwise when the fibre is untwisted, the inertia of the rotating brass bar will cause it to twist up in the opposite direction. When the brass bar remains motionless when hanging freely from the fibre, it should be withdrawn from the stirrup and the magnet inserted without allowing the fibre to twist again. The magnet is then suspended from the fibre in a box with glass sides so that the vibrations can be counted and yet air currents will be excluded (Fig. 216).

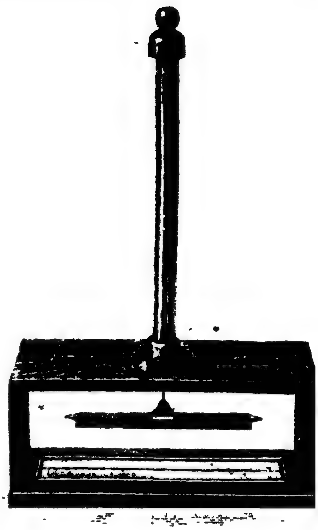

FIG. 216.—Oscillation Magnetometer.

The magnet must be allowed to swing through only a very small angle.

Determine the period of vibration of the magnet by observing the time taken to make fifty complete vibrations.

Weigh the magnet, determine its length and breadth, and calculate its Moment of Inertia I.

Calculate the value of MH from the formula

$$MH = \frac{4\pi^2 I}{T^2}.$$

(B) The Deflection Experiment.—In the second part of the determination, the deflection produced by the *same* magnet on the needle of a magnetometer is observed.

The 'end-on' position is used, placing the magnet with its axis east and west, pointing to the centre of the magnetometer.

Let

$2l$ = distance between poles of magnet,

r = distance between the centres of magnetometer and magnet.

The force on unit magnetic pole at P is in the direction OP (Fig. 216) and is equal to F. As is proved in the chapter on Magnetometry (p. 414),

$$F = \frac{2Mr}{(r^2 - l^2)^2}.$$

The magnetometer needle comes to rest under the action of the two mutually perpendicular fields, F and H, in a position making an angle θ with the meridian, given by

$$\frac{F}{H} = \tan\theta.$$

So
$$\frac{M}{H} \times \frac{2r}{(r^2 - l^2)^2} = \tan\theta,$$

or
$$\frac{M}{H} = \frac{(r^2 - l^2)^2}{2r}\tan\theta.$$

EXPT. 204. **Determination of M/H.**—Set up a magnetometer and place the magnet in the 'end-on' position as in Expt. 195. Determine the values of r, l, and θ, and calculate the value of **M/H.**

Note that $2l$ is the distance between the poles of the magnet used, while $2a$ is the distance between its ends. The poles are not exactly at the ends, so that these distances are not exactly equal. We may assume that the distance between the poles is $\frac{5}{6}$ of the distance between the ends of a bar magnet.

We have now found MH and M/H.

Let
$$MH = A$$

and
$$M/H = B.$$

Then
$$M^2 = AB, \text{ or } M = \sqrt{(AB)}$$

$$H^2 = \frac{A}{B}, \text{ or } H = \sqrt{\left(\frac{A}{B}\right)}.$$

Calculate M and H.

§ 3. DETERMINATION OF DIP

THE DIP CIRCLE

The **Dip Circle** consists of a vertical circle graduated in degrees, carrying at its centre a long needle pivoted on a horizontal pivot, so that it moves over the graduations of the vertical circular scale.

The scale and needle are enclosed in a box with glass sides, so that the needle is not affected by draughts. The whole box can be rotated about a vertical axis, the position of the box being indicated by a graduated circle on the base of the instrument.

To use the instrument, first it is levelled carefully to ensure

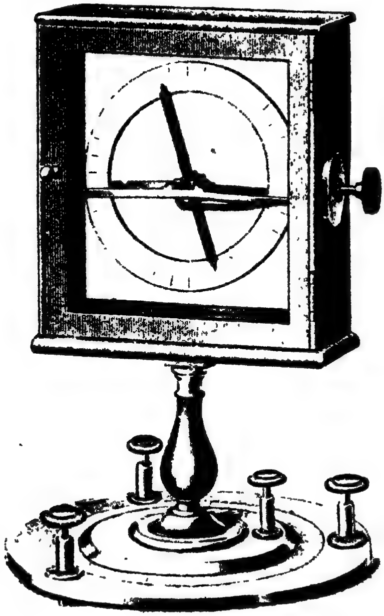

FIG. 217.—Dip Circle.

verticality of the central axis. The box is then rotated about this axis until the needle stands vertical. The plane of rotation of the needle is now exactly at right angles to the magnetic meridian. To bring it into the meridian, the box is rotated about the vertical axis through 90°, this being measured on the horizontal circle on the base of the instrument. The needle is now free to move anywhere in the magnetic meridian, and if perfectly constructed, it will take up a position along the earth's

lines of force, and its inclination to the horizontal will be the angle of dip.

EXPT. 205. **Determination of the Angle of Dip.**—Level the instrument. Rotate the box until the needle is vertical, and take the reading on the horizontal circle. Rotate the box again till the reading is increased (or decreased) by 90°. The needle now sets with its axis inclined to the horizontal.

On account of possible slight imperfections in the construction of the needle, the adjustment of the vertical scale of degrees and so on, the angle of dip cannot be relied upon to be equal to the inclination of the needle, and the following series of observations must be made :—

1. Having got the vertical circle into the meridian, read the positions of both ends of the needle. This gives two readings.

2. Rotate the whole box 180° about the vertical axis and again read both ends; this gives two more readings.

3. Remove the needle and replace it on the knife-edges supporting the pivot, but with the needle reversed back to front, *i.e.* with the pivot reversed end for end. Repeat readings 1 and 2, that is to say, read both ends and again rotate the whole box 180° about the vertical axis and again read both ends. This gives four more readings altogether.

4. Remove the needle and remagnetise it with the magnetism reversed, and start at 1 again, repeating observations as 1, 2, and 3.

This gives eight more readings or sixteen readings in all.

The mean of all these sixteen readings should be taken; this is accurately equal to the Dip at the place where the experiment is performed.

For the theory of this experiment and a discussion of the types of errors avoided or corrected for by taking this series of observations, the student is referred to a text-book of theoretical physics.

Precautions in use.—The needle must not be touched with the fingers, nor should it be brought anywhere where moisture can condense on it when being handled. It should be handled with the forceps supplied.

In placing it on the supporting knife-edges it must be put down gently—the pivot is glass-hard steel and is very brittle indeed; the knife-edges are also very brittle, being made of agate. If raising-clips are fitted (as in a balance) the pivot must be raised from the

knife-edges by these before removal from the box, and must be placed back on these before finally lowering to the knife-edges.

Care must be taken that the magnet is inserted with the proper end dipping (north end downwards in northern hemisphere), otherwise it may revolve several times and roll off the end of the knife-edges.

In remagnetising the reverse way round, the magnet must not be rubbed with a magnet. If magnets are used, the needle must be fitted in a grooved piece of wood and the magnet moved over the wood surface in the requisite direction. A solenoid carrying a current is preferable to all other means of remagnetising the bar. The current is arranged to flow in the required direction, and is switched on and off once or twice while the needle is held inside the solenoid. Considerable force will be exerted on the needle in this process, and it must be held tightly in the forceps while being remagnetised; otherwise it may be pulled out of the forceps and receive damage in falling.

Treat the instrument throughout with the same care as that demanded in the use of an accurate balance.

PART V

ADDITIONAL EXERCISES ON MAGNETISM

1. Find the direction of the magnetic axis of a circular steel plate magnetised parallel to a diameter.

2. Make a survey of the laboratory, using a compass needle to determine the presence of north or south magnetism in iron pipes, girders, radiators, stoves, etc.

3. Plot the lines of force around one pole of a long bar magnet. Do the same when a piece of soft iron is placed not far from the pole.

4. Plot the lines of force between the opposite poles of two fixed magnets. Do the same when a piece of soft iron is placed in the space between the poles.

5. Adjust the given magnet so that the field produced by it may exactly neutralise the horizontal component of the earth's field at a marked point.

6. Place two long bar magnets in the magnetic meridian, with their north poles pointing in opposite directions and 16 cm. apart. Find the neutral point between them. Hence, neglecting the earth's field, compare their pole strengths.

7. Place the given bar magnet with its north pole pointing north, and find the neutral points in its vicinity. Carefully reverse the magnet and show by means of an oscillating needle that the field at these points, with the magnet in this position, is twice as strong as the earth's field alone.

8. Magnetise the two given rods by placing them at the same time in a solenoid and passing a current through the solenoid. Then compare their magnetic moments. Repeat the experiment after heating the rods to redness and plunging them into cold water.

9. Plot a curve showing how the magnetic moment of the given electromagnet varies with the current passing through the coil.

10. Plot a curve showing how the intensity of the field of a magnet varies with distance along its axis, using a tangent magnetometer. Find the magnetic moment of the magnet. $H = 0{\cdot}185$ C.G.S. unit.

11. Find how the intensity of the field of a magnet varies with distance along the axis, using an oscillating needle, and find the magnetic moment of the magnet. $H = 0{\cdot}185$ C.G.S. unit.

12. Find the ratio of the magnetic moments of the two given bar magnets without using a third magnet.

13. Compare the horizontal components of the magnetic field at two marked points in the laboratory by using a magnetometer and the same bar magnet in each position.

14. Find the magnetic moment of the given magnet.

15. Place a short bar magnet in an oil bath so that its temperature may be varied. By means of a deflection magnetometer obtain a curve showing the relation between the magnetic moment of the magnet and the temperature, both when the temperature is rising and when it is falling.

PART VI

ELECTRICITY

CHAPTER I

ELECTROSTATIC EXPERIMENTS

§ 1. INTRODUCTORY

CERTAIN bodies when rubbed with flannel, or silk, acquire the power of attracting light bodies. In this condition they are said to be electrified, or to carry a charge of electricity..

The electrification produced by rubbing a glass rod with silk differs from that produced by similarly rubbing a rod of ebonite. There are two kinds of electrification, vitreous (or positive) and resinous (or negative). Bodies electrified in the same way repel each other, while bodies oppositely electrified attract each other.

If a brass rod is held in the hand and rubbed with flannel no electrical charge can be detected on the rod. If, however, the rod is supported by a glass handle it can be electrified and the charge can be detected. This may be explained by saying that the metal conducts away the electric charge, which passes through the body into the earth. The glass rod, if dry, does not conduct away the electric charge. Bodies which conduct well are called **conductors**; those which conduct badly are called **non-conductors** or **insulators.** Metals are good conductors, whilst glass and ebonite are insulators.

In all electrostatic experiments it is of the highest importance to avoid surface films of moisture on all insulators. Such films seriously impair their insulating properties. A small tinplate oven with a double bottom, heated by a rose burner, is very useful for keeping the apparatus dry.

§ 2. Experiments with the Gold-Leaf Electroscope

The gold-leaf electroscope is a convenient apparatus for experiments in electrostatics. A simple form of electroscope consists of a glass vessel with a brass rod passing through the insulating stopper. The top of the rod is fitted with a brass ball or disk, and a pair of gold leaves are attached to the lower end. If the leaves are given a charge the leaves will repel each other and diverge. If the insulation were perfect there would be no loss of charge and the leaves would remain inclined at the same angle indefinitely.

In modern forms of the electroscope, a single gold leaf is used, attached to a stiff strip of brass or aluminium (Fig. 219). The deflection of the gold leaf may be measured by means of a scale fixed to a mirror (to avoid any error due to parallax), or by means of a micrometer microscope.

Expt. 206. **Illustration of the Laws of Electrostatics.**— I. Touch the disk of the electroscope with the finger so as to remove any charge it may possess. Electrify an ebonite rod by friction and bring it near to the disk. The gold leaves should diverge (A, Fig. 218). The charge on the ebonite induces a charge of opposite sign on the disk, and a charge of the same sign as that on the ebonite is repelled into the gold leaves.

Try the same experiment with a glass rod.

II. To show that there are two kinds of electrification, bring up an electrified ebonite rod as in the first experiment, then bring up an electrified glass rod. The presence of the second charge should diminish the effect due to the first. By suitably adjusting their distances (B) the two may be made to nullify each other's action.[1]

III. The electroscope may be charged by conduction. The electrified rod is brought into contact with the disk of the instrument (C) and shares part of its charge with the gold leaves. These remain apart after the rod has been removed (D).

IV. To charge the electroscope by induction, bring the electrified rod near the disk without touching, then touch the disk with the finger for a moment (E). Remove the finger and *afterwards withdraw the rod* (F). The electroscope now bears a

[1] A glass rod which has been passed through a flame sometimes shows a *negative* charge when rubbed. In this case the first deflection is increased.

charge of the opposite kind to that on the rod, for when the finger touches the disk the charge of the same kind as that on the rod is repelled through the body into the earth.

V. To test the sign of a charge, charge the electroscope by induction as in the last experiment, using a vulcanite rod. Then bring up a glass rod carrying a positive charge, and notice that the gold leaves diverge farther.

Next bring up the vulcanite rod; the leaves will now collapse, the divergence diminishing more and more as the rod gets nearer.

If the rod gets very close, it *may* cause the leaves to collapse entirely and then to open out again. It is left to the student to advance an explanation of this redivergence.

Now bring up a *large* uncharged body supported on an insulating handle. Then bring up an earth-connected body (the hand of the experimenter is suitable). Note that in *both* these cases the divergence of the leaves is diminished slightly although the bodies are not charged before being brought near the electroscope.

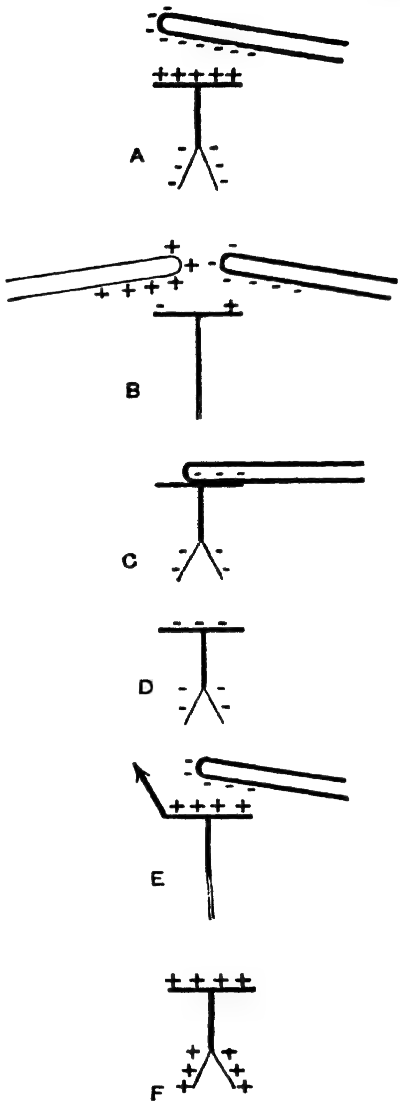

FIG. 218.—Experiments with Gold-Leaf Electroscope.

It will be seen from this that although *increased* divergence indicates a charge on the body of the same sign as that on the electroscope, we cannot conclude *with certainty* that *diminished* divergence indicates a charge of the opposite sign.

To test the sign of a charge of any kind it is necessary to have two electroscopes charged oppositely. The body to be tested is brought up to each in turn. A positively charged body gives an increased divergence with the positively charged electroscope, but a diminished divergence with the electroscope negatively charged.

A negatively charged body increases the divergence of the negatively charged electroscope, but causes a diminished divergence in the positively charged one. An uncharged body or an earth-connected body causes the divergence to be diminished in each case.

§ 3. Simple Electrostatic Apparatus

THE ELECTROPHORUS

The **Electrophorus** consists of a plate of ebonite (or resin) supported by a sole-plate of metal. On this plate of insulating material can be placed a metal disk provided with an insulating handle.

When this disk is removed, a negative charge is produced upon the surface of the ebonite by rubbing or flicking it with a dry catskin. The metal disk is then lifted by the insulating handle and placed on the electrified surface. Actual contact occurs at only a few points, while over the rest of the surface the negative electrification on the ebonite induces a positive charge on the opposed metal surface, and a negative charge is repelled to the upper surface of the disk. On touching the disk with the finger this negative charge escapes through the body to the earth, leaving the positive charge 'bound' under the attraction of the negative charge below it. If the disk is now lifted from the ebonite, it carries with it this positive charge, which may be afterwards shared with some other conductor. When the disk has been discharged by connecting it to some earthed conductor, it may be placed once more on the ebonite, and the process repeated. The charge on the ebonite is not diminished appreciably in the operations, and if the insulation were perfect the process of charging could be repeated indefinitely. The same principle is employed in the Influence Machines designed by Voss and by Wimshurst.

Expt. 207. **The Electrophorus.**—Charge an electrophorus, and use it to obtain a charge on the disk. Test the sign of the charge on the disk by the method described above. Show that it is possible to obtain sparks from the disk to

any earthed conductor (*e.g.* the knuckle of the experimenter) placed near it.

FARADAY'S ICE-PAIL EXPERIMENTS

For these experiments a metal can is provided, which can be insulated by placing it on a block of paraffin or ebonite. If the electroscope is provided with a suitable flat plate connected with the gold-leaf system, it is simpler to stand the 'ice-pail' on this plate. If the can is on a block of ebonite or paraffin, then it should be connected by means of copper wire to the disk of the electroscope.

EXPT. 208. **Faraday's Ice-Pail.**—Take a brass ball suspended by a silk ribbon, or fitted with an insulating handle, and charge it by means of an electrophorus or a small Wimshurst machine.

Lower it into the ice-pail and observe the deflection of the gold leaf. If the charged ball is well within the mouth of the vessel, the deflection will be the same whatever the position of the ball. Even if the ball is allowed to touch the inside of the can the deflection will be unaltered.

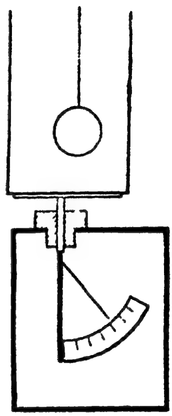
FIG. 219.—Ice-Pail on Electroscope.

This agrees with the view that a definite quantity of electricity has been introduced into the vessel, and that the effect on the electroscope depends only on the *quantity* inside the vessel.

We may make use of the ice-pail to determine whether two bodies are equally charged, and may then add together their charges (by placing them inside a hollow conductor) so as to obtain a charge double the original charge. In the same way we can give to the hollow conductor a charge which is any number of times the charge of a given body.

EXPT. 209. **The Induced Charges are equal and opposite to the Inducing Charge if this is entirely enclosed by the Conductor on which the Charges are induced.**—Place the charged ball inside the ice-pail without allowing it to touch the sides, and observe the deflection. Touch the ice-pail with the finger; the gold leaves will collapse. Remove the ball, still carrying its charge, from the pail. The deflection should be equal to the original one. The two charges induced were equal and opposite to each other; one has been removed, and

the other gives the observed deflection. *Discharge the can* and reintroduce the ball, allowing it to touch the bottom. On removal it is quite uncharged; it has given all its charge to the can. The deflection of the leaves of the electroscope is the same after the ball touches the bottom as before it touches, and is not altered when the ball is removed.

EXPT. 210. **Equal and Opposite Quantities of Electricity are produced by Friction.**—For this experiment the rubber and the body rubbed must both be insulated. Rub the two bodies together, holding them by the insulating handles, and test each one by introducing it into the ice pail. Then introduce the two together in the ice pail. If the charges are exactly equal and opposite, no deflection of the gold leaves should be observed when *both* are used, but each *alone* gives a deflection.

The operations described must be performed quickly, as there is nearly always a certain amount of leakage taking place.

In writing an account of these electrostatic experiments the results obtained should be described, and the conclusions to be drawn from them stated. It is most important that the account should be illustrated by diagrams indicating the positions of *and charges on* the various pieces of apparatus used in each of the various stages.

§ 4. CHARGE AND POTENTIAL

It is important to notice that the deflection of the leaves of an electroscope does not necessarily indicate the charge on the electroscope *as a whole*. The deflection indicates the *potential* of the electroscope always, and may be taken to indicate its *charge* only when there is no other body near to it.

Positive electricity flows from points at higher to points at lower potential, if these points are connected together by a conductor.

The test of potential is to connect the body to earth; if it gives up positive electricity its potential was positive. If it *receives* positive from the earth (or gives up negative to the earth) its potential was negative. If it neither gains nor loses electricity, it was at zero potential.

EXPT. 211. **Demonstration that the Divergence of a Gold-Leaf Electroscope indicates its Potential.**—CASE 1. Bring a charged glass rod to an electroscope; the electroscope has induced upon it two equal and opposite charges. It is *uncharged* on the whole, yet the leaves diverge. If earth-connected, positive electricity leaves the electroscope and goes to earth, therefore the electroscope was at a positive potential. *Before earth-connecting it*, it was showing a divergence, yet it was uncharged: therefore the divergence does not indicate the charge on the electroscope in this case.

CASE 2. Repeat the operation of bringing a charged glass rod to the electroscope. After earth-connecting the electroscope, keep the glass rod still near. The electroscope now has a negative charge. The leaves are, however, quite closed, therefore the divergence does not indicate the charge in this case — there is a considerable charge, yet no divergence.

CASE 3. Electrify an electroscope positively, and remove all bodies to a distance. The leaves diverge, the electroscope has a positive charge; in this case, therefore, the divergence may be taken to indicate the charge.

Consider the potentials in the three cases.

CASE 1. As already explained, the electroscope had a positive potential though zero charge (on the whole). The leaves were diverging.

CASE 2. The electroscope had a zero potential (being earth-connected) although it had a charge. There was no divergence.

CASE 3. The electroscope has a positive potential as well as a positive charge.

Thus divergence and potential go together, and we see that **the divergence of the leaves of an electroscope indicates its potential.** It only indicates the charge *as well*, when the electroscope is remote from other bodies.

The divergence only indicates the magnitude of the potential; the potential might be either positive or negative for a given divergence. This can only be tested by other means, viz.—

Bringing up a positive conductor *raises* the potential of the electroscope. If, therefore, the leaves diverge further, the electroscope has a positive potential. If they collapse a little it had a negative potential; *raising* the potential having in this latter case the meaning *diminishing its negative value.*

CAPACITY

When a charge is given to an isolated conductor, the resulting potential depends on the size and shape of the conductor; for the same charge, the larger the conductor the lower is the potential to which it is raised. The **Capacity** of a conductor is defined as the charge required to raise its potential by one unit. When a second conductor is brought near the first, the potential of the first is diminished (p. 439). The effect depends on the size of the second conductor, and if this be earth-connected the effect is usually very large, being virtually equivalent to making the earth form part of the second conductor. Such an arrangement forms a **Condenser**, which may be defined as a system of conductors placed so that the capacity of one part of the system is increased in consequence of the proximity of the other part. The capacity of a condenser is measured by the charge required on the first part to increase the difference of potential between the two parts by one unit.

THE CONDENSING ELECTROSCOPE

The condensing electroscope is an ordinary gold-leaf electroscope fitted with a disk of a size rather larger than usual. A second disk of the same size as the electroscope disk, and mounted on an insulating handle, is laid on the electroscope disk. The disks are insulated from each other by a thin coat of insulating varnish spread over the upper disk. The two disks constitute a parallel plate condenser, and when the upper disk is earth-connected the electroscope becomes a conductor of considerable capacity. That is to say, it requires a considerable charge to raise the potential of the electroscope to *unit* potential. If, therefore, the electroscope is connected to any apparatus which is maintained at a constant potential, it will take up a charge much greater than it would take up if the earth-connected disk were not upon it.

It may happen that the potential to which it is thus raised is insufficient to cause an appreciable divergence of the leaves,

and the existence of this electric potential would be undetected by the electroscope when used in the ordinary way.

If, however, we charge up the condensing electroscope while the upper disk is present, we obtain on the electroscope a charge of considerable magnitude, though the potential is too low to affect the leaves. On removing the wire connected to the electroscope disk, and immediately afterwards removing the earth-connected disk from the electroscope, the electroscope becomes a conductor of small capacity. A charge which gave it a certain potential before will now give it a potential a great many times larger. Consequently the leaves may be caused to diverge now by the charge on the electroscope, although the same charge was insufficient to affect the leaves when the capacity was greater.

EXPT. 212. **Use of a Condensing Electroscope to detect the Positive and Negative Poles of a Cell.**—Remove the upper disk from the electroscope, and touch the cap with a wire connected with one pole of any Voltaic cell, the other pole being earth-connected. Notice that no divergence can be observed.

Place the insulated disk on the electroscope disk and again touch the cap with the wire, the upper disk being earth-connected while the wire is touching the electroscope. Remove the wire—no divergence is observed; remove the upper disk—the leaves open out slightly owing to the larger potential produced by the *same* charge, now that the capacity of the electroscope has been diminished.

Test the sign of the charge, using a vulcanite rod or a glass rod.

Repeat the experiment with the other pole of the cell, meantime earth-connecting the pole previously used. Show that the charges obtained from the two poles of the cell are *opposite* in character, and that the *zinc plate* is negative in all cells in which a zinc plate is used.

Test the signs of the poles of an accumulator to see if they are correctly marked.

NOTE.—According to the view generally accepted at the present time an electric charge is due to the addition or removal of small discrete particles of negative (resinous) electricity. These negative **electrons or corpuscles** (p. 547) constitute the electric fluid in the 'one-fluid' theory of Franklin. In this book *positive* (vitreous) electricity is spoken of in a conventional way as flowing through a conductor from points at higher to points at lower potential. A flow of positive electricity in one direction may be regarded as equivalent to a flow of negative electricity in the opposite direction.

CHAPTER II

CURRENT ELECTRICITY—INTRODUCTORY

§ 1. Chemical Generation of Electricity

It can be shown by means of a condensing electroscope that when two plates of any two different metals are immersed in the same vessel of almost any liquid, one plate will acquire a higher potential than the other. When these two plates are connected *momentarily* by a wire, a discharge naturally flows from one plate to the other in consequence of their difference of potential, but the two plates are not *discharged* as a result of this. If the wire connecting them is removed, and the plates are again tested, they will be found to give a potential difference as before, the electroscope usually being insufficiently sensitive to detect any slight difference which may exist between the present and former values of this P.D.

Thus when the plates are placed in this liquid, there is a continual renewal of the charges on them, as a result of the chemical action in the cell. If a wire is connected permanently to the two poles, a *continuous* discharge or a **current of electricity** flows through the wire.

By experimenting with various forms of solutions and different kinds of plates, certain specially active *cells* have been invented by different experimenters.

In the simple cell of Volta, plates of copper and zinc are immersed in dilute sulphuric acid (1 in 10). The zinc plate tends to dissolve in the acid in accordance with the equation

$$Zn + H_2SO_4 = ZnSO_4 + 2H.$$

This is typical of the action in other primary cells.

To prevent the *polarisation* (p. 481) of the cell due to the film of hydrogen formed at the copper plate, some depolarising liquid may be employed.

TABULATED DESCRIPTION OF SOME PRIMARY CELLS

Name.	Plates. (+ −)	Exciting Liquid.	Depolarising Liquid.	Approximate E.M.F. in Volts.	Remarks.
Simple	Cu, Zn	H_2SO_4, Aq.	None	1·0	Rapid polarisation.
Daniell	Cu, Zn	H_2SO_4, Aq.	$CuSO_4$ concentrated	1·1	Constant, satisfactory.
		$ZnSO_4$, Aq.	$CuSO_4$ concentrated	1·07	No acid fumes.
Grove	Pt, Zn	H_2SO_4, Aq.	HNO_3 concentrated	1·9	Expensive, acid fumes.
Bunsen	C, Zn	H_2SO_4, Aq.	HNO_3 concentrated	1·7	Acid fumes.
Leclanché	C, Zn	NH_4Cl, satd.	MnO_2	1·4	Good for intermittent use.
Bichromate	C, Zn	H_2SO_4, Aq.	H_2CrO_4 from $K_2Cr_2O_7$	1·8	Satisfactory. Withdraw Zn plate when not in use.
Clark	Hg, Zn	$ZnSO_4$	Hg_2SO_4	1·433	Constant.
Weston	Hg, Cd	$CdSO_4$	Hg_2SO_4	1·0183	Very constant.

The electromotive force of a cell is equal to the energy drawn from the source and dissipated in the circuit when unit quantity of electricity flows round the circuit.

The practical unit for expressing electromotive force (E.M.F.) or P.D. is the **Volt,** which is 10^8 C.G.S. units. The **International Volt** is realised experimentally by taking the E.M.F. of the Weston cadmium cell as 1·0183 international volts at 20° C. The E.M.F. of the Clark cell is 1·433 volts at 15° C.

Care of Cells.—The *power* of a cell, or the rate at which it can supply current, is limited by the size of the plates, by the rate at which the necessary chemical actions can take place, and by other properties such as Internal Resistance, which will be discussed later. If a cell is overworked by *short-circuiting, i.e.* by connecting the poles together by a short piece of metal even for a moment, it will be more or less *run-down* or *polarised*, and will not work satisfactorily for some minutes, if indeed it is not damaged permanently.

It is therefore important to avoid overworking any cell in this way, especially a secondary cell or accumulator. An accumulator is made of plates of lead loaded with spongy lead on one plate and with lead peroxide on the other. Any *momentary* overload causes very rapid evolution of gas *inside* the plates as well as at the surface, and the plates buckle, or the loading is blown out from the plate. This means that the accumulator is severely damaged, and if the process is repeated may be ruined entirely.

It is mere wanton childishness to ruin these expensive cells and thereby diminish the efficiency of the laboratory for the sake of seeing a momentary flash.

§ 2. The Magnetic Action of Electric Currents

In 1819, Oersted discovered that a magnetic needle was deflected when placed near a wire carrying an electric current. The magnet tends to set itself with its axis at right angles to the direction of the current. The current gives rise to a magnetic field in the surrounding space. In the case of a long straight wire the lines of magnetic force take the form of circles. The centre of each circle is a point in the wire, and the plane of the circle is perpendicular to the wire.

Suppose the wire is at right angles to the plane of the paper and that the current is flowing into the paper at the point A. Then a positive pole would be urged round a circle, centre A, in the clockwise direction. A convenient way of expressing this result is to say that if a right-handed corkscrew is screwed so that its point travels in the direction of the current, the direction in which it is turned (or the direction in which the thumb moves) gives the direction of the magnetic force.

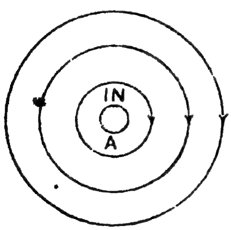

FIG. 220.—Lines of Force due to Current.

This result should be tested in a number of different cases

by examining the action of a current on a small compass needle. Connect the ends of an insulated copper wire to the terminals of a battery composed of one or two Daniell cells, and observe the deflection produced when the wire is placed in different positions with relation to the magnetic needle. Verify the fact that when the wire is doubled on itself, or twisted together so that the currents in neighbouring portions are in opposite directions, little effect is produced on the compass needle.

EXPT. 213. **Construction of a Simple Electromagnet.**—Wind an insulated copper wire round a bar of soft iron so as to form a spiral winding round the bar. Connect the ends of the wire to a battery, introducing resistance in the circuit if necessary. The iron will be magnetised by the magnetic field due to the current, and the system forms an **electromagnet.** If a cork-screw were turned so that the thumb followed the direction of the current round the turns of the spiral, the point of the cork-screw would advance in the direction of the lines of magnetic force. The lines of magnetic force run through the iron bar from the S. pole to the N., consequently the end where the point of the screw would enter the iron is the S. pole, and the end where the point of the screw would emerge is the N. pole. Test this result with the compass needle and examine the power of the electromagnet of attracting small pieces of iron.

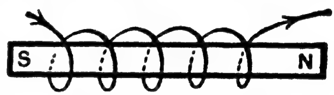

FIG. 221.—Construction of Electromagnet.

Testing the Sign of Battery Poles.—The foregoing results may be applied in order to determine the signs of the poles of a battery or other source of current. A current is passed from the source through a length of wire, taking proper precautions to avoid excessive current, and the direction in which the current flows is determined by one or other of the methods just described. Remembering that the current is said to flow from the + terminal through the external circuit to the − terminal, the sign of each terminal can immediately be given.

The signs of the terminals can also be determined by examining the chemical actions produced by the current. This question will be dealt with in a later section (p. 588).

§ 3. THE MAGNETIC FIELD DUE TO AN ELECTRIC CURRENT IN A STRAIGHT WIRE

It has been pointed out above that the lines of magnetic force due to a current in a long straight wire are in the form of circles. Each circle has its centre at a point on the wire, and the plane of the circle is perpendicular to the wire. The direction of the magnetic force round the circle, and the direction of the current are related in the same way as the directions of rotation and translation of a right-handed screw.

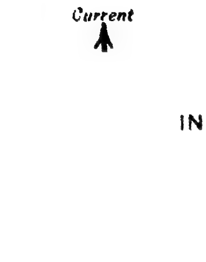

FIG. 222.—Magnetic Force due to Straight Current.

At a point where the length of the perpendicular drawn to the wire is r, the value of the magnetic force is $2C/r$, where C is the current strength in electromagnetic units.

For a definition of the electromagnetic unit of current see the theory of the tangent galvanometer (p. 459).

In an actual experiment we have to consider the magnetic field of the earth as well as the field due to the current. It is convenient to arrange for the wire to be vertical and, ignoring the vertical component of the earth's field since this has no effect on a horizontal needle, to trace lines of magnetic force in a horizontal plane.

The lines of force may be traced by means of a small compass in the same way as the lines of force due to a permanent magnet are traced.

EXPT. 214. **Plotting the Magnetic Field of a Straight Current.**—A convenient apparatus consists of a large rectangular framework with a covered copper wire passing several times round the frame so as to multiply the magnetic effect of the current (Fig. 223). One end of this wire is joined to one terminal of a small accumulator, the other to a key. The electric circuit is completed by connecting the other terminal of the accumulator to the key with a short length of platinoid wire. If direct current is used for lighting the laboratory, the current may be taken from the mains, using

a lamp in series with the apparatus to adjust the strength of the current (p. 588).

Place the frame with the vertical sides close to the edge of a table and arrange the horizontal drawing board so that it rests securely on the table.

Trace the lines of force due to the combined fields when the current is flowing either downwards or upwards. Pay

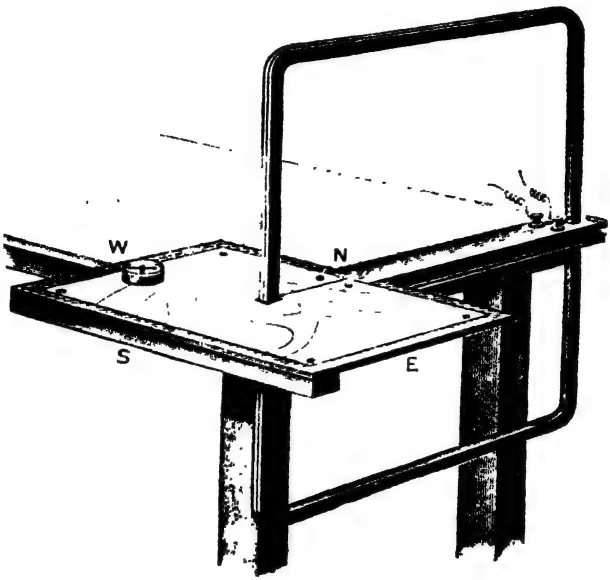

FIG. 223.—Field of Straight Current.

special attention to the lines of force (1) close to the coil and (2) near the 'neutral' point.

Having found the position of the neutral point as accurately as possible, measure the distance r from this point to the wire.

At a distance r from the wire the magnetic force due to the current is

$$F = \frac{2nC}{r},$$

where n is the number of wires down the side used, each wire carrying the same current, C electromagnetic units.

This magnetic force is balanced by the horizontal component of the earth's field, H = 0·185 gauss in Great Britain.

Hence $$\frac{2nC}{r} = 0{\cdot}185.$$

Calculate the value of C, the current in electromagnetic units, and deduce the value in amperes (one electromagnetic unit is equal to 10 amperes).

VARIATION OF THE STRENGTH OF THE MAGNETIC FIELD CLOSE TO A CURRENT FLOWING IN A STRAIGHT WIRE

It has been stated already (p. 450) that the strength of the magnetic field due to a current in a long straight wire is given by $F = 2C/r$ at a point distant r from the wire.

It is possible to show that F is inversely proportional to the distance from the wire by either of the methods previously described for comparing the strengths of magnetic fields.

EXPT. 215. **Variation of the Strength of the Magnetic Field due to a Straight Current using a Magnetometer.**—Place the wire vertically as in the previous experiment, and draw a horizontal line passing through the wire in the direction of the magnetic meridian. At some distance r from the wire along this line place a magnetometer, and note the deflection θ produced in the magnetometer when the current is flowing in the wire. Do this at various distances from the wire, and arrange a table showing corresponding values of r, θ, $\tan\theta$, and $r \tan\theta$.

The field due to the current is east and west at any point north or south of the wire, and hence the strength of the field is proportional to $\tan\theta$. The last column, $r\tan\theta$, will be found constant, and hence $\tan\theta$, or F, is proportional to $1/r$.

EXPT. 216. **Variation of the Strength of a Magnetic Field due to a Straight Current by plotting Lines of Force.**—This method is virtually the same as that of Expt. 215. Instead of using a magnetometer to observe the deflections at different points along the north and south line drawn through the wire, the field is plotted at different points where it crosses this line, using a compass needle for this purpose. The tangent to the line of force where it crosses the north and south line is drawn, and the angle between this tangent and the north and south line is the angle θ.

This is measured with a protractor, and a table drawn up as in Expt. 215.

Suitable distances are 5, 6, 7, 8, 10, 12, 15, and 20 cm. from the wire.

The Method of Oscillations.—Imagine a line passing magnetic east and west to be drawn through a vertical wire carrying a current. The field F due to the current in the wire at any point in this line is either due north or south; hence on one side of the wire the strength of the total field will be $F + H_0$, while on the other side it will be the difference between F and H_0, H_0 being the horizontal component of the earth's field.

A short heavy needle (p. 120) could be placed at some point on this east and west line, and its period of swing observed before switching on the current, *i.e.* the period of the needle could be found in the earth's field alone. Let this period be T_0.

Then $$T_0^2 = \frac{C}{H_0}, \text{ or } H_0 = \frac{C}{T_0^2}.$$

If the current were switched on, the behaviour of the needle would depend on its position and also on the direction of the current. On one side of the wire it would start to swing more rapidly than in the earth's field alone, and with its poles pointing in the same direction as at first. On this side, the field of the current and the field of the earth assist each other. On the other side the two fields are in opposition, and the swings would be slower than in the earth's field, or the needle might be reversed. If H_0 is stronger than F the needle swings less rapidly, but if F is stronger than H_0 it is turned completely round.

It is important that the needle should be used on the side where the fields assist each other: in very weak fields the torsion of the fibre has a greater percentage effect than in stronger fields, and as we take no account of the torsion, the error due to it is correspondingly increased. In the following discussion it will be assumed that the needle is placed on the side where the fields assist each other.

The strength of the composite field at any point being H, and the period T, we have

$$H = F + H_0,$$

and also
$$H = \frac{C}{T^2}.$$

Hence
$$F = H - H_0$$
$$= C\left\{\frac{1}{T^2} - \frac{1}{T_0^2}\right\}.$$

Now if F is proportional to $1/r$, we shall have $F_1 r_1 = F_2 r_2 = F_3 r_3$, etc., taking F_1, F_2, F_3, etc., as the strengths of the field at distances r_1, r_2, r_3, etc., from the wire.

If the corresponding periods of oscillation are T_1, T_2, T_3, etc., we can write

$$C\left\{\frac{1}{T_1^2} - \frac{1}{T_0^2}\right\} = F_1, \quad C\left\{\frac{1}{T_2^2} - \frac{1}{T_0^2}\right\} = F_2, \text{ etc.},$$

and therefore we can show that $F_1 r_1 = F_2 r_2$, etc., provided we show that

$$C\left\{\frac{1}{T_1^2} - \frac{1}{T_0^2}\right\} r_1 = C\left\{\frac{1}{T_2^2} - \frac{1}{T_0^2}\right\} r_2 = \text{etc.}$$

The constant C occurs in each expression, and therefore can be cancelled throughout, and we shall have proved F proportional to $\frac{1}{r}$ if we show that $r\left\{\frac{1}{T^2} - \frac{1}{T_0^2}\right\}$ is constant.

EXPT. 217. **Variation of the Strength of the Magnetic Field due to a Straight Current by the Method of Oscillations.**—Place the wire in a vertical position; draw a line passing magnetic east and west through the wire, and measure off different distances along the line, say 5, 6, 7, 8, 10, 12, 15, and 20 cm. from the wire.

Place a small oscillating needle (p. 420) at some point on the line and determine its period of oscillation *before* switching on the current. Call this T_0.

Switch on the current, observing the behaviour of the needle when this is done. If the needle swings *more rapidly than before*, and still points in the same direction, the experi-

ment may be proceeded with; *if not, reverse the direction of the current* through the wire, when the needle will be found to point as in the earth's field alone, but will swing *more* rapidly. The needle is now oscillating in a field of strength H, which is the sum of the strength of the field F due to the wire, and the horizontal component of the earth's field H_0.

Place the needle at each of the points marked along the east and west line on this side of the wire where the two fields assist each other. Observe the period of oscillation in each position.

Arrange the results of the observations as follows:—

Period of needle in earth's field, $T_0 = \ldots$ sec.

$$T_0^2 = \ldots$$

Distance from wire in cm. r.	Period of needle in sec. T.	$\frac{1}{T^2}$.	$\frac{1}{T^2} - \frac{1}{T_0^2}$.	$r\left\{\frac{1}{T^2} - \frac{1}{T_0^2}\right\}$.
5				
6				
7				
8				
10				
15				
20				

The last column will be found to be constant, thus showing that the magnetic force due to a current in a long straight wire varies inversely as the distance from the wire.

§ 4. The Magnetic Field due to a Circular Coil carrying an Electric Current

It has been shown that an electric current sets up a magnetic field in the surrounding space. An important case is that of a circular coil of wire carrying an electric current. The lines of magnetic force at all points in the plane of the coil are perpendicular to that plane. At any such point *inside* the circular boundary the direction of the line of magnetic force is related to the direction of the current in the same way as the

direction of translation to the direction of rotation of a right-handed screw (Fig. 224).

EXPT. 218. **Plotting the Magnetic Field of a Circular Coil carrying a Current.**—A convenient apparatus for this experiment consists of a circular coil fixed with its plane vertical at the centre of a horizontal board. The board is raised from the bench so that its plane cuts the coil across a horizontal diameter. Drawing-paper is fixed down to the board with pins, a slot being cut in the paper to allow it to pass over the top of the coil, and the lines of force near to the coil are traced with a compass-needle in the same way as the lines of force due to a permanent magnet are traced.

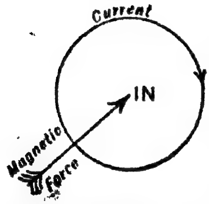

FIG. 224.—Magnetic Force due to Circular Current.

The lines of force will not represent the field of the coil *alone*, but the composite field due to the coil and the earth.

Arrange the apparatus so that the plane of the coil is in the magnetic meridian and send a current through it from accumulators or from some other *constant* source of current, adjusting the current by means of suitable resistances to a convenient value.[1] Trace the lines of force when the current is flowing round the coil, paying special attention to the lines of force (1) close to the coil and (2) near the neutral points.

EXPT. 219. **Variation of the Strength of the Magnetic Field due to a Circular Coil with the Distance along the Axis.**

(i.) **By plotting Lines of Force.**—If in the foregoing experiment the coil is placed with its plane in the magnetic meridian, the field due to the coil at all points along its axis will be east and west. The actual field is compounded of the field of the coil and the horizontal component of the earth's field, and therefore the lines of force at points along the axis will *not* be exactly east and west, but will be inclined to this direction at greater and greater angles as the distance from the coil is increased.

Find the angle between the lines of force and the magnetic north at several points along the axis of the coil, by tracing

---

[1] Current from the lighting mains can be used conveniently for this experiment wherever direct current is supplied; a lamp resistance suitable for adjusting the current to the required value is described on p. 588.

the lines for a short distance as they cross the axis at these points. Choose the points at the distances from the coil indicated in the first column of the following table. If the angle between the line of force and the magnetic north is θ, the strength of the field F due to the coil is proportional to tan θ.

Tabulate the results as below:—

Distance from coil along axis.		tan θ.
7·5 cm.		
10		
12·5		
15		
20		
25 and 30		

Plot a curve showing the variation of tan θ with distance. This indicates the way in which F varies with the distance along the axis.

(ii.) **By use of a Magnetometer which can slide along the Axis.**—The most convenient type of apparatus to

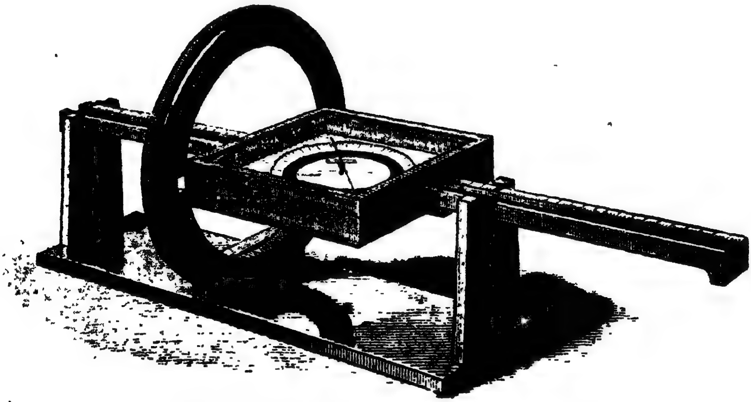

FIG. 225.—Stewart and Gee Tangent Galvanometer.

use for this purpose is a tangent galvanometer of the Stewart and Gee pattern (Fig. 225). Set up the coil with its plane

vertical and parallel to the meridian, using the needle of the magnetometer to make this adjustment.

Send round the *thin* wire coil a current sufficient to give a deflection of 75° or 80° when the magnetometer needle is exactly in the plane of the coil; keep this current constant. Slide the magnetometer box along the axis by steps of 1 cm., and note the reading of the magnetometer at each distance. Continue this motion as far as the apparatus will allow, or until the deflection is reduced to 5°.

Repeat the measurements on the other side of the coil.

Tabulate the results as below:—

Distance along axis d.	Deflection one side (θ_1).	Deflection other side (θ_2).	$\tan\theta_1$.	$\tan\theta_2$.

Plot a curve showing the variation of $\tan\theta$ on *both sides* of the coil. This curve should be symmetrical and should have a maximum value when the needle is at the centre of the coil itself.

This method is preferable to the method (i.) where the field is plotted, as it enables the readings to be carried on right up to and *through* the middle of the coil. Method (i.) must perforce stop near the plane of the coil (unless the centre is cut out), and the measurements near the coil are not very accurate owing to the rapid curving of the lines thereabouts.

CHAPTER III

APPARATUS FOR THE MEASUREMENT OF CURRENT

§ 1. THE TANGENT GALVANOMETER

BY means of the tangent galvanometer we can measure the strength of a current in **absolute electromagnetic units (C.G.S. units,** defined below). Since **one ampere,** the practical unit of current strength, **is defined as one-tenth of the C.G.S. unit of current,** it is then possible to express the strength of the current in terms of the ampere.

The tangent galvanometer is said to be an **absolute instrument,** because **its** indications can be reduced to give the value of the current in absolute or standard units. As it is designed on lines derived by theory *its indications cannot be wrong, provided the conditions demanded by theory are satisfied.* The tangent galvanometer is thus the **standard instrument for the measurement of current,** and the calibration of all other forms of current meter must be made using a tangent galvanometer as the standard for comparison.

THEORY OF THE TANGENT GALVANOMETER

The C.G.S. unit of current may be defined as that current which flowing through a wire 1 cm. long bent into an arc of a circle of 1 cm. radius produces a force of 1 dyne on unit magnetic pole at the centre.

If a current of C units be flowing through l cm. of wire bent into an arc of r cm. radius, the magnetic force at the centre is

$$F = \frac{lC}{r^2}.$$

The direction of the force is perpendicular to the plane of the circle, and is related to the direction of the current as is the direction of translation to the direction of rotation in a right-handed corkscrew.

If the wire forms one complete circle, $l = 2\pi r$, hence

$$F = \frac{2\pi r}{r^2} \times C = \frac{2\pi C}{r}.$$

For a circular coil containing n turns, the force is n times this.

In the simplest form of tangent galvanometer a circular coil is placed with its plane in the magnetic meridian so that when

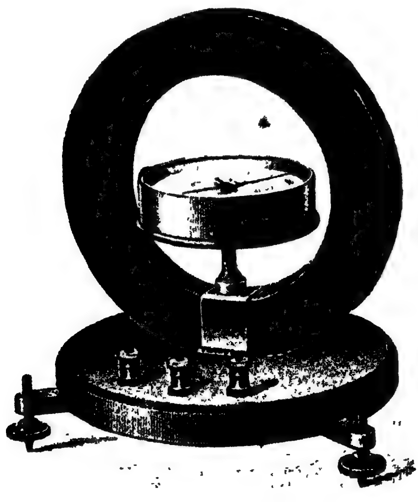

FIG. 226.—Tangent Galvanometer.

a current flows through the coil the magnetic force due to it is at right angles to the meridian. A magnetometer is placed at the

centre of the coil, the needle of which is acted on by this force F and by the horizontal component H of the earth's field.

These forces being mutually perpendicular, the needle will be deflected from the direction of the earth's field through an angle θ such that $F = H \tan \theta$, see p. 407.

If the galvanometer coil consists of n turns,

$$F = \frac{2\pi n C}{r}.$$

But, since $$F = H \tan \theta,$$

we obtain $$\frac{2\pi n C}{r} = H \tan \theta,$$

or $$C = \frac{Hr}{2\pi n} \tan \theta.$$

Since H can be measured in C.G.S. units (p. 459), this equation gives the current C in terms of quantities which can all be expressed in C.G.S. units.

The form of the tangent galvanometer is sometimes more complicated. In the general case

$$F = GC$$

G is called the **galvanometer constant.**

If C is unity, $G = F$, *i.e.* **the galvanometer constant is numerically equal to the strength of the magnetic field at the centre of the coil when unit current is flowing through it.**

Then $$C = \frac{H}{G} \tan \theta,$$

or $$C = K \tan \theta,$$

where K is called the **reduction factor**, or simply the **factor**,[1] of the galvanometer.

When $\theta = 45°$, $\tan \theta = 1$, and $C = K$, or the reduction factor is numerically equal to the current required to produce a deflection of 45°.

[1] Some writers call K the constant of the galvanometer. As K is *not* constant this cannot be recommended.

Expt. 220. **To set up a Tangent Galvanometer and to measure a Current in Absolute Units.**—Place the galvanometer in such a position that the pointer of the needle in the magnetometer box at the centre of the coil lies along the *zero line* of the magnetometer scale. If the instrument is properly designed and correctly made, the coil will now be exactly over the needle, the plane of the coil being therefore in the magnetic meridian.

In some forms of tangent galvanometer now on the market the magnetometer box has no arrangement fitted whereby it can be fixed relatively to the coil. In this case, before anything else is done, the zero line must be set as accurately as may be along the axis of the coil, and care must be taken to prevent it from being moved during the experiment. The adjustment given above must next be made.

Adjust the level of the instrument so that the needle swings freely. Send a current through one coil of the instrument from a Daniell cell, placing a resistance in series so as to give a deflection between 30° and 50°. Use a commutator to reverse the direction of the current. Read both ends of the needle, with the current flowing first in one direction then in the other. Measure the radius of the coil as accurately as possible and count the number of turns through which the current was flowing. Calculate the strength of the current in absolute units, and also in amperes.

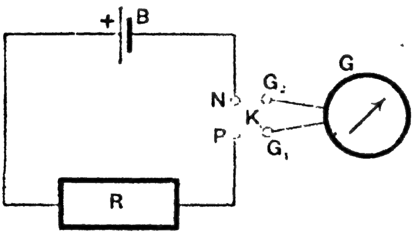

Fig. 227.—Connections for Tangent Galvanometer.

Note.—For descriptions of Commutators, Resistances and Rheostats, the student is referred to the chapter 'Notes on Electrical Apparatus' (pp. 580-588).

§ 2. Ammeters

Although the tangent galvanometer serves to determine the absolute value of the strength of an electric current, its use for practical current measurement is inconvenient for many reasons. Two of the most important objections to its use are—

(*a*) Because the deflections of the needle are not proportional to the current.

(*b*) Because the deflection for a given current depends on the external magnetic field.

The first difficulty can be got over by graduating the scale so that the readings are proportional to the tangents instead of proportional to the angles.

The second objection is more serious. Any instrument in which the determination depends on an external magnetic field is unsuited for use in the neighbourhood of large masses of iron, while in such places as Electric Supply Stations it cannot be used at all, owing to the enormous *variable* fields set up by the working of dynamos and other electrical machinery. Further, great inconvenience attends the use of the tangent galvanometer in consequence of the fact that it must be placed in a definite position relative to the field and cannot be used in any other.

Instruments arranged so as to secure *direct* readings of the strength of a current in amperes (or multiples of an ampere) are usually called *ampere-meters*, or, as the word is usually contracted, **ammeters.** These ammeters are designed in various ways: some depend on the elongation of a wire due to the heating effect of the current flowing through it; others on the attraction, or mutual turning effect, between two coils carrying the current: but the majority depend on the rotation of a small coil carrying a definite fraction of the current when placed in the strong magnetic field between the poles of a permanent magnet.

THE MOVING-COIL AMMETER

This instrument is very important, but to understand its action requires a greater knowledge of the subject than the student at this stage is supposed to possess. This lack of knowledge need not prevent its *use*, however, as the method of using it is so simple. A description of the apparatus is given in a later chapter (p. 577).

THE ATTRACTED IRON AMMETER

A form of ammeter simple to understand is the **Attracted Iron Ammeter.**

In the simplest form of attracted iron ammeter [1] there is suspended from a spiral spring a bar of iron, whose lower end just enters a long coil, or solenoid, of wire.

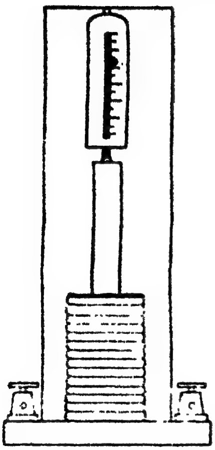

FIG. 228.—Attracted Iron Ammeter.

When a current passes round this coil, the iron bar is magnetised and is attracted some distance into the coil. The amount of motion which takes place depends on the force of attraction and the stiffness of the spring. The iron moves down until the attracting force is just equalised by the extra tension of the spring due to its elongation.

Now for a given current there is a definite pull exerted on the iron by the coil, and therefore the spring will always stretch to the same amount when this current passes round the coil. The relation between the pull on the iron and the current in the coil is, however, not simple by any means; in fact, no single law which would apply to every case could be given to express this relation. The instrument thus differs from the tangent galvanometer in that the relation between *extension* and *current* is **empirical**, *i.e.* can be found only by trial, while the tangent relation between the deflection and current in a tangent galvanometer can be predicted from theoretical considerations.

EXPT. 221. **Calibration of an Attracted Iron Ammeter.** —Set up a tangent galvanometer for use (p. 462). Connect in series with it through a reversing key, the ammeter to be calibrated, a cell capable of supplying large currents, and a rough regulating resistance (with a length of bare platinoid wire to give closer adjustment if required). Use the *thick* wire turns of the galvanometer. The connections are as in Fig. 229.

An ordinary resistance box must not on any account be used in this experiment: the large currents used would completely ruin the coils.

Both the ammeter and the regulating resistance must be

[1] See also p. 576.

kept as far as possible from the tangent galvanometer, so as to diminish the effect of the magnetic fields due to them on the needle of the galvanometer. The wires leading to the tangent galvanometer should be twisted together so that the magnetic field due to one may neutralise that due to the other. *Twin* flexible connections are very useful in this case.

Observe the reading of the pointer of the spring balance and of the needle of the galvanometer when a current is passing round the circuit.

Repeat the observations for different values of the current, choosing values which will give deflections of the galvanometer increasing by approximately 5° at a time.

Determine the number of turns (usually one or two in this experiment) in the coil of the galvanometer, and the radius of the coil. The value of H, the horizontal component of the earth's magnetic field, can be found by the method described in the magnetism course (p. 427). It may be taken as about 0·185 in London at the present time (1915).

The expression for the current in absolute units (p. 461) is

$$C = \frac{rH}{2\pi n} \tan \theta,$$

or in amperes

$$C \text{ (amp.)} = \frac{5rH}{\pi n} \tan \theta.$$

The results should be arranged in tabular form under the headings :—

Ammeter Reading.	θ.	$\tan \theta$.	C (amp.).

Plot a curve showing ammeter readings as abscissae, and currents as ordinates. This curve can then be used at any time to reduce ammeter readings to currents in amperes.

EXPT. 222. **The Calibration of an Ammeter already graduated.**—Join up in series a 2-volt accumulator, an adjustable resistance, an ammeter and the tangent galvanometer.

Note the following points :—

(1) Use the galvanometer terminals connected to the single turn of thick copper wire.

(2) Use a commutator so as to be able to reverse the current

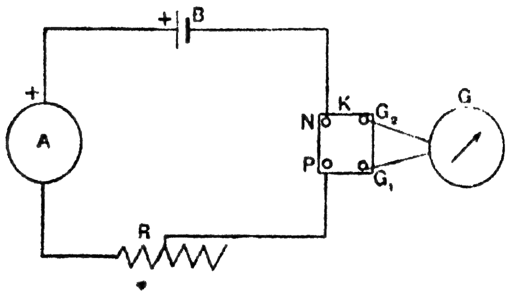

FIG. 229.—Calibration of an Ammeter.

A, ammeter reading to 3 or 5 amperes.
R, resistance of about 5 to 7 ohms.
B, 2-volt accumulator.
G, tangent galvanometer.
K, reversing switch.

through the galvanometer (not through the ammeter) and take readings on both sides of the zero.

(3) Be careful to connect the + pole of the accumulator to the + terminal of the ammeter. (This is immaterial with a hot-wire instrument.)

(4) As most ammeters contain a strong permanent magnet, the ammeter must be placed as far as possible from the tangent galvanometer.

(5) Twin wires must be used from the commutator to the ammeter, or else the two wires used must be twisted together; otherwise the field due to the current in these wires will have an appreciable effect on the galvanometer reading.

Take a series of readings of the ammeter and galvanometer for different values of the resistance. The resistance should be adjusted so as to alter the current by about half an ampere between the readings.

Measure carefully the diameter of the coil of the tangent galvanometer, using a pair of beam compasses.

Calculate the galvanometer constant G, and also the reduction factor K.

$$G = \frac{2\pi n}{r},$$

$$K = \frac{H}{G} = \frac{rH}{2\pi n}.$$

Then calculate the current through the galvanometer in absolute units and deduce the value in amperes.

$$C = \frac{rH}{2\pi n} \tan\theta$$

in electromagnetic units, and one absolute electromagnetic unit equals 10 amperes.

The table of observations should be arranged as below :—

Ammeter Reading, A, in Nominal Amperes.	Tangent Galvanometer.				$\frac{A}{C}$.
	Deflection θ.	tan θ.	C (abs. units).	C (amp.).	

DISCUSSION OF EXPERIMENTAL RESULTS

The errors of an ammeter may be divided into two types :—

(*a*) If the last column, giving the ratio of A to C, is a constant, it means that the instrument is *self-consistent*, *i.e.* that the current is proportional to the indications of the ammeter even if not actually equal to them. Any error, therefore, is a *proportional* error, and the true current can be obtained by multiplying the current indicated by the ammeter by a certain factor which is the same for all parts of the scale.

To determine the Correction Factor, calculate the mean of the approximately equal values of A/C. The reciprocal of this quantity is the correction factor, for the true current is given by multiplying the ammeter reading by the mean value of C/A.

(*b*) If the values of A/C are not constant within the limits of

experimental error, a correction table should be arranged as follows:—

Reading, A.	True Current, C.	Correction, C−A.

A correction curve should be plotted, with C − A as ordinate and A as abscissa. The ordinate corresponding with any reading gives the correction to be *added* to that reading to give the true current.

Any zero error may be included in this curve.

NOTE.—In case (*a*) the value of C is calculated on an assumed value of H. If A/C is not unity, the divergence may be due to an error in the assumed value of H. The value of H determined in the experiment on the earth's field *for the particular place where the galvanometer is used* should be taken. If H is not known with certainty, it should be determined and C recalculated before concluding that the ammeter is incorrect.

§ 3. OHM'S LAW

Ohm's Law (1827) states that when two points are taken on a linear conductor, the ratio of the difference of potential E between those points to the current C flowing through the conductor is a constant, that is, it depends only on the form, dimensions and physical condition of the conductor. This constant ratio is termed the **resistance** R of the conductor. Thus

$$\frac{E}{C} = R.$$

If E and C are measured in C.G.S. electromagnetic units, then R will also be in C.G.S. units. If practical units are employed C will be in amperes, E in volts, and R in ohms. The ohm = 10^9 C.G.S. units. For purposes of practical measurement the **International Ohm** is defined as the resistance of a

column of mercury at 0° C., 14·4521 gm. in mass, of a constant cross section, and of length 106·300 cm.

The reciprocal of the resistance is termed the **conductance.**

Ohm's Law may be extended to a complete circuit, if E now represent the electromotive force (E.M.F.) in the circuit and R the total resistance of the circuit.

Hence the current flowing round the circuit is given by

$$C = \frac{E}{R}.$$

The current has the same value at each point of the circuit, and may be measured by introducing a tangent galvanometer at any part of the circuit. The strength of the current is then given by

$$C = K \tan \theta,$$

where K is a constant called the **reduction factor**, or simply the **factor**, of the galvanometer.

Combining these two values for C we obtain

$$\frac{E}{R} = K \tan \theta,$$

or

$$\frac{E}{K} = R \tan \theta.$$

Hence if E, the electromotive force in the circuit, is constant, $R \tan \theta$ must be a constant quantity.

EXPT. 223. **An Experiment illustrating Ohm's Law and the Law of the Tangent Galvanometer.**—Join up in series with the galvanometer a 2-volt accumulator, a resistance box, and a key.* As the internal resistance of the accumulator is small, and a large current would damage the resistance coils, at least 30 ohms must always be kept in the circuit, *i.e.* R must be not less than 30 ohms. Sometimes the tangent galvanometer is provided with a number of terminals on one side of the stand. In this experiment the two terminals connected with the largest number of turns should be used, so that the current may pass through all the coils of the galvanometer. Commence by having in the circuit all the resistance in the box (when the plugs are taken out of the box, resistance is put in the

circuit), and observe the deflection of the tangent galvanometer with the current flowing first in one direction and then in the other. Take the mean of the two readings as the measure of the true deflection.

Make a series of observations, taking out in succession plugs corresponding to resistances of (say) 210, 190, 170, 150, 130, 110, 90, 70, 50, and 30 ohms.

Make a table of the results thus :—

R ohms.	Deflection θ.	tan θ.	R tan θ.

If R tan θ is constant R must be proportional to cot θ. Plot a graph, taking values of R as abscissæ and the values of cot θ as ordinates. This should yield a straight line.

The constancy of the last column of the table is a consequence of the two laws $C = K \tan\theta$ and $C \propto \frac{1}{R}$.

In taking R, the resistance in the box, as the *full* resistance of the circuit, it is assumed that the resistances of the galvanometer and of the battery are negligible. If this is not the case, a value x equal to the sum of these (assumed known) should be added to R and another column $(R+x)\tan\theta$ drawn out. This will be more nearly constant than R tan θ.

If x is not known, an approximate value for x can be obtained from the first and last numbers in the column R tan θ.

Let the first resistance be R_1 and the deflection corresponding to this θ_1, the last resistance being R_2 and the deflection θ_2. Then we know that

$$(R_1 + x)\tan\theta_1 = (R_2 + x)\tan\theta_2,$$

whence

$$x = \frac{R_2 \tan\theta_2 - R_1 \tan\theta_1}{\tan\theta_1 - \tan\theta_2}.$$

Substitute this value of x in the expression $(R+x)\tan\theta$ and calculate the value of $(R+x)\tan\theta$ for each set of observations.

The type of variation of the column R tan θ, if accurately determined, affords an interesting example of the effect of a *systematic error* involved in an experiment. It will be found that R tan θ *on the whole increases steadily* as the higher resistances are approached. This is due to the fact that the quantity x which has been neglected becomes of less and less *relative* im-

portance as the total resistance gets larger, so that the values of R tan θ approach the true constant $(R+x)$ tan θ more and more closely as R is increased.

Whenever a *regular* increase or decrease takes place in a quantity which should be constant, as one factor of the constant is altered steadily, a *systematic error* of this type should be looked for in the experiment or in the method of working out.

RESISTANCE BY THE METHOD OF SUBSTITUTION

When a resistance box, containing a number of coils of known resistance arranged in series, is available, a simple method of finding the value of an unknown resistance is that known as the **Method of Substitution**. A current from a cell or battery of constant E.M.F. is passed through the unknown resistance and through a galvanometer, and the deflection of the galvanometer is observed.

The type of galvanometer used is immaterial provided a reasonable deflection can be obtained with the resistance and E.M.F. available. If the deflection be too large, it may be diminished by *shunting* the galvanometer, that is by joining its terminals by a resistance, such as a piece of platinoid wire, so that only a fraction of the whole current passes through the galvanometer. A tangent galvanometer will serve well for this experiment in general.

The unknown resistance is then replaced by the resistance box, and the resistance of the latter adjusted till the galvanometer deflection has the same value as before. Then obviously, if the E.M.F. has remained constant, the unknown resistance must be equal to that obtained by the use of the resistance box.

EXPT. 224. **Determination of a Resistance by the Method of Substitution.**—Connect up *in series* a cell B, a galvanometer G, and the resistance R which is to be determined. If a tangent galvanometer is used, it should be set up with a commutator K, in the manner described on p. 462, with the current passing through *all* the turns of the galvanometer. Where this is not possible, the coil having the largest number of turns must be used.

The cell used may be a Daniell cell, since that gives a

constant electromotive force, or it may be a secondary cell. In the latter case, since the internal resistance of the cell is small, *great care must be taken in the second part of the experiment when the resistance box is substituted for the unknown resistance.*

Measure the deflection with the unknown resistance in the circuit, reading both ends of the needle or pointer of the galvanometer, with the current flowing first in one direction, then in the other.

Replace the unknown resistance by the resistance box, *from which all the plugs should have been removed*, and adjust the value of the resistance *down* to such a value that the mean deflection may have the same value as before. *The resistance must not be reduced below 30 ohms in any case.* Then the sum of all the numbers from which plugs are *absent* in the resistance box represents the value of the unknown resistance.

Note on Determination of Resistance by Substitution.—The degree of accuracy obtainable by this experiment is not at all large. It is an experiment whose accuracy depends on the accuracy of reading deflections, and therefore is inaccurate to the same degree as the readings of the deflections may be inaccurate, viz. to 2 or 3 per cent.

Also, the adjustment of the resistance in the resistance box is only possible in definite steps of 1 ohm (or possibly 0·1 ohm if a "decimal" ohm box is used). The value of the equivalent resistance substituted for the unknown can never be correctly adjusted except in the rare cases when the unknown resistance is an integral number of ohms (or tenths of an ohm).

Furthermore the limits of resistance for which the method is at all suitable depend largely on the galvanometer used. For resistances between 30 and 70 ohms an ordinary tangent galvanometer is useful. Above 70 ohms or thereabouts, a more sensitive type of galvanometer must be used. The method is entirely unsuited for *low* resistances.

The only way to test whether the unknown resistance is of magnitude suitable for determination by this method is to connect it up to the roughest form of galvanometer available. If the deflection is small, 5° or so, the method may be used, but a more delicate galvanometer must be employed. If the deflection is between 10° and 70° the galvanometer first chosen may be used. If the deflection is above 70° with the roughest galvanometer available, the method is quite unsuitable for this particular resistance and another way must be used (see Wheatstone's Bridge, p. 496).

RESISTANCES IN SERIES AND IN PARALLEL

If resistances R_1, R_2, R_3, etc., are connected in *series*, their equivalent resistance R is the *sum* of the separate resistances.

Resistances connected in *parallel*, however, have an equivalent resistance *less than any one* of the constituent resistances; the total *conductance* in this case is the sum of the conductances of the constituent resistances, *i.e.*

For resistances in Series

$$R = R_1 + R_2 + R_3 + \text{etc.}$$

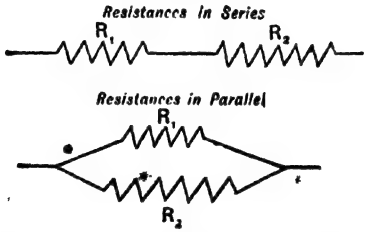

FIG. 230.—Resistances in Series and in Parallel

For resistances in Parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \text{ etc.}$$

EXPT. 225. **An Experiment on Resistances in Series and in Parallel.**—Measure the value of two separate resistances R_1 and R_2 by the method of substitution. Next place the two resistances in series with one another, and measure the resultant resistance, R, also by the method of substitution.

Verify the result that $R = R_1 + R_2$. Finally arrange the two resistances in parallel with one another and measure the equivalent resistance, S, in the same way.

Verify the result that $\frac{1}{S} = \frac{1}{R_1} + \frac{1}{R_2}$.

GALVANOMETER SHUNTS

When a resistance of S ohms is placed in parallel (*i.e.* as a shunt) with a galvanometer of resistance G ohms the current flowing through the galvanometer is reduced in general. When, however, a constant P.D. is applied to the galvanometer termi

nals, shunting the galvanometer has no effect on the current flowing through it.

It is assumed that the total current flowing round the circuit is unaltered by the introduction of the shunt. This will be practically true provided the resistance of the remainder of the circuit is large compared with the resistance of the galvanometer.

Let C = total current flowing round the circuit,
C_1 = current through the galvanometer,
C_2 = current through the shunt,

then $C = C_1 + C_2$.

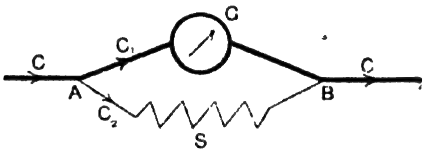

FIG. [illegible].—Principle of Galvanometer Shunt.

Let E = difference of potential between A and B. By Ohm's Law

$$E = C_1 G,$$
$$E = C_2 S.$$

So
$$C_2 S = C_1 G.$$

Hence
$$\frac{C_2}{C_1} = \frac{G}{S}.$$

Add 1 to each side,
$$\frac{C_2}{C_1} + 1 = \frac{G}{S} + 1,$$

or, finally,
$$\frac{C_2 + C_1}{C_1} = \frac{G + S}{S} = \frac{C}{C_1}.$$

Hence if the ratio C/C_1 is found, the value of the resistance of the galvanometer G can be found in terms of S.

If a tangent galvanometer be used, the current is given by the equation $C = K \tan \phi$, where K is the reduction factor and ϕ the deflection of the galvanometer due to the current C.

Denoting the deflection without the shunt by ϕ, and the deflection when shunted by ϕ_1, we have

$$\frac{C}{C_1} = \frac{K \tan \phi}{K \tan \phi_1}.$$

But

$$\frac{C}{C_1} = \frac{G+S}{S},$$

therefore

$$\frac{G+S}{S} = \frac{\tan \phi}{\tan \phi_1},$$

whence

$$G = S\left(\frac{\tan \phi}{\tan \phi_1} - 1\right).$$

The student must deduce this from the above.

EXPT. 226. **Determination of the Resistance of a Galvanometer by Shunting.**—Connect up, as shown in Fig. 232, a secondary cell B, a reversing switch K, and a resistance R (which must be *at least* 10 ohms) with the galvanometer G and the shunt S.

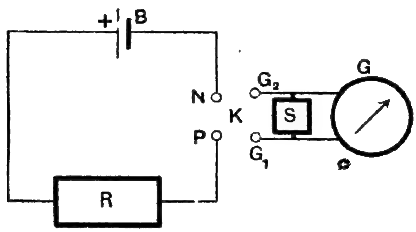

FIG. 232.—Resistance of Galvanometer by Shunting.

Observe the deflection of the galvanometer before connecting the shunt to it, then connect up the shunt and find the deflections corresponding with *various* shunt resistances; a suitable range of S for an ordinary tangent galvanometer would be from 1 to 20 ohms.

The current must be reversed for each reading and the mean of the deflections taken as the true value of ϕ_1.

Arrange the observations in a table as shown :—

S.	ϕ_1.	$\tan\phi_1$.	$\frac{\tan\phi}{\tan\phi_1}-1$.	$S\left(\frac{\tan\phi}{\tan\phi_1}-1\right)$.
1				
2				
3				
4				
5				
7				
10				
15				
20				
∞	$\phi=$			

When no shunt is connected, the shunt resistance is infinite, the deflection in this case is the full deflection ϕ.

The last column will be approximately constant, its mean may be taken as the value of G.

NOTE.—The method of working out the value of G in this experiment is based on the assumption that the total current is not appreciably altered when the shunt resistance is connected up. Unless R is at least 20 times the value of G, this assumption is not sufficiently near to the truth. If, therefore, the value of the numbers in the last column is greater than 5 per cent of R, the experiment should be repeated, using a resistance R of the requisite magnitude.

When G is only 5 per cent of R the maximum variation of current cannot exceed 5 per cent, even when G is entirely short-circuited, and the errors in the deflections observed may introduce errors of this magnitude into the result. The *best* value of G corresponds with a shunt S which gives $\tan\phi_1 = \frac{1}{2}\tan\phi$.

The value of G may also be calculated from the expression

$$\frac{RG}{S(R+G)} = \frac{\tan\phi}{\tan\phi_1} - 1,$$

even when R is less than 20 times G.

CHAPTER IV

ELECTROMOTIVE FORCE AND INTERNAL RESISTANCE OF A CELL

§ 1. Simple Discussion of the Action of a Voltaic Cell

The following discussion is not to be taken as a theory of the fundamental electrolytic and chemical actions which are concerned in the action of a voltaic cell, but is to be used merely as a useful *working hypothesis* of the action in so far as it affects the P.D. between the terminals, etc.

CELL ON 'OPEN CIRCUIT'

Two plates of different metals immersed in a suitable solution at once acquire a difference of potential. In the following treatment the case of a simple cell will be considered, the terms copper and zinc being used to indicate the positive and negative plates respectively, though the general account of the happenings detailed will apply to *any type of cell whatsoever.*

As soon as the plates are put in the liquid, positive electricity begins to move through the liquid towards the copper. This positive electricity may be supposed to come from the zinc plate, which is accordingly left negatively charged. The motion of the positive electricity is due entirely to the chemical nature of the cell, and the electromotive force producing the flow of electricity may be termed an **Electromotive Force of Chemical Action or a Chemical E.M.F.**

This chemical E.M.F. urges positive electricity from the

zinc plate to the copper plate through the liquid in the cell, that is to say, **the E.M.F. of a cell acts from the negative pole to the positive pole of the cell.**

This statement is universally true for all cells, and it must be noted that the term the *E.M.F. of a cell* means simply the force driving electricity through the cell, and as such it can only be used with reference to the interior action of the cell.

The positive electricity carried across the cell from the zinc to the copper causes the potential of the copper plate to be raised above that of the zinc, and there are now *two* forces acting on any electric charge inside the cell. A positive charge inside the cell is urged from the zinc to the copper by the E.M.F. of the cell, and is urged from the copper to the zinc in consequence of the *potential difference* between these two, this P.D. having been acquired as a result of the action of the chemical E.M.F. of the cell.

The P.D., so far from being identical with the E.M.F. of the cell, as is often imagined to be the case, is only a result of the action of this E.M.F.; inside the cell the P.D. and the E.M.F. act in opposition.

The plates are not connected externally in any way, since the cell is on open circuit, and therefore the potential difference continues to rise, due to the accumulation of electricity on the two plates. It cannot rise indefinitely, however, for there is only a limited E.M.F. acting inside the cell. The potential difference will rise to such a value V′ that the tendency of the positive electricity to flow from zinc to copper under the E.M.F. of the cell is just balanced by its tendency to flow from copper to zinc due to the P.D. When this balance between the E.M.F. and the P.D. occurs, no motion of electricity will take place in either direction through the cell, and all action ceases.

Thus **on open circuit,** or generally, **when no current is flowing in either direction through the cell, the P.D. between the plates of the cell will be equal to the E.M.F. of the cell.**

Again, it must be pointed out that the two quantities are not identical, the P.D. on open circuit, V′, acts so as to send

positive electricity from the copper to the zinc, while the E.M.F. of the cell, E, which only exists inside the cell, acts from the zinc to the copper.

On open circuit $$V' = E.$$

It is impossible to measure E directly; we can only measure an

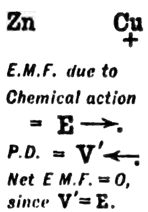

FIG. 233.—Cell on Open Circuit.

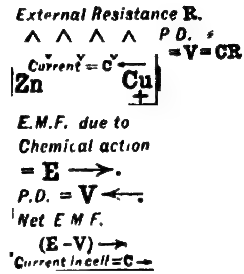

FIG. 234.—Cell on Closed Circuit.

E.M.F. by the P.D. it creates, therefore to measure the E.M.F. of a cell we measure the P.D. across its terminals when on open circuit, the value of this P.D. (V′) being equal to the E.M.F. of the cell, E.

CELL ON 'CLOSED CIRCUIT'

Outside the Cell.—Suppose the plates of the cell to be connected by a wire of resistance R. The electricity at once begins to flow through this wire under the action of the P.D. between the plates. There is no chemical action in the wire at all, and hence the P.D. is the only force driving electricity through it, and therefore the electricity will flow from copper to zinc through the wire.

At once, as soon as the wire is connected up, the potential difference between the two plates will begin to diminish owing to the passage of electricity from one plate to the other. If V be the value of the potential difference at any instant, the current flowing through the wire will be

$$C_1 = \frac{V}{R},$$

C_1 representing the current in the *external circuit, i.e.* the current in the wire.

Inside the Cell there is still the E.M.F. of the cell acting, and the value of this will not have altered in the slightest (polarisation effects will be considered later) since it is a property of the chemical constitution of the cell. Its action is not now opposed by the potential difference V′, but by a diminished P.D., V. The E.M.F. will therefore begin to drive electricity through the cell again, from zinc to copper, the *net* driving force being the difference between the E.M.F. of the cell and the value of the now diminished P.D., V. If the resistance of the cell (its *internal* resistance) is B, the current flowing inside the cell from zinc to copper will be

$$C_2 = \frac{E - V}{B}.$$

Thus, we have going on simultaneously a flow of electricity away from the copper plate through the *external* circuit equal to

$$C_1 = \frac{V}{R}$$

units of electricity per second, and a flow of electricity towards the copper plate from the inside of the cell equal to

$$C_2 = \frac{E - V}{B}.$$

As V gets smaller and smaller, the rate of loss of electricity from the copper plate, C_1, will diminish, and the rate of gain of electricity, C_2, will increase. When these two become equal, V will become steady again, though of course less than V′, and we shall have

$$C_1 = C_2,$$

and

$$\frac{V}{R} = \frac{E - V}{B}.$$

Thus, when a cell has its external circuit completed through a simple resistance R, the P.D. between the plates falls to some value V, such that the current inside the cell from zinc to copper

is equal to the current flowing outside the cell from copper to zinc. The relation between the new P.D. (V), the E.M.F. (E) of the cell and the internal and external resistances being

$$\frac{V}{R} = \frac{E - V}{B}.$$

This relation can also be proved by a *different method* *(p. 487)*.

It will, of course, be realised that the adjustment of the P.D. to the value $V' = E$ is almost instantaneous, and the P.D. falls to its steady value V in an extremely minute fraction of a second when the external circuit is closed.

When a current is driven through a cell from copper to zinc (or from positive to negative) the potential difference *applied* has to be greater than E, because it overcomes E and also overcomes the resistance of the battery. The student is recommended to investigate this case in the same manner as above, and to show that the current driven through the cell or battery from positive to negative is given by

$$C = \frac{V - E}{B},$$

where V is the applied P.D. This result is useful in charging accumulators.

Effect of Polarisation.—Polarisation occurs in a cell due to any change in the chemical constitution of the cell. If too large a current is taken from the cell, the liquid round the zinc becomes 'used up,' or the oxidising agent near the positive plate is unable to cope with the rapid generation of hydrogen there, and so the plate becomes coated with hydrogen. The chemical constitution of the liquid just near the plates is thereby changed, and the plates themselves are altered in character. The E.M.F. of the cell is changed in consequence and is not restored to its original value until diffusion of the liquid and oxidation of the hydrogen has reproduced the original conditions. It is assumed in the foregoing discussion that the cell is never overloaded to such an extent as to produce polarisation.

§ 2. Comparison of Electromotive Forces of two Cells

SUM AND DIFFERENCE METHOD USING A GALVANOMETER

This method enables us to compare the electromotive forces of two cells, but does not give an absolute measure of the electromotive force.

In addition to the cells or batteries to be compared, a galvanometer or some other instrument for measuring the strength of a current is required, together with a resistance to adjust the current to a suitable value. Let E_1 be the E.M.F. of the first cell, the resistance of which is B_1, and E_2 the E.M.F. of the second cell, the resistance of which is B_2. Let G be the resistance of the galvanometer and R the resistance of the rest of the circuit. None of these resistances need be known, but they must *all* be constant during the whole of the experiment.

The cells are first connected in series with the resistance and the galvanometer, arranging the cells so as to *assist* each other: the E.M.F. in circuit now is the sum of the E.M.F.s of the two cells.

By Ohm's Law and the definition of resistance

$$\frac{\text{E.M.F. in Circuit}}{\text{Resistance of Circuit}} = \text{Current flowing round circuit},$$

i.e.
$$\frac{E_1 + E_2}{R + G + B_1 + B_2} = C_1.$$

C_1 is the current flowing in the circuit, and is measured by the deflection of the galvanometer.

One of the cells is now reversed—for preference this should be the *weaker*, say E_2, though it is immaterial which cell is reversed if the galvanometer is used with a reversing commutator.

The E.M.F. now in circuit is $E_1 - E_2$, and the current will have some value C_2, given by

$$\frac{E_1 - E_2}{R + G + B_1 + B_2} = C_2.$$

C_2 is measured by the deflection of the galvanometer and then, *since none of the resistances have been altered*, we have

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{C_1}{C_2},$$

or

$$\frac{E_1}{E_2} = \frac{C_1 + C_2}{C_1 - C_2}.$$

Expt. 227. **Comparison of E.M.F.s by the Sum and Difference Method using a Tangent Galvanometer.**—The method may be used to compare the E.M.F.s of a Leclanché and of a Daniell cell, or to compare either of these with that of an accumulator.

Set up a tangent galvanometer G with a commutator K as described on p. 462, connecting in series with the galvanometer, a resistance box, R, *from which all the plugs have been removed.* The current should go through all the coils of the galvanometer.

Arrange the two cells B_1, B_2 in series so as to assist each other, and connect them so as to send a current through the galvanometer and the resistance box in series, the connections being made so that the current through the galvanometer can be reversed (Fig. 235).

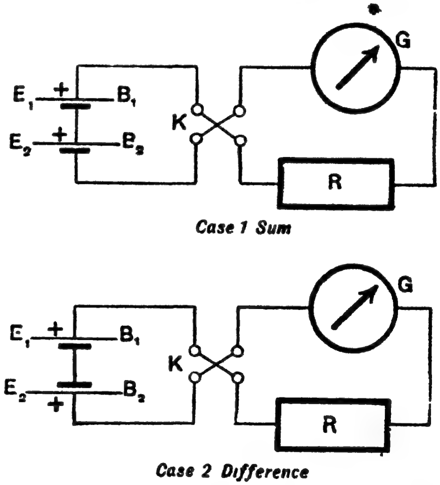

Fig. 235.—E.M.F.s by Sum and Difference.

Reduce the resistance of the box by inserting plugs until the deflection of the galvanometer is about 60° to 70°: *In no case must the resistance of the box be reduced to less than* 30 *ohms.*

Read the deflection of the galvanometer with the current passing through it, first in one direction, then in the other: let the mean of the readings be θ_1. Then the current C_1 is given by

$$C_1 = K \tan \theta_1,$$

where K is the reduction factor of the galvanometer (this need not be known).

Reverse the weaker of the two cells as shown in Fig. 235, but do not alter any other connection in the circuit; *if possible avoid shaking the cells* while disconnecting B_2 and connecting it up reversed, otherwise their internal resistances will be changed.

If θ_2 is the mean deflection observed with B_2 reversed,

$$C_2 = K \tan \theta_2.$$

Thus, since
$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{C_1}{C_2} \qquad \text{(p. 483)}$$

we have
$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{K \tan \theta_1}{K \tan \theta_2} = \frac{\tan \theta_1}{\tan \theta_2}.$$

From this it follows that

$$\frac{E_1}{E_2} = \frac{\tan \theta_1 + \tan \theta_2}{\tan \theta_1 - \tan \theta_2}.$$

Calculate in this way the ratio of the E.M.F.s of the two cells used.

If one cell is a Daniell, with a solution of $ZnSO_4$ as the exciting liquid, its E.M.F. may be taken as 1·08 volts, and the E.M.F. of the other cell can be calculated on this assumption from the ratio obtained by experiment.

THE POTENTIOMETER

The **potentiometer** is an apparatus used for the comparison of electromotive forces. It is usually in the form of a long uniform wire stretched on a flat board, which is fitted with a sliding key so that contact may be made with any desired point on the wire. When the wire is very long, it is often arranged in zigzag fashion on the board so as to economise space, or a number of parallel wires may have their ends so connected by thick pieces of copper that a current may be passed through them in series. In considering the theory of the apparatus it will be simpler to think of a single straight wire (Fig. 236).

A constant battery S (which may consist of one or two secondary cells) is used to send a steady current through

the uniform wire AD. The point A is connected with the positive pole of the battery, so that there is a fall of potential from the point A to the point D. If the wire be uniform, the potential will diminish in a regular way as we pass from A to D.

The object of the experiment is to compare the E.M.F.s of two cells, which may be called E_1 and E_2. The first cell has its positive terminal connected to the point A, its negative terminal is connected through a galvanometer G to the sliding key, which makes contact with a point P on the potentiometer wire. By moving the key backwards and forwards, a point P_1 is found such that no deflection of the galvanometer is observed when the key is pressed down. When this is the case no

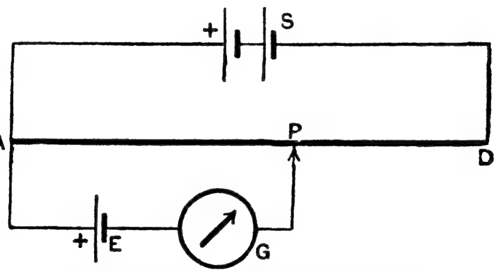

FIG. 236.—Principle of the Potentiometer.

current flows through the galvanometer. If no current flows through the galvanometer, the potential of the point P_1 must be the same as the potential of the pole of the cell connected to the galvanometer, *i.e.* the fall in potential through the cell must be exactly equal to the fall of potential along the wire between A and P_1. There is, however, no current flowing through the cell, therefore the P.D. between the plates is equal to the E.M.F. of the cell (p. 478). The condition that is satisfied, therefore, is that the electromotive force E_1 of the cell under test is exactly equal to the difference of potential between A and P_1.

The same process is then carried out with the cell E_2, and a point P_2 is found, such that the electromotive force E_2 exactly balances the difference of potential between A and P_2.

Hence $$\frac{E_1}{E_2} = \frac{\text{Diff. of potential between A and } P_1}{\text{,, ,, ,, A and } P_2}.$$

Now if the steady current through the potentiometer wire be C,

P.D. between A and $P_1 = C \times$ Resistance of AP_1 and
P.D. between A and $P_2 = C \times$ Resistance of AP_2.

Thus $$\frac{E_1}{E_2} = \frac{\text{Resistance of } AP_1}{\text{Resistance of } AP_2}$$

$$= \frac{\text{Length of } AP_1}{\text{Length of } AP_2},$$

assuming the wire to be uniform.

The comparison of E.M.F.s is carried out by comparing the lengths of the potentiometer wire necessary to secure a balance.

EXPT. 228. **Comparison of the E.M.F.s of two Cells by means of a Potentiometer.**—Connect a constant cell or battery S to the ends of a potentiometer wire AD, the positive terminal of the battery being connected to A. To A connect the positive terminal of one of the cells E_1 to be compared, connecting its negative terminal to a galvanometer. The sliding contact P which moves along the potentiometer wire is connected to the other terminal of the galvanometer, and P is moved along the wire until a point is found where no deflection is produced in the galvanometer when P is depressed so as to make contact with the wire. The length AP_1 on the potentiometer wire is then measured. The second cell E_2 is then substituted for E_1, and the length AP_2 determined.

Since there may be some change taking place in one or other of the cells in use in the experiment, it is necessary to repeat the observations, using first one and then the other of the two cells under test. It is accordingly convenient to introduce a two-way switch, so that it may be possible to change quickly from one cell to the other. By changing over rapidly in this way, the adjustments can be made in a very short time, and thus there will be less chance of error due to variation of the steady current in the wire.

The apparatus would then be arranged as in Fig. 237. K represents the two-way switch which connects A to E_1 or E_2

at will. Determine the mean values of AP_1 and AP_2, and calculate the ratio of E_1 to E_2.

As a confirmation of the first comparison connect the two

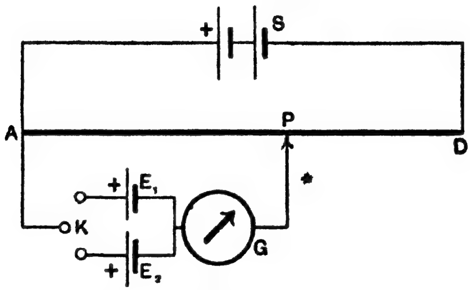

FIG. 237.—Connections for Potentiometer.

cells in series (*a*) so as to assist one another; (*b*) so as to oppose one another, and compare the resultant E.M.F.s. If l_1, l_2 are the readings on the potentiometer wire corresponding to these two cases we have

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{l_1}{l_2}$$

and therefore

$$\frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2}.$$

NOTE.—It is obvious that if the *negative* pole of the constant cell were connected to A_1 and the *negative* pole of the cells under test were also connected to A_1, the experiment could be carried out just as well: the *rise* of potential from A to P along the wire would be equal to the E.M.F. of the cell when no current flows in the galvanometer.

§ 3. Measurement of the Internal Resistance of a Battery

The internal resistance of a battery can be measured by means of a voltmeter and a suitable resistance. If a cell with an internal resistance of B ohms be connected to a wire of resistance R ohms, then according to Ohm's Law the current, C, will be given by the equation

$$C = \frac{E}{R + B},$$

where E is the E.M.F. of the cell in volts.

In an ordinary voltmeter the resistance is very large, so that the current passing through the coils is extremely small. It may, indeed, be assumed that no current whatever flows through the voltmeter. Theoretically it would be better for the present experiment to use an *electrostatic* voltmeter, which would measure the potential difference between the terminals of the cell without taking any current.

The potential difference between the terminals when these are not connected together is V′ volts, V′ being equal to the E.M.F. of the cell (E). When they are connected by a wire it will be less than V′.

As this point sometimes causes difficulty it may be useful to consider an analogous case. Suppose we have an endless pipe through which water can be driven by means of a turbine, T (Fig. 238). When the stop-cock S is closed the turbine will develop a certain pressure which can be measured by means of a vertical pipe as shown. When the stop-cock is opened, the pressure indicated by the level in the pipe will diminish and as the aperture is increased will diminish still further. The pump has a mechanical effect which might be called a water-motive-force. The column of water measures the pressure difference produced. The tap and tube constitute a resistance in an external circuit.

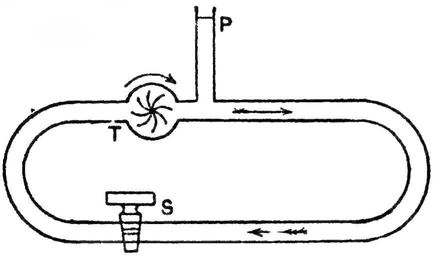

FIG. 238.—Hydro-dynamic Analogy.

The electrical analogy is worked out in detail in the introductory part of this chapter, to which the student is referred (pp. 477-481).

Let V denote the P.D. between the terminals of the cell when they are connected together by a resistance R. Then the current through R by Ohm's Law is V/R, being due entirely to

the P.D. between the terminals. But the current in the circuit is $E/(R + B)$, and so

$$\frac{V}{R} = \frac{E}{R + B},$$

$$\therefore V = \frac{ER}{R + B} = \frac{E}{1 + B/R}.$$

Thus V is less than E, but if R is very large compared with B, the difference $E - V$ will be very small.

If R is not very large, and we can find E/V, we can calculate B by means of the equation

$$\frac{E}{V} = \frac{R + B}{R}, \text{ or } B = R\left(\frac{E}{V} - 1\right).$$

For an alternative proof of this see the introductory part of this chapter (pp. 477-481).

To determine B, then, we compare the E.M.F. of the cell with the P.D. between its terminals when it is short-circuited by a wire of known resistance R, which should not be much greater than B.

If R is taken equal to B, $V/E = \frac{1}{2}$, so that the potential difference between the terminals is only one-half that on open circuit.

EXPT. 229. **Determination of the Internal Resistance of a Cell by Use of a Voltmeter.**—Connect the terminals of the cell to the voltmeter. In using a moving coil voltmeter* be careful to connect the cell so that the current enters the voltmeter at the terminal marked +. If the connection be made incorrectly, the pointer may be bent and the instrument damaged.

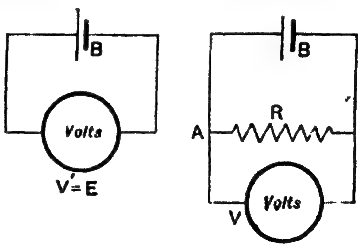

FIG. 239.—Internal Resistance of Cell.

Observe the deflection, which gives the value of V′, the P.D. on open circuit.

NOTE.—It is assumed that no current passes through the voltmeter, therefore the circuit is still 'open' and $V' = E$.

Then connect the terminals of the cell with various resistances, noting the corresponding deflections of the voltmeter in each case. A resistance box may be used, if care is taken not to pass the current through it for more than two or three minutes at a time.

The resistances chosen should be of such magnitudes that some give deflections greater than, and others less than, half the original deflection V′. About six different resistances should be used, *three* giving values above V′/2 and three below.

It is convenient to start with a resistance of 10 ohms in the external circuit, and to work upwards or downwards as required.

Tabulate the observation as below:

P.D. on open circuit V′ (= E) = . . .

R	V	E − V	$\left(\frac{E-V}{V}\right)R$.

NOTE.—The internal resistance, and even the E.M.F., of most batteries varies a good deal with the current they are sending in consequence of temporary changes in the liquid near the plates (p. 481). B is consequently a somewhat indefinite quantity.

In the above method it is assumed that E does not change when the cell is short-circuited. In some forms of Leclanché the cell *polarises* very rapidly when short-circuited, and its E.M.F. rapidly falls. In such cases the method is not applicable. This method should not be applied to a secondary cell; to obtain accurate readings it would be necessary to make R too small, the cell would be damaged, and the resistance box might be 'burnt out.'

This method being a deflection method possesses the defects inherent to all methods depending on the observation of deflections. The accuracy of the method is therefore not very great. It is, however, an extremely instructive experiment when taken in conjunction with the discussion of the action of a cell dealt with on pp. 477-481. In any case the resistance of a cell is such a variable quantity that the *order of magnitude* of the resistance is the result really aimed at, and for this the experiment is quite good. When one considers that shaking a cell or substituting a new zinc plate for an

old one may reduce the resistance of the cell by half in some cases, it will be realised that any method which is correct to within 20 per cent is good enough for this purpose. If the resistance of a particular cell is required very accurately, Mance's method, using a Post Office Box, should be employed (see Wheatstone's Bridge, p. 511).

INTERNAL RESISTANCE OF A SECONDARY CELL

A secondary cell has a very low internal resistance. The method of finding the internal resistance already described is not suitable in such a case because the current requisite to produce a measurable fall of P.D. would be too great, and would damage the cell. This is because a voltmeter has to be used which covers the whole range up to the full E.M.F. of the cell, and as the variation of P.D. obtainable with the biggest current allowable is only 1 or 2 per cent of this maximum, the measurements are not exact. The following method, which is applicable to any cell of low internal resistance, overcomes this difficulty, and as a very sensitive voltmeter may be used, the change of P.D. may be measured accurately.

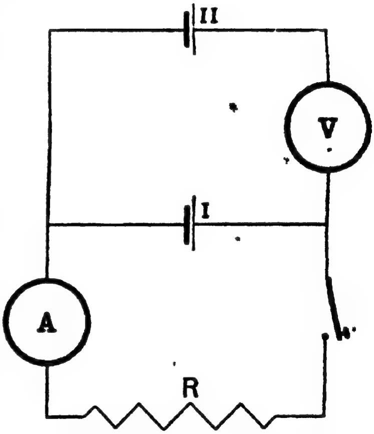

FIG. 240.—Internal Resistance of Secondary Cell.

EXPT. 230. **Determination of the Internal Resistance of a Secondary Cell.**—Connect two *similar* cells in parallel as in Fig. 240, with a sensitive voltmeter across their + terminals. In series with one of the cells, connect a resistance R and an ammeter A, including a key K in the circuit. When the key K is *open*, the voltmeter will indicate no P.D., since the cells are similar. Depress the key K and take the reading of the voltmeter (v) and of the ammeter (C).

The current C flows from cell I only, the voltmeter resistance being supposed infinite. If B be the resistance of cell I, the P.D. across its terminals falls an amount CB which is registered on the voltmeter as v. Hence $B = v/C$.

If the E.M.F. (E) of cell I be known and R be also known, C may be taken as E/R, and $B = vR/E$. The ammeter may then be dispensed with.

INTERNAL RESISTANCE OF A CELL BY MEANS OF A POTENTIOMETER

It has already been pointed out in the comparison of E.M.F.s by the Potentiometer that the P.D. between the terminals of a cell can be measured by balancing it against the P.D. between two points on a wire carrying a current. If we connect up a potentiometer as shown in Fig. 236, and adjust the contact to

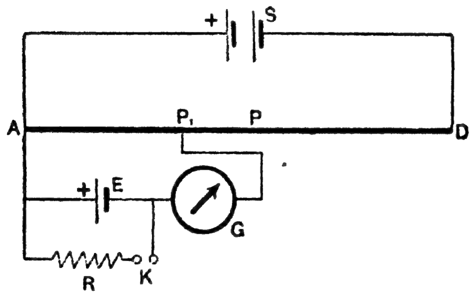

FIG. 241.—Internal Resistance by Potentiometer.

some point P such that no current flows through the galvanometer, the P.D. between P and the negative plate of the cell is zero (otherwise a current would flow through the galvanometer). A is at the same potential as the positive plate of the cell, therefore the P.D. between A and P is the same as the P.D. between the plates of the cell.

In the case considered, there is no current flowing through the cell E, and therefore this P.D. V′ is equal to the E.M.F. of the cell (pp. 477-479).

If, now, we short-circuit the cell through a resistance R (Fig. 241), the P.D. between the cell terminals is reduced to some value V given by the relation (pp. 487-489)

$$\frac{E}{V} \text{ or } \frac{V'}{V} = \frac{R+B}{R},$$

B being the internal resistance of the cell. Thus, the point P will now be at a higher potential than the negative plate of the cell E. Hence the galvanometer will now be deflected if contact is made at P. Balance will, however, be restored if we make contact at some point P_1, nearer to A than P. The P.D. between A and P must be equal to the reduced P.D., V, now existing between the two plates of the cell.

Now
$$\frac{V}{V'} = \frac{AP_1}{AP},$$

if the wire is uniform.

Hence
$$\frac{R}{R+B} = \frac{AP_1}{AP} = \frac{l_2}{l_1}.$$

B can therefore be calculated from the known values of R, l_1, and l_2, for

$$B = R\left[\frac{l_1 - l_2}{l_2}\right].$$

EXPT. 231. **Determination of the Internal Resistance of a Cell by means of a Potentiometer.**—Determine the internal resistance of a Daniell cell by the method described.

DISCUSSION OF THE POTENTIOMETER METHOD OF DETERMINING THE INTERNAL RESISTANCE OF A CELL

This method of measuring Internal Resistances is not much more suitable than the voltmeter method previously described. The cell has to be kept short-circuited through the resistance R for a considerable time, viz. the time taken to find the balance-point P_1. During this time it is discharging at a considerable rate, and is rapidly becoming polarised. The result of this is very confusing if its cause is not realised. If the point P_1 is found and the cell disconnected from the resistance for a moment, an entirely different balance-point may be found on reconnecting and testing for balance, in consequence of the cell having recovered somewhat while disconnected.

For accurate work a tapping key should be inserted in the resistance circuit as shown in Fig. 241. This should be depressed

momentarily while P_1 is being sought, and released again as soon as C is raised from the wire for adjusting to a fresh position along AB.

The second key must, of course, be depressed *before* C, and not raised until *after* C has been raised from contact with the wire.

By the use of this second key greater accuracy is obtainable, but the method possesses all the defects due to polarisation to as great a degree as the voltmeter method: its main advantage is that it is a *null* and not a deflection method. In theory it has other advantages over the voltmeter method: the voltmeter has always an appreciable current flowing through it although supposed to have none, hence E′ is never measured correctly by the voltmeter. In the present method the current in the cell is certainly zero when the cell is not short-circuited, and therefore E is obtained accurately.

The greater difficulties met with in use, and the confusion entailed owing to the 'drift' of P_1 towards A due to polarisation, render the potentiometer method of measuring internal resistance suitable only for advanced students.

CHAPTER V

MEASUREMENT OF RESISTANCE

§ 1. Ohm's Law

Ohm's Law states that when two points are taken on a linear conductor the ratio of the difference of potential, E, between those points to the current, C, flowing through the conductor is a constant. This constant ratio is termed the resistance, R, of the conductor. $E/C = R$. The reciprocal of this ratio is the conductance.

The most direct method of measuring resistance is to measure the two members of the quotient, difference of potential, and current. If the difference of potential in volts is measured by a voltmeter, and the strength of the current in amperes by an ammeter, the resistance will be obtained in ohms.

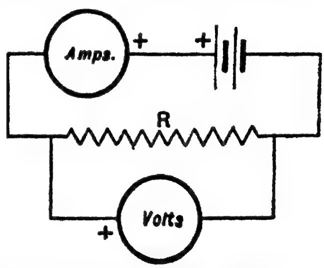

Fig. 242.—Measurement of Resistance by Ammeter and Voltmeter.

Note carefully that the ammeter is connected *in series* with the resistance to be measured, while the voltmeter is connected across the ends of the resistance, so that, with a moving-coil instrument, the coil of the voltmeter is *in parallel* with the resistance. The terminals marked + on the ammeter and the voltmeter must be connected to the + pole of the battery. In this method the resistance of the conductor is measured while a current is flowing through it. The method is therefore applicable in cases where other methods fail; for instance, we can measure in this way the resistance of an incandescent electric lamp while it is glowing (p. 539).

The method is a rough method only, though very convenient indeed in many cases. It depends on the observed deflections of the ammeter and voltmeter, and is thus not so accurate as a *null* method of resistance measurement. If the ammeter and voltmeter have not been calibrated, the result may be erroneous owing to errors of graduation.

It should be used only when an approximate value is required.

§ 2. Wheatstone's Bridge

The comparison of resistances can be carried out in a convenient way by the arrangement known as Wheatstone's Bridge

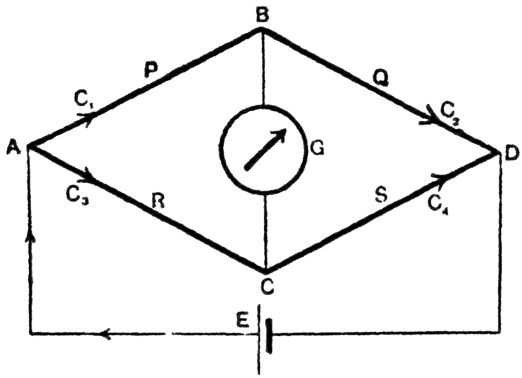

Fig. 213.—Principle of Wheatstone's Bridge.

This consists of four resistances P, Q, R, and S joined together so as to form four sides of a quadrilateral ABDC. If the two corners A and D are joined to the terminals of a cell, a current entering at A divides and flows partly along ABD and partly along ACD. There must be a fall of potential as we pass along ABD, and also as we pass along ACD. By properly adjusting the resistances P, Q, R, and S, the potential at the point B may be made to have the same value as the potential at C. When this is so, no current would flow through a galvanometer joined to the points B and C. We now proceed to find the condition that must hold between the resistances for this to be the case.

Let the currents through P, Q, R, and S be C_1, C_2, C_3, and C_4 respectively, and the potentials at A, B, C, and D be V_A, V_B, V_C, V_D respectively.

Applying Ohm's law to each branch or arm of the bridge in turn we obtain

$$V_A - V_B = C_1P \quad . \quad . \quad . \quad (1)$$
$$V_A - V_C = C_3R \quad . \quad . \quad . \quad (2)$$
$$V_B - V_D = C_2Q \quad . \quad . \quad . \quad (3)$$
$$V_C - V_D = C_4S \quad . \quad . \quad . \quad (4)$$

But in the case considered $V_B = V_C$, so that the left-hand side of equation (1) becomes identical with the left-hand side of equation (2).

Therefore $$C_1P = C_3R \quad . \quad . \quad . \quad (5)$$

Similarly from (3) and (4)

$$C_2Q = C_4S \quad . \quad . \quad . \quad (6)$$

Dividing (5) by (6) gives

$$\frac{C_1P}{C_2Q} = \frac{C_3R}{C_4S} \quad . \quad . \quad . \quad (7)$$

But if no current flows along BC, $C_1 = C_2$ and $C_3 = C_4$ and equation (7) reduces to

$$\frac{P}{Q} = \frac{R}{S} \quad . \quad . \quad . \quad . \quad (8)$$

Properties of Conjugate Conductors.—The battery might have been placed in the arm joining B and C, and the galvanometer in the arm joining A and D, and exactly the same condition would have been required for no current through the galvanometer. The two arms AD and BC are then said to be **conjugate** arms of the bridge. Two arms of a network of conductors are said to be conjugate arms if the current in either arm is entirely independent of any E.M.F. in the other. A cell in either of the arms BC or AD would send no current through the other, therefore BC and AD are conjugate arms of the network. The condition that BC and AD should be conjugate arms is that $P/Q = R/S$.

Determination of the Resistance of a Wire.—Equation (8) shows that if we know the ratio of two of the resistances (R to S say) and the actual value of a third (Q), then the fourth resistance (P) is determined when B and C are at the same potential.

Determination of the Resistance of a Galvanometer—Thomson's (Kelvin's) Method.—The resistance of a galvanometer also can be found by Wheatstone's Bridge. The galvanometer is connected in the arm AB, the resistance of the

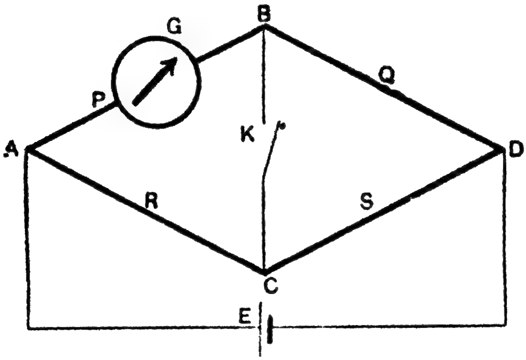

FIG. 11.—Resistance of Galvanometer.

galvanometer G being the value of P in this case. As a constant current flows along AB, a steady deflection is produced in the galvanometer. When the resistances are related in such a way that $P/Q = R/S$, B and C will be at the same potential, and on connecting B and C no current will flow through BC.

If the condition $P/Q = R/S$ is *not* satisfied, some current will flow along BC if these points are connected. Hence in this case the distribution of current through the rest of the network will be altered. Consequently the current in the galvanometer will be altered. Thus, unless B and C are at the same potential, the galvanometer deflection will be altered when B and C are connected. The degree of alteration will depend on the current which flows along BC, therefore to ensure sensitiveness the

resistance BC should be made as small as possible, a short piece of copper wire being generally used.

The resistances are adjusted until connecting B and C does not alter the steady deflection of the galvanometer, and G (= P) is calculated from the relation which then holds:

$$\frac{P}{Q} = \frac{R}{S}, \text{ i.e. } \frac{G}{Q} = \frac{R}{S}.$$

Determination of the Internal Resistance of a Cell—Mance's Method.—Suppose a cell is placed in the arm AB, the

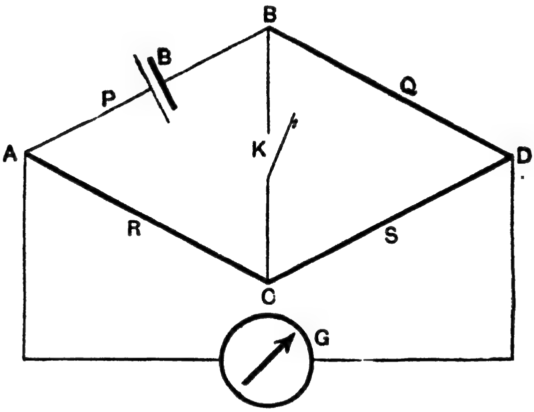

FIG. 247.—Resistance of Battery.

resistance of the cell being B. Then if $B/Q = R/S$, the arms BC and AD are conjugate arms of the network, and any E.M.F. introduced in the arm BC will not affect the current in AD.

There will be a steady current through the arm AD due to the E.M.F. in AB, and the galvanometer will therefore be deflected permanently. If the condition $B/Q = R/S$ is satisfied, this deflection will not be affected by any E.M.F. introduced in the arm BC, and so we can test if this relation holds by introducing an E.M.F. in the arm BC: this can be done by connecting a cell across the points B and C.

The *magnitude* of the E.M.F. introduced in the arm BC is quite immaterial, except in so far as it affects the sensitiveness of

the test. It can be shown that introducing a cell of small E.M.F. and of very low resistance is as sensitive a test as a cell of larger E.M.F. but greater resistance would offer. We therefore introduce in the arm BC a 'cell' of infinitesimal E.M.F. and very low resistance by merely connecting across BC with a copper wire. This gives a satisfactory means of detecting any error in the adjustment of the resistances. When no change in the deflection of the galvanometer is produced on connecting B and C with a copper wire, the relation $B/Q = R/S$ holds good.

THE SLIDE-WIRE BRIDGE

The method of Wheatstone's bridge may be applied by using the apparatus known as the slide-wire bridge. As the wire employed is frequently one metre in length, this is also often termed the metre bridge.

A long uniform wire is stretched on a base-board between the two points A and D. The ends are connected by means of thick copper strips, of negligible resistance, to the terminals F and L. HK is another thick copper strip, provided with terminals at H, B,

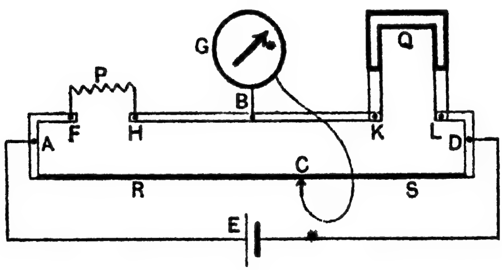

FIG. 246.—Slide-wire Bridge.

and K. The unknown resistance P is joined to the terminals F and H, so as to complete the circuit in the gap between them. A known resistance of suitable magnitude (that is, not very different from P) is joined to the terminals K and L. In making these connections short thick wires or flat strips of copper must be used, so as not to introduce additional unknown resistances. A movable key or jockey, by means of which contact may be made with the slide-wire at any point desired, slides along the base-board. Its position may

be read off on a fixed scale. The 'ratio arms' of the bridge are the two parts into which the slide-wire is divided by the movable key.

In carrying out the measurement a cell is connected to the end terminals A and D, a key being included sometimes in this part of the circuit so that the current can be cut off when no observation is being made. A galvanometer is connected to the central terminal B and to the movable contact C. An astatic galvanometer is frequently used in elementary work. The object of the manipulation is to find the point at which contact must be made with the slide-wire so as to give no deflection of the galvanometer needle. It is best to start by noting the direction in which the needle moves when contact is made, first near one and then near the other end of the slide-wire. If these deflections are in opposite directions the point sought for must lie somewhere between them. If the deflections are in the same direction in the two cases, it indicates that one of the resistances P or Q is very much greater than the other, or that there is a faulty connection in some part of the apparatus. It is impossible to get an accurate result unless P and Q are at least of the same order of magnitude. Assuming that suitable values have been chosen, the point on the slide-wire for no deflection should be found somewhere in the central portion of the wire. The 'balance point' should be in the middle 'third' of the wire in all cases.

Much time can be saved in carrying out the experiment by learning how to increase and how to decrease the deflection of the needle. Suppose that when contact is made near one end of the slide-wire the deflection is clockwise. Then to increase the swing the contact should be made whenever the needle is swinging in the clockwise direction, and should be broken when the needle is swinging in the opposite direction. To diminish the swing and bring the needle to rest, contact should be made when the needle is swinging in the counter-clockwise direction, and should be broken when the needle is swinging in the clockwise direction.

Having determined as accurately as possible the point on the slide-wire corresponding to no deflection, the distances $AC = l_1$ and $CD = l_2$ are measured.

Then
$$\frac{P}{Q} = \frac{R}{S}$$
$$= \frac{l_1}{l_2},$$

assuming the wire to be uniform.

Hence
$$P = Q \times \frac{l_1}{l_2}.$$

RESISTIVITY OR SPECIFIC RESISTANCE

The resistance of a wire of length 1 cm. and of cross-sectional area 1 sq. cm. is called the **resistivity** or the **specific resistance** of the material of which the wire is composed.

Let X be a wire 1 cm. in length, and having a cross section of 1 sq. cm. The shape of the cross section is immaterial. Let the resistance of this wire be S ohms. Let Y be a second wire of

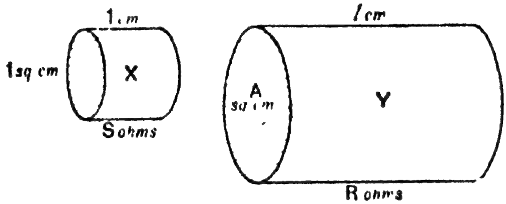

FIG. 347.—Specific Resistance.

the same material, l cm. in length, and having a cross section of A sq. cm. Let its resistance be R ohms. Since the resistance of a wire is directly proportional to its length, and the length of Y is l times the length of X, the resistance of Y will be l times that of X in consequence of the difference in length. Again, the resistance of a wire is *inversely* proportional to the area of cross section; hence, since the cross-sectional area of Y is A times the cross-sectional area of X, the resistance of Y will be $1/A$ times that of X in consequence of the difference in cross section.

Consequently $$R = \frac{lS}{A},$$

or $$S = \frac{RA}{l}.$$

So if we can measure R, A, and l we can determine S, which is the resistivity of the material. The resistivity may be expressed in ohms per unit length of a wire of unit cross section, the dimensions of resistivity being ohms × cm.

Expt. 232. Determination of the Specific Resistance of a Wire, using a Slide-wire Bridge.—To measure the resistivity select a wire about a metre in length, free from kinks. Measure its length[1] to the nearest millimetre, and measure the diameter very carefully with a micrometer screw gauge. As the area of cross section A depends on the square of the diameter ($A = \pi d^2/4$ for a wire whose cross section is circular), an error in this measurement is serious; the percentage error in d is doubled in the square of d, so that the result is wrong to *twice* the extent of the error of d. Express the length in centimetres, and the area of cross section in square centimetres.

Next connect the wire across one of the gaps P in the thick copper strip of the slide-wire bridge, connecting a 'decimal-ohm' box across the corresponding gap Q on the other side of the bridge (Fig. 246). Thick copper connectors must be used for joining the box to the sides of the gap, and these must be cleaned where they fit under the connecting screws. The connections for the battery and the galvanometer are shown in the same figure. A Daniell cell is suitable for this purpose, with some form of simple astatic galvanometer. Adjust the decimal-ohm box to a resistance of 1 ohm. Slide the jockey along the slide-wire, making contact with the wire at various points; *the jockey must not make contact with the wire while it is being moved along*, otherwise the wire will be worn unevenly in various parts and the accuracy of the bridge destroyed.

Find two points at which definite deflections are produced, but in opposite directions at the two points; the balance-point must lie between these two.

By working between these two points, successive pairs of points can be found which give deflection in opposite directions, each pair being closer together than the previous pair. It may happen finally that two points are found at which the deflection produced in the galvanometer is inappreciable, though at any point beyond either of them a deflection can be observed. The balance-point may be taken as the centre of that part of the wire between these points.

The resistance of the wire P can be calculated by means of the expression

$$P = \frac{l_1}{l_2} Q,$$

[1] The length required is not the whole length of the wire, but the length which is between the terminals when the resistance is being measured.

Q being the value of the resistance in the resistance box, l_1 the length of the wire from A to C, and l_2 the length of the remainder from C to D.

The point C should be in the middle 'third' of the slide-wire; if this is not the case a different value of Q must be taken to bring C into this part of the wire. Three different values of Q must be used in any case, and the corresponding balance points found, the values of P being calculated for each case. If the experiment has been performed accurately these three values of P will be the same to a very close approximation; their mean is taken as the true value of P.

From the value of the resistance thus obtained, and the dimensions of the wire already determined, calculate the specific resistance of the material of the wire from the expression

$$S = \frac{RA}{l}$$

EXPT. 233. **Determination of the Resistance of a Galvanometer.**—For this experiment connect up the slide-wire bridge as in Fig. 248. Q is a decimal-ohm box adjusted

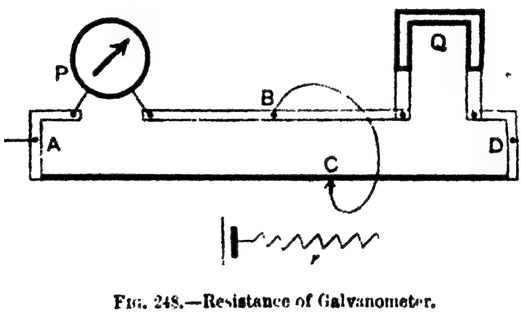

FIG. 248.—Resistance of Galvanometer.

to 1 ohm at first, r is a large resistance which need not be known. In this case the galvanometer itself forms the unknown resistance P in one of the arms of the bridge. The connection BC is made with a piece of copper wire. It is clear that as soon as the battery is connected to the points A and D, a current must flow through the galvanometer and cause a deflection of the needle. The bridge is balanced when this deflection is unaltered upon making contact between the points B and C by means of the jockey key. When this condition is satisfied no current flows in BC, and the potential at B must be the same as that at C, and as in the previous case

$$\frac{P}{Q} = \frac{R}{S}.$$

If a key be used in the battery branch, it should be a plug key and not a tapping key, since it is necessary to get a steady current through the bridge before making contact at C. If the steady deflection of the galvanometer be large, the alteration produced by making contact at C will be very small. A large deflection may be reduced by employing a smaller current. This is secured by introducing a resistance box r in the battery branch, using sufficient resistance to give a convenient deflection. In the case of a mirror galvanometer with a moving needle, the controlling magnet may be used to bring the spot of light back on to the scale.

The method is not easy to carry out, the difficulty being that the galvanometer is deflected considerably the whole of the time, and often the needle is in a position where the sensitiveness of the galvanometer is only small. As a result of this, the adjustment of the point C may be altered appreciably without making any noticeable change in the deflection; consequently the accuracy obtainable is not very great when a metre bridge is used. Considerably greater accuracy is obtained with a P.O. box if suitable means are adopted (Expt. 236).

EXPT. 234. **Determination of the Resistance of a Cell.**—The cell whose resistance is to be measured is placed in the

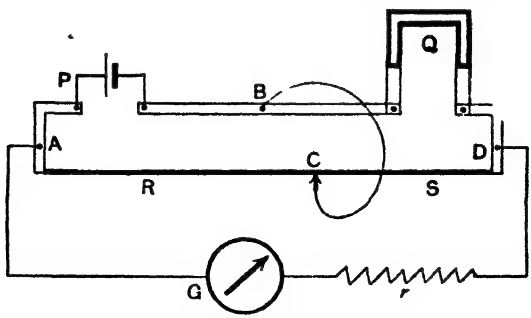

Fig. 249.—Resistance of Cell.

position of the unknown resistance P, so that it forms one arm of the bridge. The galvanometer is connected directly to the two extreme points of the bridge A and D. In this case also a steady current flows through the galvanometer as soon as the

connections have been made. The jockey C is adjusted so that no alteration is produced in the deflection of the galvanometer by making connection at C. When this condition is satisfied P/Q = R/S (p. 499). If the steady deflection of the galvanometer be too large, a resistance r must be introduced in series with the galvanometer so as to diminish the current passing through it. The connections are the same as those used when determining the resistance of a galvanometer, except that the galvanometer and battery are interchanged. Q is adjusted to 1 ohm to start with, and the point C is found where the steady deflection of the galvanometer is unaltered on depressing the jockey.'

If the point C is not in the middle third of the wire when Q is 1 ohm, a different value of Q must be used, choosing the value so as to bring C near to the middle of the wire.

The resistance of the battery is then calculated from the expression

$$B = Q \cdot \frac{l_1}{l_2}.$$

THE POST-OFFICE BOX

In the account of Wheatstone's bridge we have seen that when the bridge is balanced the four resistances P, Q, R, S, which form four sides of a quadrilateral figure, satisfy the relation

$$\frac{P}{Q} = \frac{R}{S}.$$

If we know the ratio of P to Q, and R is a known resistance, then the fourth resistance S, previously unknown, is determined.

In the Post-Office Box we find sets of resistance coils representing three of the arms of the bridge, namely, two *ratio arms*, P and Q, and an *adjustable arm* for the known resistance R. The fourth arm S is the unknown resistance whose value is to be found.

The distinguishing characteristic of the ratio arms P, Q is that they are two portions of the box, usually comprising together the whole of one bar, with identically similar sets of resistances, 10, 100, 1000, and sometimes 10,000 ohms in each. The whole of the rest

of the resistances in the box comprise the third arm R of the bridge. The actual arrangement of the resistances in the box differs in different patterns, but the student should have little difficulty in identifying the ratio arms and the adjustable arm, and then determining the points corresponding to A, B, C, D in Fig. 243. In this diagram the battery is connected to the terminals A and D, and the galvanometer to the terminals B and C. It should, however, be noted that the positions of the battery and galvanometer

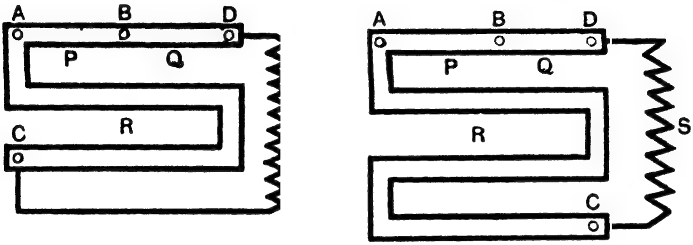

FIG. 250.—Two Forms of Post-Office Box.

may be interchanged without affecting the final result, that is, the battery may be connected to the terminals B and C and the galvanometer to A and D. A tapping key must be inserted in the battery circuit, and a second in the galvanometer circuit. In some forms of Post-Office Box these keys are included in the box, and the connections between the keys and the corners of the bridge are indicated by white lines traced on the ebonite cover of the box. The battery key must be depressed *first*, to avoid self-induction effects.

Another arrangement sometimes used is a double key which makes contact first for the battery, and subsequently for the galvanometer.

The unknown resistance should be connected to the points C and D by means of short thick wires, or flat copper strips, of small resistance.

A mirror galvanometer, which may be either of the moving needle or of the moving coil type, is generally used with the Post-Office Box. For a description of these instruments see the section on galvanometers, pp. 568-575.

The top of the box is usually marked at different points with the cryptic letters C, Z, E, L, G, or perhaps a letter B is used instead of C and Z. These mean respectively Carbon, Zinc (or Battery), Earth, Line, and Galvanometer.

If the box be connected up by the aid of these letters, the experiment usually works successfully, but little benefit is gained

from it. The student should avoid using them, and should rely on the connections worked out by himself with the aid of suitable diagrams.

The Galvanometer Shunt.—If a very sensitive galvanometer be used, it must be provided with a **shunt**, so that its sensitiveness may be varied to suit the requirements of the moment. In its simplest form the shunt consists of a resistance which is placed in parallel with the galvanometer (Fig. 231). A delicate galvanometer is provided usually with a shunt box containing a number of resistances which may be marked 1/9, 1/99, 1/999. This means that the resistances in the box are respectively 1/9, 1/99, and 1/999 of the resistance of the galvanometer. In the present experiment the plug belonging to the shunt should be placed at the outset in the hole marked 1/999. In this case only 1/1000 of the current in the galvanometer circuit passes through the coils of the galvanometer, and consequently the instrument is not very sensitive.

When an approximate balance has been obtained, the plug may be shifted to the hole marked 1/99, or that marked 1/9. For the final adjustment the plug may be removed altogether so that the full current passes through the galvanometer, and the arrangement is as sensitive as possible.

EXPT. 235. **Determination of the Resistance of a Wire by Means of a Post-Office Box.**—Connect up the wire, the battery, and the galvanometer to the P.O. Box after working out the proper connections by the aid of the foregoing description and diagrams (pp. 496-498, 506-508).

When the connections have been made, remove the plugs marked 10 in the ratio arms of the bridge. The ratio of P to Q is then 10 : 10, that is a ratio of equality; and for the bridge to be balanced, R, the adjustable arm, must be equal to S, the unknown resistance. Shunt the galvanometer with the 1/999 shunt if a shunt box is supplied, and see that all the plugs in R are in place. Make contact momentarily with both keys and note the direction in which the mirror of the galvanometer is deflected. As this deflection is probably very great, it is usually better in this stage of the experiment to watch the mirror itself instead of the spot of light on the scale. Next remove the plug in R marked 'infinity,' and again note the deflection. It should be in the opposite direction to that previously observed. From these observations the experimenter should construct a simple rule to be borne in

mind during the remainder of the determination: "When the deflection is to the right (or to the left, or away from me, as the case may be) the resistance of the adjustable arm is too large."

Replace the infinity plug and take out another (say 1000 ohms) and notice the deflection, seeing whether the resistance is too large or too small. Determine in this way limits between which the resistance must lie, and proceed till the value of the resistance S is found correct to the nearest ohm. Suppose S is found to be between 6 and 7 ohms.

To determine the next decimal place, that is to find the value of S correct to 0·1 ohm, remove the plug from the 100-ohm coil in P and insert the plug in the 10-ohm coil. Then the resistance in P is 100 ohms, that in Q is still 10 ohms. The ratio of P to Q is 100 : 10 or 10 : 1. Consequently, when the bridge is balanced, R must be 10 times S. Adjust the resistance R till an approximate balance is obtained. If S lies between 6 and 7 ohms, R must lie between 60 and 70 ohms. Suppose it is found to lie between 63 and 64 ohms. Then the value of S lies between 6·3 and 6·4 ohms.

Next remove the plug from the 1000-ohm coil in P and insert the plug in the 100-ohm coil. Then the resistance in P is 1000 ohms, that in Q is still 10 ohms. The ratio of P to Q is 1000 : 10 or 100 : 1. Consequently, for a balance R must be 100 times S. It is clear that R must now lie between 630 and 640 ohms. Suppose it is found to lie between 638 and 639 ohms. Then the value of S must lie between 6·38 and 6·39 ohms.

With a sensitive galvanometer yet another decimal figure may be found by noting the resting-points of the spot of light on the scale when R is adjusted to 638 and to 639 ohms, and using the method of 'proportional parts.' If, for example, 638 ohms give 6 mm. on one side, and 639 ohms give 9 mm. on the other side of the galvanometer zero, a difference of 1 ohm in R causes a change of 15 mm. Therefore a change of 6 mm. means 0·4 ohm, *i.e.* R = 638·4 ohms and S = 6·384 ohms.

Determine in this way the resistance of a coil to within 0·1 per cent.

Determine also the resistance of a piece of wire, and calculate the specific resistance of the material of the wire.

When the unknown resistance S is large, it may not be possible to find its value correct to the $\frac{1}{100}$th part of an ohm, or even to the $\frac{1}{10}$th part of an ohm, with the coils supplied in the ordinary Post-Office Box. In such a case it should be noted that, as a rule,

the bridge is most sensitive when the four arms of the bridge are approximately equal.

When the unknown resistance is very large, it may be necessary to make the resistance Q 10 times or even 100 times the resistance P in order to obtain a balance by adjusting R. Then the resistance S will be 10 times (or 100 times) the resistance R.

EXPT. 236. **Determination of the Resistance of a Galvanometer.**—The principle of the method is the same as with the slide-wire bridge (p. 504). The galvanometer forms the fourth arm of the bridge, as the unknown resistance S. A tapping key is not used in the battery branch. As soon as the connections are completed the spot of light usually goes right off the scale. The use of a galvanometer shunt is not admissible in this case.

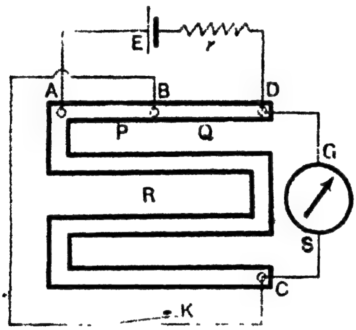

FIG. 271.—Resistance of Galvanometer.

The chief difficulty met with in this form of experiment is due to the great sensitiveness of the galvanometer. This causes it to stay obstinately pressed against one side of the case, however much the controlling magnet is turned. To avoid this trouble, the following means should be employed:—*First*, introduce a considerable resistance in the battery arm; a resistance adjustable up to 10,000 ohms may be used conveniently if the galvanometer is a specially sensitive one. *Secondly*, use ratio arms of the lowest possible resistance, both equal to 10 ohms if possible. *Thirdly*, reduce the sensitiveness of the galvanometer by lowering the control magnet until the period of swing is very short. See the notes on galvanometers, p. 570. (This third method is not possible in the case of suspended-coil galvanometers.)

It will now be found that the spot of light does not move much beyond the scale, if at all, and it can be brought back by use of the control magnet, or by turning the top of the suspending fibre in a suspended-coil galvanometer.

Make the requisite adjustment of R so that when the key K is pressed there is no change in the position of the spot of light.

It will generally be found that R can be varied over a

wide range without affecting the position of the spot, as the arrangement is now so insensitive. The sensitiveness may now be increased again *very gradually*, first by raising the control magnet. On reaching the limit of increased sensitiveness—consistent with control of the position of the spot—obtainable by this means, the ratio arms may be *both* increased to 100 ohms each. This allows a bigger current to pass through the galvanometer, and a large increase of deflection results. Correct this deflection by rotating the control magnet, but do not *lower* the magnet again; then adjust R as before to give no change in the position of the spot on pressing the key.

With a galvanometer of the suspended-coil type, the ratio arms are increased at once to 100 and the increased deflection corrected by further twisting the fibre.

The adjustment will now be more delicate than before. The sensitiveness is increased still further by increasing the ratio arms to 1000. Not until the extreme limit of sensitiveness has been reached in this way must the resistance r be diminished. If necessary, the ratio arms may be adjusted later to give 10 1 in the case of galvanometers of resistance lower than 1000 ohms, the same precautions being taken as to sensitivity and control.

The battery and key *must* be connected as shown in Fig. 251 if these means of reducing the sensitiveness are employed, otherwise reducing the resistance of the ratio arms will not have the desired effect.

If the experiment be carried out carefully as described above, the resistance of the galvanometer can be determined quickly and conveniently. This method enables a student to carry out this experiment with a gratifying sense of certainty.

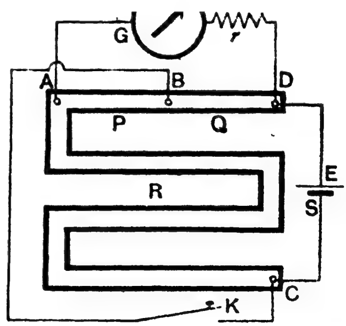

FIG. 252.—Resistance of Battery.

EXPT. 237. **Determination of the Resistance of a Battery.**—The battery forms the unknown arm S of the bridge. Keys are not used for the battery or the galvanometer, but a tapping key is employed in the diagonal branch BC. The methods available for diminishing the steady deflection of the galvanometer are exactly the same as those

described in the determination of the resistance of the galvanometer. The simplest statement of the arrangement is to say that the positions of the battery and of the galvanometer are merely interchanged, every other connection, including the resistance r, being left absolutely the same as in the measurement of the galvanometer resistance; and the adjustments are made in exactly the same way.

When P, Q, and R are adjusted so that the galvanometer deflection remains unaltered when the key K is depressed, the battery resistance is given by

$$\frac{P}{Q}=\frac{R}{B}.$$

§ 3. Wheatstone's Bridge: Carey Foster's Method

The ordinary slide-wire pattern Wheatstone's Bridge is not susceptible of very great accuracy when employed in the usual way. It is impossible to find the position of the balance-point to within 1 mm., and with a slide-wire 1 m. long this uncertainty introduces a possible error of $\frac{1}{250}$ at least. If the balance-point be not at the middle of the wire, the uncertainty of the result is greater than this. A longer slide-wire can be used if desired, and the relative magnitude of an error of 1 mm. is correspondingly reduced; but the use of a slide-wire longer than 1 m. is inconvenient.

In Carey Foster's arrangement the *effective* length of the slide-wire is increased without actually using a wire of more than the normal length, by introducing resistances in series with the wire, one at each end. The connections are as indicated in Fig. 253. The arms of the bridge, P and Q (Fig. 243), are the resistances R_1 and R_2 respectively, the remaining arms R and S being composed of X plus a length l_1 of bridge-wire, and Y plus the remainder of the bridge-wire (l_2).

When the bridge is balanced, we have the relation

$$\frac{P}{Q}=\frac{R}{S}$$

stated as

$$\frac{R_1}{R_2}=\frac{X+l_1\rho}{Y+l_2\rho},$$

where ρ is the resistance of 1 cm. of the bridge-wire. If X and Y are together equal to about ten times the resistance of the bridge-wire, the terms $l_1\rho$ and $l_2\rho$ are of the order of 10 per cent

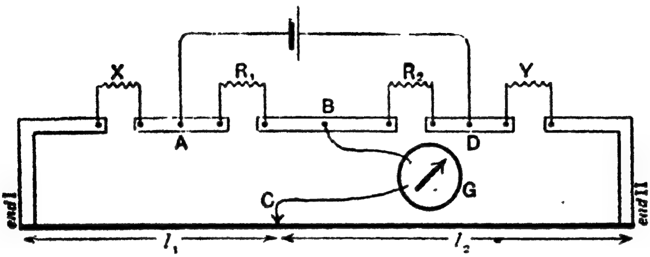

FIG. 253.—Carey Foster's Method.

of X and Y. Any error in reading l_1 is thus reduced to about $\frac{1}{10}$th of the relative magnitude it has when the bridge is used in the ordinary way, an error of 1 mm. corresponding with an error of $\frac{1}{2500}$ in the result instead of $\frac{1}{250}$.

End Corrections.—If the bridge-wire is soldered imperfectly at the ends where it joins the copper strips, the joint may introduce an appreciable resistance into the arms R and S, and the true ratio R_1/R_2 should be expressed as

$$\frac{R_1}{R_2} = \frac{X + l_1\rho + \lambda_1\rho}{Y + l_2\rho + \lambda_2\rho},$$

λ_1 and λ_2 being the **end corrections** expressed as equivalent lengthenings of the two parts of the wire.

METHOD OF ELIMINATING END CORRECTIONS IN CAREY FOSTER'S ARRANGEMENT

By the following method of working, the effects of these end corrections can be eliminated when using the slide-wire bridge in Carey Foster's arrangement.

The ratio R_1/R_2 is expressed as

$$\frac{R_1}{R_2} = \frac{X + (l_1 + \lambda_1)\rho}{Y + (l_2 + \lambda_2)\rho}$$

if we take into account the end corrections.

If now the resistances X and Y are interchanged, a new balance point may be found at a point whose distances are l_1' and l_2' from the ends of the wire, and we have

$$\frac{R_1}{R_2} = \frac{Y + (l_1' + \lambda_1)\rho}{X + (l_2' + \lambda_2)\rho}$$

From these two values of $\frac{R_1}{R_2}$ we can obtain

$$\frac{R_1}{R_1 + R_2} = \frac{X + (l_1 + \lambda_1)\rho}{X + Y + (l_1 + l_2 + \lambda_1 + \lambda_2)\rho} \quad . \quad (1)$$

and

$$\frac{R_1}{R_1 + R_2} = \frac{Y + (l_1' + \lambda_1)\rho}{X + Y + (l_1' + l_2' + \lambda_1 + \lambda_2)\rho} \quad . \quad (2)$$

The denominators of these fractions are identical, since $l_1 + l_2 = l_1' + l_2'$, hence

$$X + (l_1 + \lambda_1)\rho = Y + (l_1' + \lambda_1)\rho,$$

or

$$X = Y + (l_1' - l_1)\rho.$$

This method of working is only suitable for the comparison of two resistances X and Y provided they are nearly equal, as for comparing a home-made resistance with a standard resistance to which it is supposed to be equal. This is, however, a type of experiment which frequently has to be performed in practice, and Carey Foster's arrangement is a convenient method of carrying it out with simple apparatus. One great advantage of the method lies in the fact that R_1 and R_2 need not be known accurately: it is only necessary that they should be approximately equal and absolutely constant. They should have about the same value as X and Y.

It is worthy of note that in this method of comparing two resistances, the resistances themselves are never compared *directly* in the whole experiment.

EXPT. 238. **Construction of a 1-ohm Coil.**—Determine the resistance of a piece of manganin or constantan wire by means

of a metre bridge (p. 500). Calculate what leng of the wire must be used to make a resistance of 1 ohm.

If very great accuracy be not required, cut off a piece a few centimetres longer than this. Solder it to two stout copper strips, or to two terminals mounted on the top of a wooden bobbin. Redetermine its resistance, and shorten the wire to the required amount by twisting the middle part of the loop together, soldering the twisted part of the wire when the adjustment is correct.

If very great accuracy be required, cut off a length of wire about *10 per cent longer* than the calculated length for 1 ohm. Solder it to two terminals as already described, and redetermine its resistance very accurately. Calculate what length of similar wire would have to be connected *in parallel* with it in order that the resistance of the two together, when connected in parallel, shall be 1 ohm.

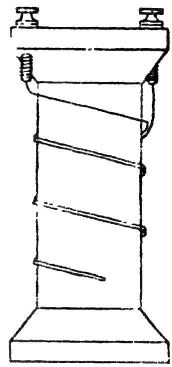

FIG. 254.—1-ohm Coil.

If the above instructions have been carried out carefully, the length required for the parallel wire should be about ten times as long as the piece originally cut off.

Cut off this length, and solder it in parallel with the first length across the terminals of the bobbin. Take each loop of wire, and draw it out so that the halves of the loop lie close together and parallel. Wrap the two loops separately round the bobbin, taking the halves of each loop round and round in the same direction. Fix the double end of each loop to the wood of the bobbin with sealing-wax, taking care not to make too sharp a bend at the end of the loop.

Wrap tape round the coils thus formed and immerse the whole in molten paraffin wax.

It is desirable to test the resistance before wrapping and waxing the coil, if great accuracy is required. If the resistance is not quite exact, the final adjustment is made by twisting together the end of the loop to the middle of the *longer* wire, soldering together the twisted end when the adjustment is exact.

EXPT. 239. **Standardisation of a 1-ohm Coil by Carey Foster's Bridge.**—Connect two nearly equal resistances across the middle gaps of a slide-wire bridge. Two coils of approximately 1 ohm each are suitable for this purpose; these constitute the resistances R_1 and R_2 (Fig. 253).

In the extreme gaps insert the resistance to be standarised, and a standard resistance of 1 ohm; the standard resistance is represented by Y in Fig. 253, and the other resistance is X.

Find the point of balance when a galvanometer and a cell are connected as in Fig. 253,—let this be l_1 cm. from the end I of the wire as shown.

Interchange the resistances X and Y, and again find the balance point: this will probably be in a different position, say at a distance l_1' from the end I.

Then $$X = Y + (l_1' - l_1)\rho,$$

where ρ is the resistance of the bridge-wire per cm.

Determination of ρ.—Remove the coils from the end gaps of the bridge, and close the first gap with a thick copper strip. Then $X = 0$. In the other gap insert a decimal-ohm box, connecting it to the terminals of the bridge by copper strips of negligible resistance. Use a resistance of 0·1 ohm in this box for Y. Determine the point of balance, and let it be x_1 cm. from the end I of the wire. Interchange the positions of the copper strip and the decimal-ohm box, and let the new position of the balance-point be y_1 cm. from the same end of the wire.

Then by substituting in the general equation

$$X = Y + (l_1' - l_1)\rho,$$

we find $$0 = 0{\cdot}1 + (y_1 - x_1)\rho,$$

so that $$\rho = \frac{0{\cdot}1}{x_1 - y_1}.$$

Similarly if 0·2 ohm is used for Y, and the corresponding balance points are x_2, y_2 cm. from the end of the wire,

$$\rho = \frac{0{\cdot}2}{x_2 - y_2}.$$

Since the difference between x_2 and y_2 is greater than the difference between x_1 and y_1, this result is more accurate than the former.

By using still larger values for Y still greater accuracy is secured, but Y must be less than the resistance of the slide-wire itself.

Calculate the resistance of the bridge-wire per cm., and use the result to find the resistance of the coil under test in terms of the standard ohm.

The same method may be used to find the resistance of the British Association Ohm in terms of the Legal Ohm.

COEFFICIENT OF INCREASE OF RESISTANCE WITH TEMPERATURE

The ratio of the difference of potential between two points of a wire to the current through the wire is constant only when the temperature is constant. In other words, the resistance of a wire varies with temperature, and, in general, the resistance at a higher temperature is greater than the resistance at a lower temperature. The increase of resistance per degree rise of temperature is approximately constant for the same wire. The **Coefficient of Increase of Resistance with Temperature is the Increase of Resistance per degree, divided by the Resistance at 0° C.**

Thus, if R_0 = resistance at 0° C.,

and R_t = resistance at t° C.,

the *mean* value of the coefficient over this range is

$$a = \frac{R_t - R_0}{R_0 t}.$$

Thus, if a is constant,

$$R_t = R_0 (1 + at).$$

To determine a, it is necessary to measure the resistance at two different temperatures. It is convenient to select the freezing point and the boiling point of water as the temperatures of observation. The value then obtained for a is the mean value between 0° C and 100° C.

The accuracy with which a is determined depends on the accuracy with which the *change* in resistance may be measured. Since $R_t - R_0$ is the *small* difference between two large quantities, each resistance must be measured most carefully. Thus, if the change of resistance be $\frac{1}{10}$ of R_0, an error of 0·1 per cent in R_0 or R_t becomes an error of 1 per cent in $R_t - R_0$. For this reason Carey Foster's method (p. 512) of using Wheatstone's Bridge is used for this determination.

EXPT. 240. **Determination of the Temperature Coefficient of Resistance for Platinum.**—A small coil of fine platinum wire, having a resistance of about 1 ohm, is fitted with thick copper leads and placed inside a glass tube closed at the lower end. The leads outside the tube are of flexible wire. An

exactly similar set of leads simply soldered together at the lower ends is also provided. These are called the **compensating leads.** The ends of the leads are fitted with copper connecting forks, those connected with the platinum being lettered PP and those attached to the compensating leads being lettered CC. A metre bridge with four gaps is required. In the two inner gaps of the bridge connect two *exactly equal* resistances R, R (1-ohm coils are suitable). Connect the terminals marked PP in one of the outer gaps. Connect one of the C terminals to one side of the other gap, the other C terminal to a terminal of an adjustable resistance box S containing ohms and decimals of an ohm.

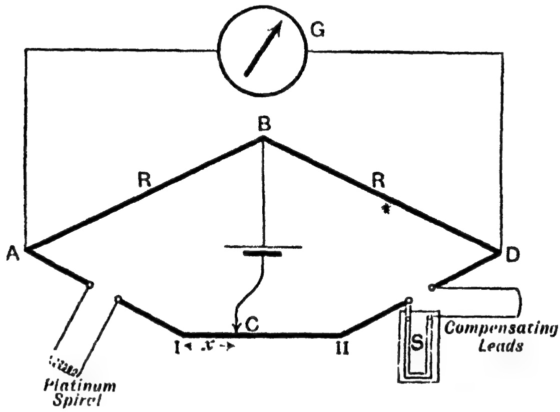

FIG. 255.—Temperature Coefficient of Resistance.

The second terminal of this resistance box must be connected to the other side of the gap by a thick copper strip. Connect a mirror galvanometer to the terminals A and D, and the battery to B and C.

The battery is connected to the tapping key on the slide-wire to obviate heating effects due to the current, for with this arrangement the current only flows during the moment the adjustment is being tested. The adjustment for balance is correct when there is no *immediate* deflection on depressing the tapping key. If the key is kept down for a short time, the platinum becomes heated by the passage of the current, its resistance changes, and the balance is no longer correct.

The arrangement described forms a Wheatstone's network, shown diagrammatically in Fig. 255. When the point of balance is found,

$$\frac{R}{R} = \frac{P+r+xp}{r+S+(100-x)\rho},$$

where P = resistance of platinum coil,
r = resistance of connecting (or compensating) leads,
x = distance of point of balance from one end of bridge-wire,
ρ = resistance of 1 cm. of bridge-wire.

Now $$\frac{R}{R} = 1,$$

consequently $$P + r + x\rho = r + S + (100 - x)\rho$$

or $$P = S + (100 - 2x)\rho.$$

NOTE.—The value of P should be calculated from the result obtained when S is adjusted so that the balance point is as near as possible to the middle of the slide-wire.

Determination of ρ.—Place the tube containing the platinum coil in melting ice, and obtain a balance point with S equal to 1 ohm; let this be x_1 cm. from the end of the wire. Alter S to 1·1 ohms and find the new balance point; let this be x_2 cm. from the end. Alter S to 1·2 ohms, and let the balance point be x_3 cm. from the end.

Then
$$P = 1 + (100 - 2x_1)\rho,$$
$$P = 1{\cdot}1 + (100 - 2x_2)\rho,$$
$$P = 1{\cdot}2 + (100 - 2x_3)\rho.$$

Hence $$1 + (100 - 2x_1)\rho = 1{\cdot}1 + (100 - 2x_2)\rho,$$

i.e. $$\rho = \frac{0{\cdot}1}{2(x_2 - x_1)}.$$

Similarly, from the first and third observations,

$$\rho = \frac{0{\cdot}2}{2(x_3 - x_1)}.$$

The second value is probably the more correct, though the average might be taken for use.

Determination of P at the Freezing Point and the Boiling Point.—The resistance of the platinum spiral can be calculated from the observations already made, since the value of ρ has been determined.

The resistance at the boiling point is determined by finding a point of balance when the tube containing the coil is placed

in a hypsometer and surrounded by steam. The value of S must be adjusted so that the point of balance comes on the bridge-wire and as near the middle as possible. The temperature of the boiling point should be corrected for atmospheric pressure at the time of the experiment.

Calculate the mean value of the temperature coefficient of resistance between the freezing point and the boiling point.

The mean coefficient for any given range of temperature may be determined by finding the resistance R_1 and R_2 at the limits of the range. The temperatures t_1 and t_2 may be found by using a mercury thermometer to take the temperature of the bath in which the platinum spiral is immersed.

Then $$R_1 = R_0(1 + at_1) \text{ and } R_2 = R_0(1 + at_2).$$

Hence, by division, $$\frac{R_2}{R_1} = \frac{1 + at_2}{1 + at_1},$$

giving $$a = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}.$$

§ 4. Comparison of Resistances by the Fall of Potential Method

When two resistances are included in the same circuit so that the same current is flowing through each, we can compare the resistances if we can compare the potential differences between the ends of the two resistances.

Let DE and FG be two resistances, whose values are r_1 and r_2 respectively, and let them be included in a circuit containing

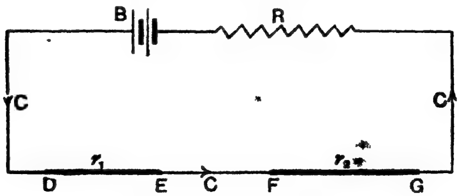

Fig. 256.—Comparison of Resistances.

any other resistance R and a battery B. Then the same current C flows through each part of the circuit.

If V stands for the potential at any point, we have, in accordance with Ohm's Law,

$$V_D - V_E = Cr_1$$

and

$$V_F - V_G = Cr_2.$$

Consequently

$$\frac{V_D - V_E}{V_F - V_G} = \frac{r_1}{r_2}.$$

Hence, if we can compare the fall of potential between D and E with that between F and G, we can find the ratio of r_1 to r_2. This method is specially suitable for low resistances.

EXPT. 241. **Comparison of two low Resistances.**—Connect in series a 2-volt cell, a large adjustable resistance R, and the two low resistances r_1 and r_2, as in Fig. 256.

If we determine the ratio of the differences of potential between DE and FG, we can find the ratio of r_1 to r_2: if either of these is known, the value of the other can be found.

In order to compare the fall of potential in one case with that in the other, all that is necessary is to connect the points D and E to a *high-resistance* galvanometer (virtually a voltmeter) and observe the deflection, and afterwards to connect the points F and G to the same galvanometer and again observe the deflection. A double-pole throw-over switch is convenient for this experiment (p. 584).

If d_1, d_2 are the deflections (assumed small) in the two cases,

$$\frac{V_D - V_E}{V_F - V_G} = \frac{d_1}{d_2}.$$

Hence

$$\frac{r_1}{r_2} = \frac{d_1}{d_2}.$$

Compare in this way the resistance of a metre of copper wire of about 20 standard wire gauge, and a standard resistance of 0·1 ohm. Find the resistance of the copper wire in ohms, and from this and the dimensions of the wire determine the specific resistance of copper.

This is a very useful method of comparing resistances, and deserves more attention than it has received. The principle is applicable in the case of the comparison of very small resistances. In some cases a quadrant electrometer may be used in place of the high-resistance galvanometer. It is, of course, a deflection method, and is therefore not so accurate as Kelvin's Double Bridge, which is a null method. This experiment is too advanced to be described in this book.

§ 5. Measurement of High Resistances

The ordinary form of Post-Office Box may be used to measure resistances up to 1,000,000 ohms. By having resistances of 10 ohms and 1000 ohms in the ratio arms P and Q (Fig. 250) the unknown resistance may be 100 times as large as the resistance of the adjustable arm, which is usually not greater than 10,000 ohms. Thus the unknown resistance may be 1,000,000 ohms. To measure a resistance greater than this a modification of the substitution method may be employed. A battery of constant E.M.F. is used to send a current through the high resistance placed in series with a sensitive galvanometer, and the deflection is observed. The same battery is then connected through a large adjustable resistance to the galvanometer, but the galvanometer is now *shunted* so that only a known fraction of the total current passes through it. If it be possible to obtain the same deflection as before, the unknown resistance can be calculated. If the adjustable resistance be not large enough to make this possible, the deflection obtained with the largest available resistance is noted, and the calculation is carried out on the assumption that the deflection is proportional to the current through the galvanometer. Another method that may be used in such a case is to vary the applied E.M.F. in a known manner, as by varying the number of cells in a battery composed of cells of equal E.M.F.

In measuring high resistances the various parts of the apparatus must be insulated carefully: thus no wires should be allowed to touch the table, as the resistance of the wood may be comparable with the resistance that is being measured.

Expt. 242. **Measurement of the Resistance of a Carbon Strip.**—A suitable high resistance for this experiment may be constructed by fixing two terminals to an ebonite base, and drawing lines from one terminal to the other with a black-lead (graphite) pencil The apparatus should be provided with a cover to protect the carbon strips from damage.

Connect the unknown resistance in series with a battery giving an E.M.F. of 6 or 8 volts, a plug key and a sensitive mirror galvanometer. Observe the deflection d_1 produced when the whole of the current passes through the galvanometer.

Remove the unknown resistance, and in its place put an adjustable large resistance—a Post Office Box will serve the purpose. Shunt the galvanometer with a coil having a resistance $\frac{1}{999}$ of the galvanometer resistance, so that only $\frac{1}{1000}$ of the total current passes through the galvanometer. Adjust

the resistance so that the deflection obtained is the same as that previously observed. Then as the *total* current is now 1000 times that in the first part of the experiment, the unknown resistance must be 1000 times the resistance in the resistance box.

If it be impossible to obtain in this way a deflection as small as that observed in the first part of the experiment (d_1), note the deflection (d_2) obtained with 10,000 ohms in the box and the galvanometer shunted so that $\frac{1}{1000}$ of the total current passes through it. Since the deflection may be assumed proportional to the current, and the current is inversely proportional to the resistance, the unknown resistance has the value $1000 \times 10{,}000 \times \frac{d_2}{d_1}$.

Calculate the unknown resistance from the deflections observed.

CHAPTER VI

ELECTROLYSIS—ELECTROCHEMICAL EQUIVALENTS

§ 1. Electrolysis

LIQUIDS which are decomposed when an electric current passes through them are termed **electrolytes**, and the process of decomposition is called **electrolysis**. Solutions of salts and acids in water, and certain compounds, when fused, are decomposed by the passage of a current, the products of decomposition appearing only at the plates where the current enters or leaves the electrolyte. These plates are termed the **electrodes**, that where the current enters the electrolytic cell being called the **anode**, the other the **kathode**. Thus, inside the electrolytic cell, or **voltameter**, as it is often called, *the current flows from the anode to the kathode.* The metallic (electropositive) ions, including hydrogen ions, are carried along with the current towards the kathode. In his "Experimental Researches" Faraday writes: "I propose to distinguish such bodies by calling those *anions* which go to the anode of the decomposing body; and those passing to the *cathode, cations*; and when I shall have occasion to speak of these together, I shall call them *ions*."

It was proved by the experiments of Faraday that the mass M of a radicle set free by the passage of a current is directly proportional to the quantity of electricity, Q, which has passed through the voltameter. But $Q = Ct$, where C is the strength of the current and t the time during which the current has passed. Hence M is proportional to Ct.

If the same current be passed through several voltameters in series containing different electrolytes, the number of ions that enter into chemical action in a given case is proportional to the **chemical equivalent** of the ion concerned. The chemical equivalent is the mass of an ion or radicle, which will replace, or combine with, one part by mass of hydrogen. In the case of an element the chemical equivalent is equal to the atomic mass divided by the valency. Thus in the case of copper, whose atomic mass is about 63, the chemical equivalent in a cuprous salt like cuprous chloride ($CuCl$) is 63, because copper here is univalent, while in a cupric salt like cupric chloride ($CuCl_2$) the chemical equivalent is $63/2$ because copper here is divalent.

The **electrochemical equivalent**, e, of an ion is the mass in grams which is set free by the passage of unit quantity of electricity. It follows from this definition that

$$M = eQ = eCt.$$

It also follows from the statements above that the electrochemical equivalent is directly proportional to the chemical equivalent, or the electrochemical equivalent of an ion is equal to the chemical equivalent of the ion multiplied by the electrochemical equivalent of hydrogen.

The **Practical Units** of Quantity or Current of Electricity are frequently defined in terms of the amount of chemical action produced by the passage of electricity through an electrolyte. Thus, the **International Coulomb**[1] has been defined as the quantity of electricity which liberates 0·001118 gm. of silver from a neutral solution of nitrate of silver in water. Again, a current of one **International Ampere**[1] is that current which liberates per second 0·001118 gm. of silver from a neutral solution of nitrate of silver in water. Thus, the electrochemical equivalent of silver is, from this definition, 0·001118 gm. per coulomb. The chemical equivalent of silver (referred to

[1] The *Legal* or *International* Coulomb and Ampere defined in this way differ by a very small amount from the Coulomb and Ampere defined by means of the magnetic action of a current.

hydrogen) is 107·02; hence the electrochemical equivalent of hydrogen is 0·00001045 gm. per coulomb.

The quantity of electricity required to liberate one *gram-atom* of any univalent element is called one *Faraday*, and is equal to $107·88 \div 0·001118 = 96500$ coulombs approximately.

One gram-atom means that mass for which the number of grams is the same as the number representing the atomic ma of the element. Taking the atomic mass of xygen as 16, on gram-atom of silver contains 107·88 gm.

§ 2. Determination of Electrochemical Equivalents

ELECTROCHEMICAL EQUIVALENT OF HYDROGEN

When a current of electricity is passed between platinum electrodes through a dilute solution of sulphuric acid in water, decomposition takes place, and oxygen is evolved at the anode, hydrogen at the kathode. In this case the current can be made to pass continuously only if the electromotive force available is greater than about 1·5 volts, for the products of decomposition on the electrodes act like the plates of a cell. This cell has an E.M.F. which *opposes* the passage of the current, the **Back E.M.F.** set up being about 1·5 volts. If the strength of the current passing through the solution be extremely small, it may happen that the hydrogen liberated is dissolved by the water and no visible evolution of bubbles of gas takes place, but with a stronger current bubbles should be evolved freely and the water soon be saturated with gas, so that all the hydrogen liberated subsequently may be collected.

Other solutions may be used in place of the sulphuric acid solution, *e.g.* in preparing very pure hydrogen a solution of barium hydroxide $Ba(OH)_2$ is frequently employed.

In order to determine the electrochemical equivalent of hydrogen it is necessary to pass a known current for a known time, to collect the hydrogen evolved, to measure its volume under known conditions, and to calculate its weight.

To measure the current a tangent galvanometer may be used,

for this measures the current in *absolute* units. It is, however, often more convenient to use a moving coil ammeter, reading, say, from 0 to 3 amperes, which has been standardised already by comparison with a tangent galvanometer (p. 466). Care must be taken to connect the terminal of the ammeter marked + to the positive terminal of the battery. A current of from 1 to 2 amperes is usually convenient, and a suitable current may be obtained by varying the number of cells in the battery, or by introducing sufficient resistance in the circuit when the current is taken from the electric-light mains where direct current is supplied.[1] Several different forms of apparatus have been constructed for collecting the gas evolved. Two only will be referred to here.

I. Apparatus for Mixed Gases.—The glass portion of the apparatus is constructed as in Fig. 257. The gas evolved from both electrodes is collected in one and the same tube, which is graduated in c.c. Two volumes of hydrogen are evolved to one volume of oxygen, so that of the total volume of gas collected only two-thirds are hydrogen. The graduated glass tube can be refilled with water by tilting the apparatus on its side repeatedly. In this way the apparatus can be prepared for use expeditiously without the risk of dilute acid being splashed about on the laboratory table.

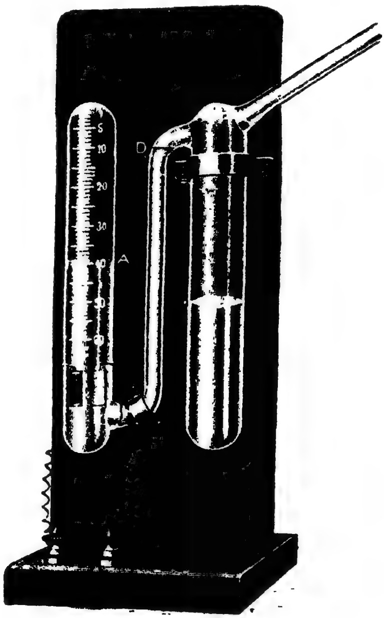

Fig. 257.—Voltameter for Mixed Gas

EXPT. 243. **Determination of the E.C.E. of Hydrogen. I.**—The method of making the connections is shown in Fig. 258.

When these connections have been made, complete the circuit for a few moments by means of the plug key, in order to see that a suitable deflection is obtained on the ammeter, and that bubbles of gas are given off from the electrodes. If the current is obtained from a battery of

[1] For a convenient type of lamp resistance see p. 588.

storage cells, a piece of resistance wire (platinoid or manganin) may be introduced as one of the connecting wires, in order to adjust the current to the strength desired. Before beginning the actual experiment remove the bubbles of gas that have collected in the graduated tube during the preliminary adjustments.

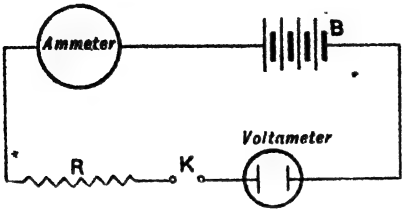

FIG. 278.—Determination of Electrochemical Equivalent.

Start the current and the seconds hand of a stop-watch, or clock, at the same instant, and allow the evolution of gas to continue till the graduated portion of the tube is filled with the mixed gases.[1] Readings of the ammeter must be taken every half minute, and from these readings the mean value of the current must be calculated. When the full amount of gas has been collected, stop the current and the stop-watch at the same instant, and find the value of t, the time in *seconds* during which the current has been flowing. Determine also the volume, v c.c., of gas collected under the conditions of the experiment, remembering that only two-thirds of the total quantity of gas is hydrogen.

This volume, v c.c., requires to be corrected for temperature and pressure, that is, we require to find what the volume would be at 0° C. and under a pressure of 760 mm. of mercury.

Correction for Temperature.—This correction is most easily applied by using the *absolute scale* of temperature, since the volume of a given mass of gas is directly proportional to its absolute temperature. To convert temperatures on the centigrade scale to temperatures on the absolute scale it is only necessary to add 273° to the reading of the centigrade thermometer. The volume of the gas at 0° C. (or 273° A.) would be

$$v \times \frac{273}{T},$$

where T is the temperature of the room on the absolute scale.

Correction for Pressure of Aqueous Vapour.—Let P be the actual pressure in mm. of mercury of the gaseous mixture in the tube at the end of the experiment.

[1] The evolution of gas must not be continued long enough for the electrodes to become uncovered by liquid; if this happens there is some risk of a serious explosion taking place with this form of apparatus.

Let B be the height of the barometer in mm. The pressure P in the tube differs from the atmospheric pressure on account of the difference in water level between A and D (Fig. 257).

Let h = the difference in level in mm. between A and D. The pressure due to the difference of level between A and D $= h/13{\cdot}6$ mm. of mercury, since the density of mercury is 13·6. Hence $P = B + h/13{\cdot}6$.

The total pressure P is made up of a pressure P′ due to the hydrogen and oxygen, and p due to the aqueous vapour in the tube. The space in the tube is saturated, therefore p = saturation vapour pressure of water in mm. of mercury at the temperature of the room.

Thus, $P' = P - p$, or $P' = B + h/13{\cdot}6 - p$, and consequently the volume of the hydrogen and oxygen together under standard pressure (760 mm.) at 0° C. is

$$v_0 = v \times \frac{273}{T} \times \frac{P'}{760}$$

$$= v \times \frac{273}{T} \times \frac{B + h/13{\cdot}6 - p}{760}.$$

The volume of the hydrogen is two-thirds of this volume.

The mass of 1 litre of hydrogen at 0° C. under 760 mm. of mercury is 0·08987 gm., so that we may take 1 c.c. of hydrogen under these conditions as having a mass of almost exactly 0·00009 gm. Hence the mass of the hydrogen $M = \frac{2}{3}v_0 \times 0{\cdot}00009$.

The electrochemical equivalent of hydrogen may then be calculated from the equation $e = \frac{M}{Ct}$.

The experiment may be repeated with different values of C.

II. Apparatus for Separate Gases.—In this apparatus the oxygen and hydrogen are collected separately in two inverted gas collecting tubes, or burettes. These are filled with water before the commencement of the experiment and placed in position over the platinum electrodes (Fig. 259).

EXPT. 244. **Determination of the E.C.E. of Hydrogen. II.**—Make the electrical connections as in Fig. 258, and fill the collecting tubes with water.

The resistance of the voltameter varies very considerably with the depth to which these glass tubes are immersed, so

that the strength of the current may be adjusted by altering the position of one of the tubes. In making this adjustment great care must be taken to see that all the bubbles of gas are caught within the glass tube; if the tube is not at a sufficient depth in the liquid some bubbles may escape. The procedure is very similar to that already described, but in this case the hydrogen evolved at the kathode is collected till the tube containing it is filled to the level of the water in the voltameter. By adjusting the level of the water inside the tube so as to be the same as that of the water outside, h becomes zero, and no correction for difference of level is needed.

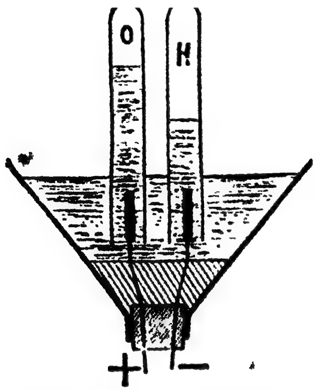

FIG. 259.—Voltameter for Separate Gases.

The aqueous vapour present must be corrected for. If the atmospheric pressure is B, and the volume V is read at atmospheric pressure, we have present in the tube this volume of hydrogen at a pressure $B-p$, p being the saturation vapour pressure of aqueous vapour at the temperature of the water.

It is also necessary to correct for temperature.

The volume of the hydrogen, reduced to 0° C. and 760 mm. pressure, will be given by

$$V_0 = V \times \frac{273}{273+t} \times \frac{B-p}{760},$$

where t is the temperature (centigrade) of the water in the voltameter.

ELECTROCHEMICAL EQUIVALENT OF COPPER (CUPRIC)

When a current of electricity is passed between copper electrodes through a solution of a copper salt in water, copper is dissolved from the anode but is deposited on the kathode. The amount of chemical change taking place is proportional to the quantity of electricity passing through the voltameter. It is found, however, that trustworthy results cannot be obtained from the loss in weight of the anode, as small pieces become

loosened and detach themselves from the anode mechanically, so that in quantitative experiments it is always the increase in weight of the kathode that is determined. Our present object is to determine the electrochemical equivalent of copper, that is the weight of copper deposited by the passage of one coulomb of electricity. We must therefore measure the strength of the current by some absolute instrument such as the tangent galvanometer, and observe the time in seconds during which the current flows. In many experiments, however, the value of the electrochemical equivalent is assumed, and the copper voltameter used to measure the strength of the current flowing.

The Copper Voltameter consists of a glass jar containing a solution of copper sulphate made by dissolving about 20 parts by weight of crystallised copper sulphate in about 80 parts of water. The solution is made slightly acid by the addition of 1 per cent of strong sulphuric acid. The anode is made of two similar copper plates placed parallel to one another and supported by a bridge of ebonite which rests on the top of the glass jar. The plates are both connected to one terminal fixed to the ebonite. The kathode is a copper plate of about the same size as one of the anode plates. It is placed between the latter and is attached to a brass block on the ebonite by a single binding-screw, so that it can be removed easily for weighing. As both anode and kathode are composed of the same metal as that which is liberated by the current, there is no back E.M.F. due to polarisation, and the smallest applied E.M.F. will bring about the deposition of copper. But if the current is very small, a very long time is required to give an amount that could be weighed accurately. On the other hand, when the current is too strong, the copper is liable to scale off the plate. To get a firm and smooth deposit of copper the current should not exceed one ampere for each 50 or 60 sq. cm. of surface. It is therefore necessary to find the area of the two sides of the kathode and to calculate approximately

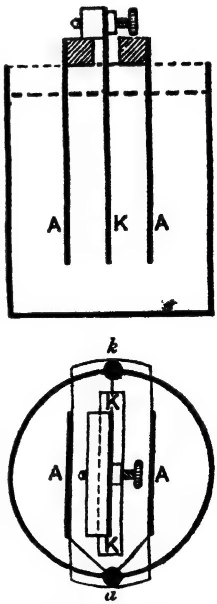

FIG. 260.—Copper Voltameter.

the deflection of the galvanometer corresponding to this maximum current. If the kathode be a rectangular plate about 5 cm. wide and be immersed to a depth of 10 cm., the current to be used should be about 2 amperes, and the deposition should be continued at least half an hour.

EXPT. 245. **Determination of the E.C.E. of Copper.**—Connect the apparatus as in Fig. 261, taking care that the anode is connected to the positive pole of the cell.

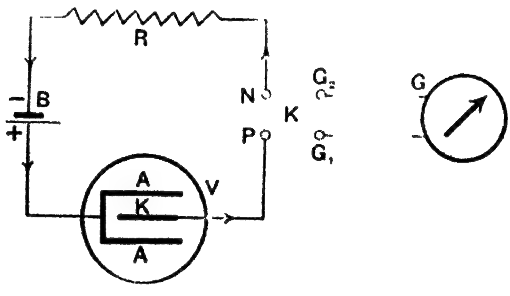

FIG. 261.—E.C.E. of Copper.

B is the battery for which a single secondary cell is sufficient.

V is the voltameter.

R, an adjustable resistance which may be a length of platinoid wire.

K, a commutator.

G, a tangent galvanometer.

The tangent galvanometer usually employed is one with a single coil containing either *a single turn*, or two turns close together, of thick copper wire. All iron must be removed from the neighbourhood of the galvanometer, which must be set up so that the plane of the coil is parallel to the magnetic meridian, and the reading is zero. The wires connecting the galvanometer to the commutator should lead away from the galvanometer and should be twisted together, so that the magnetic fields they produce may not affect the needle; twin flexible leads are convenient for this purpose.

The kathode must first be well cleaned by rubbing it with sand and water with a rag, or with a piece of sand-paper or pumice-stone. It is washed with water and placed in the voltameter so that the current may be adjusted to the desired

value by altering the value of the resistance R. When the current has passed for a minute or two, the circuit is broken and the kathode removed for examination. The part immersed should now be covered with a smooth salmon-coloured deposit of copper. If the plate is dark in colour it shows that the connections have not been made correctly. The red-coated plate must be washed and dried carefully. The excess water should be removed with blotting-paper without rubbing, and the plate may then be gently warmed at some distance *above* the flame of a Bunsen burner or a spirit lamp. Great care must be taken that the surface is not oxidised in the process. The kathode, when dry and cold, is weighed on an accurate chemical balance *to within one milligram*, and is replaced in the voltameter. The current is started at a known instant and allowed to flow for at least half an hour. The deflection of the galvanometer is noted during the first five minutes. At the end of this time the current through the galvanometer is reversed quickly and the deflection again noted. At the end of another five minutes the current is reversed again, and so on.

Record the results as follows :—

Time.	Mean Deflection.	Tangent.
0 mins.		
	+35·0	0·7002
5		
	−36·5	0·7400
10		
	−36·0	0·7265
15		
	−36·0	0·7265
20		
	+35·5	0·7133
25		
	−35·0	0·7002
30		
		0·7178 mean

The mean current in amperes during the experiment is given by the formula

$$C = 10 \times \frac{H}{G} \times \tan\theta,$$

where C is the mean current in amperes,

tan θ is the mean tangent of deflections,

G is the magnetic field at the centre of the galvanometer due to 1 C.G.S. electromagnetic unit of current,

and H is the earth's horizontal field (=0·185 C.G.S. units in London in 1915).

Now for a circular coil and *a single turn*[1] of radius r cm.

$$G = \frac{2\pi}{r}.$$

Hence $\qquad C \text{ (in amperes)} = \dfrac{10rH}{2\pi} \tan\theta.$

Measure r, the mean radius, in cm. of the single coil of thick wire as accurately as possible with a beam compass or a pair of callipers, and calculate C.

The kathode is carefully washed and dried as before, and then weighed again. The increase in weight is found. Let this be M grams. Then the electrochemical equivalent

$e = \dfrac{M}{Ct}$ = the number of grams deposited per coulomb.

[1] For a Helmholtz galvanometer (p. 566) of two coils, one turn each, at a distance apart equal to the radius r of either coil,

$$G = \frac{8{\cdot}99}{r}.$$

Hence, in this case.

$$C \text{ (in amperes)} = \frac{10rH}{8{\cdot}99} \tan\theta.$$

CHAPTER VII

THE HEATING EFFECT OF AN ELECTRIC CURRENT

§ 1. Joule's Law

The difference of potential between two points in an electric circuit is equal to the work done in carrying unit quantity of electricity from one point to the other. Hence, if a quantity of electricity Q is carried between two points when the P.D. is E, the work done is $W = EQ$.

If a steady current C flow for time t, $Q = Ct$, and therefore $W = ECt$.

If E is in volts, C in amperes, t in seconds, W is in joules (1 joule = 10^7 ergs).

When the energy of an electric current is not used in doing mechanical work or in chemical action it appears as heat in the conductor. According to Joule's law the heat produced is equivalent to a certain amount of mechanical energy, in accordance with the equation

$$W = JH.$$

If W is in joules, H in calories, $J = 4{\cdot}2$ very nearly, for 1 calorie is equivalent to $4{\cdot}2 \times 10^7$ ergs.

Then $$JH = ECt.$$

Expt. 246. **Determination of the Mechanical Equivalent of Heat by an Electrical Method.**—In order to test the accuracy of this result it is necessary to measure the heat produced by the passage of a given current, for a given time, under a known difference of potential.

The heat produced is measured by immersing a coil of resistance wire carrying the current in a large calorimeter (capacity about half a litre) containing a liquid of known specific heat. In the case of water a certain amount of electrolysis takes place, but provided the potential difference employed does not exceed 8 or 10 volts, and the resistance of the immersed spiral is low (about 0·5 ohm), this does not materially affect the result. The spiral is connected by thick copper leads to two terminals fixed to the wooden cover of the calorimeter. The cover is provided with a hole for the thermometer and another for the stirrer. *Efficient stirring is most important in this experiment.*

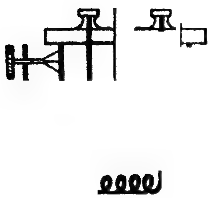

FIG. 262.—Calorimeter Heating Coil.

The calorimeter is weighed empty, and again when full of water.

The electrical quantities are measured most conveniently by using an ammeter and a voltmeter. The apparatus should be connected as in Fig. 263.

B is a battery comprising 4 or 5 secondary cells.

K is a plug-key.

A is the ammeter for currents up to about 15 or 20 amperes.

V is a voltmeter reading to about 5 volts.

C is the calorimeter.

R is a resistance consisting of a wire frame rheostat, or a bare resistance wire.

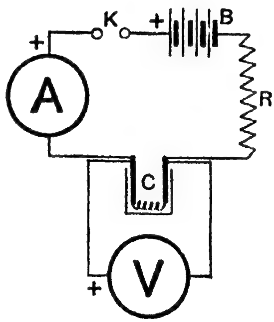

FIG. 263.—Heating Effect of Current.

It is advisable to use a current of from 8 to 12 amperes so that an appreciable change of temperature may be obtained in two or three minutes. Having connected up the apparatus, using *thick* copper wires to make the connections in the main circuit, adjust the strength of the current to a suitable value. Wait a few minutes for the temperature of the calorimeter to become steady, stirring occasionally, then take the reading t_1.

Start the current when the seconds hand of a watch passes

the 60 mark, and allow the current to flow for say 3 minutes, stirring thoroughly meanwhile. Take readings of the ammeter and the voltmeter every half minute. Stop the current after a known interval of time and read the final temperature θ_2.

Let m = mass of calorimeter (inner vessel only),
M = mass of water,
s = specific heat of metal of calorimeter,
θ_1 = initial temperature,
θ_2 = final temperature.

Then the heat given to the calorimeter and its contents is

$$H = (M + ms)(\theta_2 - \theta_1).$$

Calculate the value of J in the equation

$$JH = ECt,$$

noting that the time must be measured in seconds.

The experiment may be repeated for a different value of the current.

Instead of measuring E with a voltmeter, the resistance R of the immersed spiral may be measured by means of the slide-wire bridge, and the value of J found from the equation

$$JH = C^2Rt.$$

NOTE.—This last equation is true whether work is being performed by the current or not, the *portion* of the applied potential difference required to overcome a resistance R being CR if a current C is being passed, however the rest of the P.D. is being employed. The heating effect therefore is measured always by C^2Rt, and is often referred to by electrical engineers as 'the C^2R loss.'

§ 2. THE EFFICIENCY OF AN ELECTRIC LAMP

The principal units employed for the scientific measure of *energy* are :—

The **erg** = 1 dyne centimetre.

The **joule** = 10^7 ergs.

The **calorie,** or heat unit of energy = $4\cdot2 \times 10^7$ ergs = $4\cdot2$ joules.

The **Board of Trade Unit (or kelvin)** = the energy supplied by an engine working for a period of 1 hour, the power of the engine being 1 kilowatt (see below).

[Calculate the number of ergs in the kelvin.]

The units employed for measuring *power* or *activity* (rate of doing work) are the following:—

The C.G.S. unit = 1 erg per second.

The **watt** = 1 joule per second.

The **kilowatt** = 1000 watts.

The British horse-power = 33,000 ft. lbs. per min. = 746 watts.

The measurement of electric power depends on the measurement of *current* and *difference of potential*; the *energy* depends on these and also on the *time*.

There is said to be unit difference of potential between two points when unit amount of work has to be done against the electric forces, in carrying unit quantity of electricity from one point to the other. This quantity may be carried by unit current flowing for unit time: the quantity carried by a current C in t seconds is $Q = Ct$.

If we use C.G.S. electromagnetic units the work done is measured in ergs. If we use practical units the work done is measured in joules. For the difference of potential between two points is one **volt** when one **joule** of work has to be done in carrying one **coulomb** of electricity from one point to the other against the electric forces.

If a steady current of one **ampere** flow between two points between which the P.D. is one **volt**, the rate of doing work is one joule per second, or one **watt**.

1 Coulomb = 10^{-1} E.M.U. (Abs.).
1 Ampere = 10^{-1} E.M.U. (Abs.).
1 Volt = 10^{8} E.M.U. (Abs.).
1 Ohm = 10^{9} E.M.U. (Abs.).

When electrical energy is used for purposes of illumination it is important to know the relation between the rate at which energy is supplied and the candle-power produced. Electrical engineers often speak of the number of watts per candle-power as the *efficiency* of the source of light. It would be more correct to term this the *inefficiency*, and to use the term efficiency to denote the candle-power per watt.

Expt. 247. Determination of the Efficiency of an Electric Lamp.—The candle-power of the lamp under test may be measured by one of the methods described in the Chapter on Photometry, p. 302.

To measure the rate at which energy is supplied to an incandescent lamp, it is necessary to measure the current passing through the lamp and the difference of potential between its terminals.

Connect up the apparatus as in Fig. 264.

L is the lamp.

R is an adjustable wire-frame resistance.

A is the ammeter, connected *in series.* V is the voltmeter, connected *in parallel* with the lamp.

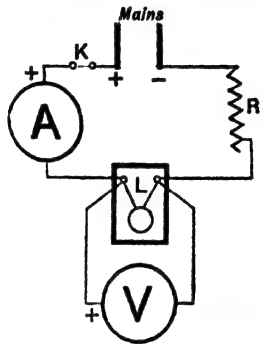

Fig. 264.—Efficiency of Electric Lamp.

Care must be taken to see that connection is made to the proper terminals for both ammeter and voltmeter, before switching on the current. Note the readings of the ammeter and the voltmeter for a particular value of the resistance R.

Measure the candle-power of the lamp in this condition.

Take a series of readings in this manner for different values of the resistance R. Finally cut out the whole of the resistance R, and take a set of readings when the lamp is supplied with current at the voltage for which it was designed.

1. Calculate the number of watts per candle at the specified voltages.

2. Calculate also the candle-power developed per watt.

3. Calculate the resistance of the lamp when glowing at different candle-powers.

4. Calculate the heat produced in the lamp in calories per second.

Tabulate the results, and illustrate them by means of graphs.

It is instructive to carry out experiments of this kind both with a metal filament and a carbon filament lamp.

The variation of the resistance of the carbon filament with temperature (estimated *qualitatively* by the colour) differs in an interesting way from that of the resistance of the metal filament.

CHAPTER VIII

INDUCED CURRENTS—ELECTROMAGNETIC MACHINES

§ 1. Electromagnetic Induction

In 1831 Faraday showed that an electric current was produced in a closed circuit whenever the number of lines of magnetic induction passing through the circuit was changed. Such a current is termed an **induced current.**

The alteration in the number of lines of magnetic induction may be due to

(1) Starting or stopping currents in neighbouring conductors,

(2) Variations in the strength of those currents,

(3) The motion of conductors carrying currents, or

(4) The motion of permanent magnets with reference to the circuit under consideration.

The results in all these cases may be summarised in the statement, due to Faraday and Neumann, that **the E.M.F. induced in a circuit is equal to the rate of decrease of the number of lines of magnetic induction passing through the circuit,** the positive direction of the E.M.F. being related to the positive direction of the magnetic induction in the same way as the direction of rotation to the direction of translation in a right-handed screw.

An *increase* in the number of lines gives rise to a *negative* E.M.F.

Expt. 248. **Illustration of the Laws of Electromagnetic Induction.**—These laws are illustrated in the present experiment by the aid of two coaxial coils. One coil, the **primary,** is connected in series with a secondary cell, an adjustable resistance, and a key; the other, the **secondary,** is connected with a galvanometer.

Before connecting the coils it is necessary to determine in which direction the wire is wound in each of them. In

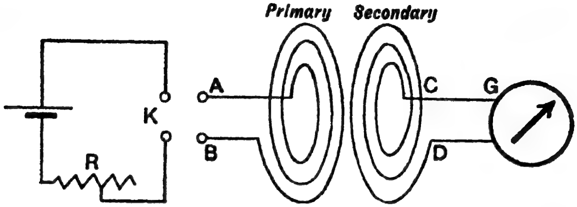

FIG. 265.—Apparatus for Induced Currents.

R=resistance. G=galvanometer. K=reversing switch.

the following description it is assumed that the axis of each coil is vertical.

To find the Direction of Winding in the Coils.—The best method of finding the direction of winding in the coils is as follows:—Mark the terminals of the primary coil A and B, and the terminals of the secondary coil C and D. Connect the positive terminal of the battery to A, and the negative terminal of the battery through a rough regulating resistance, R, to B. Bring a compass needle close to the upper end of the coil, and note the way in which the needle points. If the north end of the needle points towards the top of the coil, this end acts like the south pole of a magnet, *i.e.* lines of force enter the coil at this end.

This means that the current in the coil circulates clockwise, as viewed from above, when the current enters at A. The reverse holds good if the south pole of the needle points to the top of the coil. In this way the direction of circulation of the current in the primary can be determined when the current enters at A.

Now connect the cell to the secondary coil in a similar way, connecting the positive of the cell to the terminal marked C. Determine the polarity of the top of the coil in this case, and from it deduce the direction of circulation of the current in the secondary when the current enters at C.

Let the direction of circulation in the secondary be clockwise, as viewed from above, when the current enters the secondary at C.

To find the Direction of the Secondary Current corresponding with any Deflection of the Galvanometer.—The direction of motion of the needle of the galvano-

meter must now be found for a current flowing round it in some known direction.

Mark the terminals of the galvanometer E and F. Connect E to the positive terminal of the cell, connecting F *momentarily* with the negative terminal of the cell through as large a resistance as the regulating resistance will give.

Suppose the north pole of the galvanometer needle to go to the east, then this north pole moves east when the current enters at E, and hence the direction of circulation of the current in the galvanometer can be found from the motion of the needle.

Connect up the secondary coil to the galvanometer so that C is connected with E, and D with F.

Then on the above supposition, a deflection of the north pole of the needle towards the east means that the current enters the galvanometer at E, *i.e.* an easterly deflection indicates a current circulating from D to C in the secondary, since the current leaves the secondary at C.

In all the following description it will be assumed that the coils are viewed from above. This means that an easterly deflection of the north pole of the galvanometer needle indicates an anti-clockwise circulation of the current in the secondary, since the circulation is clockwise (by supposition) when it enters the secondary at C.

Demonstration of the Laws of Electromagnetic Induction.—Having made these preliminary observations and obtained the result expressed in the last sentence, connect up the primary coil with its terminal A connected with the positive terminal of the cell, and with B connected to the negative with a large adjustable resistance in series.

The current in the primary will now circulate round the coil in some known direction: suppose that this direction is clockwise.

Then carry out the following experiments, noting in each case the direction of circulation of the current in the secondary as shown by the deflection of the galvanometer needle:—

1. *Current started in Primary.*—The deflection of the needle is east. Therefore a *clockwise* current started in the primary induces an *anti-clockwise* (*inverse*) current in the secondary.

2. *Current stopped in Primary.*—The deflection of the needle is west. Therefore a *clockwise* current stopped in the primary induces a *clockwise* (*direct*) current in the secondary.

Note the deflection as before and deduce the direction of the secondary current induced in each of the following cases:—

3. Current in primary suddenly increased.
4. Current in primary suddenly diminished.
5. Secondary suddenly moved away from primary, keeping primary current constant.
6. Secondary suddenly brought up to primary again.
7. Current in primary suddenly reversed.

It will be found that switching on the current produces an effect of the same kind as 3 and 6.

Switching off produces an effect of the same kind as 4, 5 and 7. We can thus deduce another Law of Induced Currents:—

The induced current in the secondary coil is always in such a direction as to oppose any change in the magnetic field through the secondary. It only lasts while the change is being produced.

In the suppositions made, switching on the current introduces *downward-directed* lines of force. The induced current circulates anti-clockwise and thus creates *upward-directed* lines of force, which however only persist for a moment, the induced current dying away almost instantaneously.

> Verify that the above law holds good for all changes in the magnetic field, however produced.
>
> This may be done by bringing up a bar-magnet and finding the direction of the induced current:—
>
> (*a*) When the north pole of the magnet is inserted in the coil, *i.e.* the magnet is inserted with its north pole downwards.
>
> (*b*) When the north pole is withdrawn suddenly.
>
> (*c*) When the magnet is inserted with its south pole downwards.
>
> (*d*) When the south pole is withdrawn suddenly.
>
> Place a bundle of iron wires in the coil, and repeat the first series of experiments. Show that the effects produced are always of the same kind as with air in the coil, but that the induced currents are of much greater magnitude.

The last result is explained by saying that iron is more permeable than air to magnetic lines. If H denote the strength of the magnetic field, or the number of C.G.S. lines per sq. cm. in air, and B the corresponding quantity in iron, the ratio of B to H is called the **Permeability**, μ, of the iron. Thus

$$\mu = \frac{B}{H}.$$

The term magnetic **Flux** is applied to the number of the magnetic lines passing through the iron. The C.G.S. unit of magnetic flux is the **maxwell.** The maxwell corresponds to one C.G.S. magnetic line.

THE INDUCTION COIL

The object in view in the construction of an induction coil is the production of a high induced electromotive force that is mainly unidirectional. Let M denote **the coefficient of mutual induction** of two coils, *i.e.* the number of times the secondary is threaded by lines of magnetic induction when unit current flows in the primary.[1] Then if a current C flow in the primary, the number of times the secondary is threaded by lines due to this current is $N = MC$.

But the induced E.M.F. = Rate of decrease of N,
= Rate of decrease of MC,
= M × (Rate of decrease of C),

provided M is a constant quantity.

Thus to make the induced E.M.F. large, the two factors on the right-hand side must be made large. To make M large, the secondary coil is wound with a large number of turns, and an iron core is used to concentrate the magnetic lines. To make the rate of decrease of the current large, the original current in the primary must be large and it must be cut off very rapidly.

Thus the essential features of the induction coil are :—

(1) A primary coil of a small number of turns of thick wire, and therefore of low resistance.

(2) A secondary coil of a large number of turns, involving the use of fine wire and consequently large resistance.

(3) A core consisting of a bundle of soft iron wires.

(4) An interrupter, to break the primary current as rapidly as possible.

In most cases the coil is fitted with a condenser, with its

[1] If there are n turns in the secondary, each line threads the circuit n times.

plates connected to the two points of the primary circuit between which the break takes place.

Fig. 266 shows the construction of a Ruhmkorff's coil fitted with a hammer contact-breaker. The battery B is connected to the primary P through the commutator K, connection being completed through the point of the screw A and the back of the hammer-head H. The hammer-head is mounted on a spring D, the tension in which can be adjusted by the insulated screw T. When the current passes round the primary, the iron core is magnetised and attracts the soft

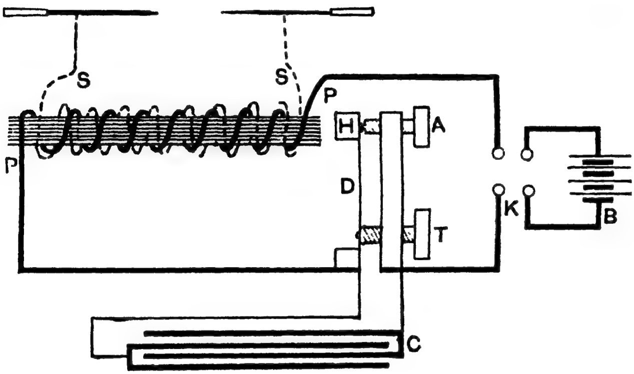

FIG. 266.—Ruhmkorff's Coil.

iron H, which flies forward and breaks the contact. The magnetic field is thus destroyed, and an E.M.F. is induced in the secondary due to the sudden withdrawal of the magnetic lines. The hammer-head, being no longer attracted, returns to its former position and makes contact once more. C is a condenser in the base of the apparatus. It is introduced so that the E.M.F. due to the self-induction of the primary may charge up the condenser instead of producing a spark at the point of break. In this way the primary current is reduced to zero more rapidly, or the rate of diminution of the current is increased. An E.M.F. is also induced in the secondary on making contact, but this is much smaller than the E.M.F. at

break, since the current in the primary takes some time to rise to its full value in consequence of self-induction.

NOTE.—When the contact is broken, a further phenomenon is that, as the condenser and the primary of the Ruhmkorff coil form an **oscillating system**, an oscillating E.M.F. is induced in the secondary.

EXPT. 249. **The Induction Coil.**—It is assumed that the induction coil is fitted with the ordinary hammer-break. Release the lock-nut on the screw carrying the platinum point, and withdraw the screw so that there is no contact with the platinum on the hammer-head. Turn the adjusting screw T till there is no extra tension on the spring D. Connect the terminals of the coil to a suitable battery, introducing a piece of lead wire fusing at 10 amperes to prevent an excessive current from damaging the coil. With a coil of medium size a battery giving an E.M.F. of about 8 volts will be suitable. Turn the handle of the commutator into the *on* position. Advance the screw carrying the platinum point till contact takes place with the hammer-head. The interrupter should commence to work, and when the terminals of the secondary are set at a small distance apart, sparks should pass between them. The screw should be clamped in this position by means of the lock-nut. The stiffness of the spring may now be adjusted by turning the milled head. Occasionally the platinum points fuse together and the action ceases. Should this happen, momentarily reverse the current; this will often start the interrupter working again. If it does not, switch off the current and turn back the screw carrying the platinum point. After long working the platinum contacts become pitted and may need polishing with fine emery paper, but this should never be attempted by the student.

Determine the maximum length of spark that can be obtained from a given setting of the spring. Assuming that the spark length is approximately proportional to the P.D., and that 30,000 volts are required for a spark 1 cm. long, calculate the E.M.F. induced in the secondary of the coil.

Connect the coatings of an insulated Leyden jar to the secondary terminals, and examine the character of the spark.

Examine the discharge through a 'vacuum tube' connected to the terminals of the coil. With a moderate vacuum, near the positive electrode is a long luminous column known as the **positive column**, while the negative electrode is surrounded by a blue luminous layer known as the **negative glow**. With a high

vacuum these effects disappear, but the glass walls of the tube exposed to the negative electrode become fluorescent under the bombardment of kathode rays (negative electrons) proceeding from that electrode.

An X-ray bulb may also be examined. The saucer-shaped kathode must be connected to the negative terminal of the coil. The anode and anti-kathode are connected to one another and to the positive terminal. If the commutator is in the right position, that half of the bulb which is in front of the anti-kathode will fluoresce with a green glow. The anti-kathode is bombarded by the kathode rays, and the X-rays originate at the point of impact. Their effect may be detected by a fluorescent screen or a photographic plate. The skin should not be exposed unduly to the X-rays.

THE TRANSFORMER

The induction coil is a particular instance of a general type of apparatus known as a **Transformer.** A typical case is Faraday's Ring Transformer, in which an iron ring is provided with two windings as in Fig. 267. The current in the primary, P, gives rise to lines of magnetic induction which form a closed magnetic circuit in the iron ring. When the strength of the current in the primary is varied, an E.M.F. is induced in the secondary, S. The magnitude of this E.M.F. depends primarily on the material of the core, and on the relative numbers of turns in the primary and the secondary.

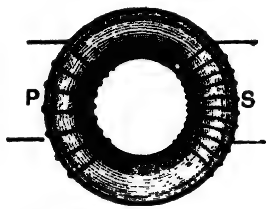

FIG. 267.—The Transformer.

When an alternating current is sent through the primary, an alternating E.M.F. is induced in the secondary. If the number of turns in S be greater than the number of turns in P, the voltage at the ends of S will be greater than that at the ends of P approximately in the ratio of the number of turns. Neglecting energy losses, the current is diminished in the same ratio as the voltage is increased. This arrangement is called a *step-up* transformer. The converse of this is the *step-down* transformer, in which the voltage is decreased while the current is increased.

EXPT. 250. **The Ring Transformer.**—Connect the primary coil of a ring transformer through a commutator to a battery of accumulators, placing in the circuit a rough regulating resistance and an ammeter. Connect the secondary coil to a ballistic galvanometer (p. 572).

Observe the *throw* of the galvanometer produced by a sudden reversal of the current in the primary.

Repeat the observations for different values of the current in the primary, and plot a curve showing the relation between the throw of the galvanometer and the strength of the primary current.

The throw of the galvanometer is proportional to the quantity of electricity passing through it, *i.e.* to the change in the number of lines of magnetic induction passing through the iron ring. The current in the primary is a measure of the magnetising force producing these magnetic lines. Thus the curve serves to indicate **the permeability** of the iron to lines of magnetic induction, for different values of the magnetising force.

THE EARTH INDUCTOR

When a coil rotates in a magnetic field, there is induced in the coil an E.M.F. which is *alternating* in character (Curve I., Fig. 269). With a uniform rate of turning, the induced E.M.F. has its maximum value when the plane of the coil passes through the plane of the field, and is zero when the coil is in a position perpendicular to the plane of the lines of force.

Methods of measuring the Induced Current.—An ordinary galvanometer connected to such a coil would give no deflection if the coil were rotated always in one direction unless a commutating device were fitted. An arrangement convenient for commutating the current produced by such a coil is to connect the terminals of the coil to two halves of a split cylinder of brass on an insulating cylinder mounted on the axle of the coil (Fig. 268). Against this, at opposite ends of a diameter, two springs press, the springs being fixed to the

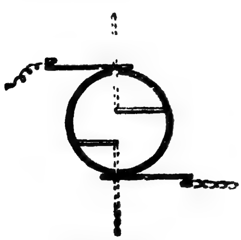

FIG. 268.—Commutator for Direct Current.

framework supporting the axle of the coil. As the coil rotates, these springs make contact alternately with the two parts of the split cylinder, and are therefore connected alternately to the two terminals of the coil. By suitably arranging the position of these springs or *brushes*, the position of commutation can be made to correspond with the position where the E.M.F. is zero, and thus a *rectified* or *unidirectional* current is sent through the *external* circuit (the apparatus connected to the brushes), and it is produced by the *alternating* E.M.F. generated in the coil (Curve II., Fig. 269).

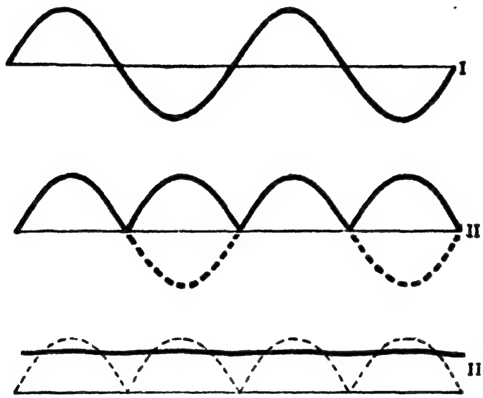

FIG. 269.—E.M.F., etc., due to Earth Inductor.

This current, passing through a galvanometer, would produce a deflection which would be practically constant, the deflection corresponding with the *mean* value of the current: the galvanometer deflection does not follow the variations of current, on account of the inertia of the moving system (Curve III., Fig. 269).

In some cases no commutator is fitted to the coil, the ends of the coil being connected to 'slip rings,' from which the current is taken by a pair of brushes, B_1, B_2 (Fig. 270). When this is the case the coil must be used with a hot-wire milliammeter or millivoltmeter if rotated continuously, as it gives

alternating current. An alternative method is to use it with a ballistic galvanometer, the deflection of the galvanometer being observed when the coil is rotated suddenly through half a revolution. Starting when the coil is perpendicular to the field, it is turned rapidly through half a revolution, *i.e.* until it is again perpendicular to the field but with its faces reversed, and the deflection of the galvanometer produced by this motion is observed. This deflection is proportional to the total number of lines cut by the coil, *i.e.* to the intensity of the field perpendicular to the face of the coil in the initial position.

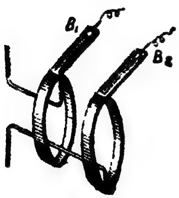

FIG. 270.—Slip-rings for Alternating Current.

EXPT. 251. **Determination of the Angle of Dip by Means of an Earth Inductor.**—*Assuming the coil fitted with a commutator*, place the coil of the earth inductor so that its plane is vertically east and west when in 'commutating position,' *i.e.* when the brushes are not in contact with *either* side of the split cylinder. This ensures that the current is commutated as it passes through its zero value.

Connect the brushes to the terminals of a sensitive galvanometer, placing *in series* with the galvanometer a large adjustable resistance. A suspended-coil galvanometer is most suitable for this experiment, as the oscillations of the moving coil are destroyed very quickly, since it is short-circuited by the coil of the earth inductor and the series resistance. *It is of no use whatever to shunt the galvanometer*, unless a *series* resistance is also used, as the E.M.F. induced is a fixed quantity for a given field and speed, and the same current would flow through the galvanometer whether shunted or not, being driven by the same P.D. Rotate the coil at a speed which can be maintained steady for a little while, and adjust the resistance until the galvanometer deflection is about *half* the maximum deflection which can be measured: a convenient rate of revolution is from 60 to 80 turns per minute. Endeavour to maintain a steady speed while measuring the rate of revolution with the aid of a watch. Place the watch so that the seconds hand can be observed with ease while the coil is being rotated. Turn the coil at such a rate that the deflection given by the galvanometer is constant, then find the time required for 100 revolutions.

With a little practice quite consistent results can be obtained. It is desirable that the student should practise making these measurements *alone*, as this offers a useful opportunity for acquiring facility in making several types of observations simultaneously.

Having become accustomed to the method of carrying out the experiment, the following observations should be taken: (1) Find the time taken for 100 revolutions of the coil, and observe the mean deflection produced when the coil is rotating

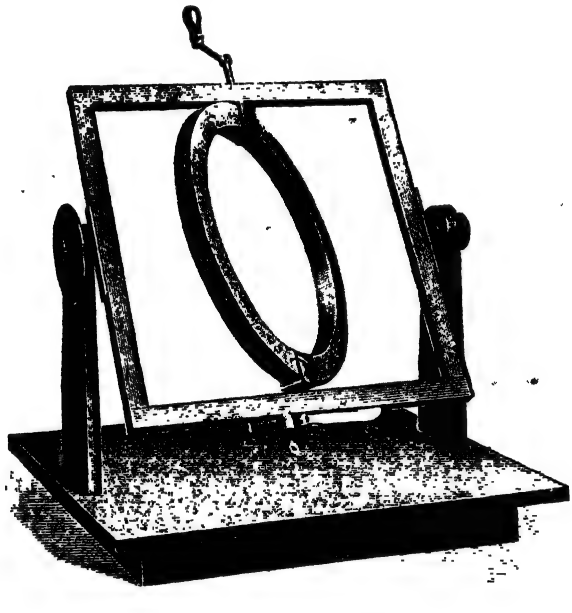

FIG. 271.—The Earth Inductor.

about a vertical axis, and the commutator reverses as the plane of the coil is east and west. Let the time for 100 revolutions be t_1 and the deflection δ_1. Three separate determinations should be made of these quantities. (2) Turn the coil so that its axis is horizontal, and take a similar set of observations when the commutator reverses as the coil passes through the horizontal position. If necessary, the direction of rotation of the coil should be opposite to that in the first case, so as to get the deflection in the same direction as before. The resistance of the circuit must not be altered at all. Let the time for 100 revolutions be t_2 and the deflection be δ_2.

In any single experiment δ is proportional to the induced current, and therefore to the induced E.M.F., since the resistance is maintained constant. The induced E.M.F. is proportional to

$$\frac{n}{t} \times \begin{Bmatrix} \text{Field strength perpendicular to} \\ \text{coil in commutating position} \end{Bmatrix},$$

where n is the number of revolutions in time t.

Thus if H and V are the horizontal and vertical components of the earth's field respectively, we have

$$\delta_1 = K\frac{100}{t_1}H,$$

and
$$\delta_2 = K\frac{100}{t_2}V,$$

i.e.
$$\frac{V}{H} = \frac{\delta_2 t_2}{\delta_1 t_1}.$$

From the observations made determine the ratio of the vertical and horizontal components of the earth's field. This ratio is the tangent of the angle of dip ϕ, and hence the angle of dip can be found from the expression

$$\tan \phi = \frac{V}{H} = \frac{\delta_2 t_2}{\delta_1 t_1}.$$

As a check on this result the axis of rotation of the coil is placed in various positions in the magnetic meridian and the coil rotated as rapidly as possible. In one position no deflection of the galvanometer will be observed, however rapidly the coil is rotated. This means that the field perpendicular to the plane of the coil in the commutating position is zero, *i.e.* that the axis of rotation lies along the angle of dip. Measure the inclination of the axis of rotation of the coil to the horizontal in this position, and compare it with the result obtained in the above experiment.

EXPT. 252. **Estimation of the Relative Accuracy of this Type of Experiment.**—If the coil be rotated so that its plane is perpendicular to the angle of dip when in the commutating position, the field measured when the coil is rotated will be the earth's *total* field. The deflection δ_3, obtained when the coil makes 100 revolutions in t_3 seconds in this position, will be given by

$$\delta_3 t_3 = K\,100\,T,$$

where T is the strength of earth's total field.

Now $$T^2 = H^2 + V^2,$$

therefore $(\delta_3 t_3)^2$ should be equal to $(\delta_1 t_1)^2 + (\delta_2 t_2)^2$, which can be found from the results already obtained.

Find $(\delta_1 t_1)^2$ and $(\delta_2 t_2)^2$, and compare their sum with the value of $(\delta_3 t_3)^2$ obtained in the present experiment. The closeness of their agreement may be taken as a measure of the relative accuracy of the experiment.

NOTE.—If a lamp and scale are used for observing the galvanometer deflection, and the galvanometer is of the suspended-coil type, the deflection for H should be made about 20 cm. on the scale. The oscillations of the galvanometer coil will die down very rapidly, and there should be no difficulty in obtaining the deflections accurately to 2 mm. By taking three sets of determinations, the accuracy obtainable should therefore be of the order of 0·5 per cent in the deflections. The period for 100 turns should be accurate to about one second in each case; and as the time for 100 turns is about one minute, and three observations are made, the error should not be greater than 1 per cent in any of the values of t.

The possible error in any value of $(\delta t)^2$ should therefore not exceed 3 per cent with careful work, and hence the largest error possible in the equation $(\delta_1 t_1)^2 + (\delta_2 t_2)^2 = (\delta_3 t_3)^2$ should not be greater than 7 or 8 per cent. As the errors will probably eliminate each other to some extent, the actual agreement will be found probably to be within 3 per cent in most cases.

EXPT. 253. **Ballistic Method of experimenting with the Earth Inductor.**—Similar experiments can be carried out with a ballistic galvanometer whether the coil is fitted with a commutator or not, the galvanometer being connected to the coil without any additional resistance, and the ballistic swing determined when the coil is rotated through half a revolution. The deflections of the galvanometer, when the coil is rotated through half a revolution, are proportional to the strengths of the field perpendicular to the coil in the initial positions.

Thus, starting with the coil plane vertically east and west, the first swing of the galvanometer being Δ_1, we have

$$\Delta_1 \propto H.$$

With the coil plane horizontal, the first swing of the galvanometer being Δ_2, we should have

$$\Delta_2 \propto V.$$

Hence $$\frac{\Delta_2}{\Delta_1} = \frac{V}{H} = \tan\phi.$$

If Δ_3 be the deflection when the coil plane is perpendicular to the angle of dip, we should find

$$\Delta_3^2 = \Delta_2^2 + \Delta_1^2 \text{ approximately.}$$

When the axis of the coil is along the lines of force, there will be no deflection on rotating the coil through half a revolution.

§ 2. Electromagnetic Machines—Dynamos and Motors

A **dynamo,** or a **motor,** consists of a coil or of a system of coils called the **armature,** so mounted that it can revolve in a magnetic field. In the dynamo, the armature is forced to rotate, and a current is taken from the machine, the current being produced by the E.M.F. induced in the armature due to its motion in the field. In the motor, a current is supplied from an external source, and motion of the armature results from the force exerted by the magnetic field on the coils carrying the current. In direct-current machines the current is led into, or taken away from, the armature coils by means of brushes and a commutator, similar in principle to that described under the earth inductor.

THE DYNAMO (DIRECT CURRENT)

The magnetic field in which the armature rotates may be produced either by permanent magnets or by electromagnets. In the former case the dynamo is known as a **magneto-machine** (Fig. 272). In the latter case the dynamo itself is used generally to supply its own *excitation current* or *field current,* the current being taken from the armature round the field coils. The residual magnetism of the field magnets is sufficient to *start* the generation of current, and the induced current *builds up* the magnetic field as the speed increases: the requisite energy is supplied by the driving power. If the whole of the current from the armature passes through

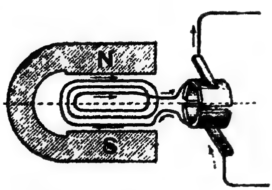

FIG. 272.—Magneto-machine.

the field coils, the machine is said to be **series-wound** (Fig. 273). If the field coils are connected across the brushes so as to be in parallel with the external circuit, it is called a **shunt-wound**

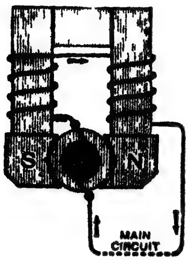

FIG. 273.—Series Winding.

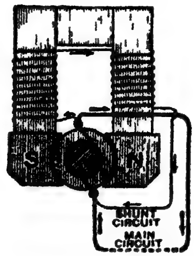

FIG. 274.—Shunt Winding.

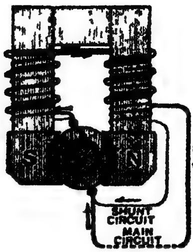

FIG. 275.—Compound Winding.

machine (Fig. 274). A combination of the two systems is used largely, and is known as **compound-winding** (Fig. 275); a compound-wound machine is designed to give a steady P.D. in the external circuit for widely different loads when the speed is constant.

For a given field strength the E.M.F. (E.) induced in the armature coils is proportional to the speed of revolution, but the P.D. between the brushes will not necessarily obey quite the same law. If different loads are thrown on the machine, *i.e.* if different amounts of current are taken from the machine, there will be a variation in the P.D. between the brushes due to the resistance of the armature. If the resistance of the armature is R, and a current C flows through the armature, an amount CR of the induced E.M.F. will be absorbed in driving the current through the armature coils, and the P.D. available at the brushes will only be

$$V = E - CR.$$

THE MOTOR

Any machine which may be used as a dynamo can be employed as a motor by supplying current to it. Thus we have

series-wound, shunt-wound, and compound-wound motors as well as dynamos.

The way in which the current in a motor varies is one of the most important points to deal with, and most of the phenomena associated with motors may be considered in this connection.

When the armature is rotating, it is a conductor cutting lines of force. There is therefore induced in it an E.M.F. (E.) proportional to the rate of revolution and to the field strength, and this E.M.F. is in opposition to the current which causes the motion, *i.e.* is opposed to the applied P.D. (V). The current C flowing through the armature is given by the equation

$$C = \frac{V - E}{R},$$

being driven by the excess of the applied P.D. over the back E.M.F. induced by the motion.

Variation of Current with Speed.—With a constant field, E is proportional to the rate of revolution of the armature; hence, *if the speed be diminished, E is diminished and the current increases.*

Variation of Current with Load.—When a bigger load is put on the machine, *i.e.* when more mechanical work is done by it, the amount of energy supplied to the machine must be increased, *i.e.* C must increase if the P.D. of supply be constant.

Variation of Speed with Load.—If the load be increased, C must increase as above; for this to be possible E must diminish according to the equation

$$C = \frac{V - E}{R}.$$

Hence, as the load increases, the speed of the motor will diminish provided the field strength is constant, but the *power* or *rate of working* will be increased in consequence of the increase in C. This is not necessarily the case for compound-wound motors; in these the field coils are generally wound so that increased load causes a diminished field, and therefore E diminishes, the current increasing to the required amount without alteration of speed.

Variation of Speed with Excitation for a Given Load.—For a given load, VC must be (approximately) constant, and hence the speed will adjust itself till the required current is flowing. If the strength of the field in which the armature rotates is increased, the induced E.M.F. will increase accordingly for a given speed. Thus, C will reach its required value at a lower speed than before, and the motor will run *slower* for increased excitation. Reduced excitation will demand a higher speed before E reaches the value which reduces C to its required magnitude, and thus the motor runs *faster* for a given power when the excitation is reduced.

EXPERIMENTS WITH A MAGNETO-DYNAMO

EXPT. 254. **Variation of the E.M.F. of a Magneto-dynamo with Speed.** — Couple the armature shaft of a magneto-dynamo to the shaft of a variable speed motor, and to a speed indicator, by flexible springs. Connect across the brushes of the dynamo a voltmeter of suitable range, and note the voltage indicated by the voltmeter at various speeds. Draw a curve showing the relation between E.M.F. and speed. Since the field is produced by a permanent magnet the excitation is constant, and the E.M.F. of the machine should be exactly proportional to the speed.

EXPT. 255. **Variation of the Terminal P.D. of a Magneto-dynamo with Load at Constant Speed.**—Couple the machine with a motor and speed indicator as in Expt. 254. Connect the brushes to an ammeter and a variable resistance in series, and also connect a voltmeter across the brushes. Run the machine at a constant speed, and adjust the variable resistance so that different currents are taken from the machine. Note corresponding readings of the ammeter and the voltmeter.

Plot a curve showing the variation of terminal P.D. with load (current), and deduce the resistance of the armature.

NOTE.—The value of the resistance obtained in this way is usually rather higher than the true value. The voltage drop across the terminals when the current is increased is not entirely due to internal resistance; the field is actually weakened by the field produced by the armature current, or, as it is called, by the 'armature reaction.'

Other types of experiment will suggest themselves, and the

student is advised to consider for what purposes such a machine is specially suitable.

A similar set of experiments might be performed with dynamos of other types, in which the field is produced by the current generated in the dynamo itself. Since this excitation current will vary with the speed, and in the case of a *series*-dynamo with the load also, the curves will not be the same as those obtained with a magneto-machine.

EXPERIMENTS WITH A MAGNETO-MOTOR

EXPT. 256. **Variation of the Speed of a Magneto-motor with its applied P.D.**—Couple the armature shaft to a speed indicator. Connect in series with the armature a variable resistance and a battery of cells, and across the terminals of the machine connect a voltmeter. Alter the resistance in series with the machine and note the readings of the voltmeter and the speed indicator.

Plot a curve showing the relation between the speed and the P.D. applied between the brushes.

EXPT. 257. **Variation of Power, Speed, and Load. Efficiency of a Magneto-motor.**—Connect the motor to a battery of cells in series with an ammeter and a resistance, and also connect a voltmeter across the terminals of the machine. Attach to the armature shaft a speed indicator.

By means of a brake-band operating on a large pulley attached to the armature shaft, similar to that described in the determination of the mechanical equivalent of heat by Callendar's apparatus (p. 377 and Fig. 189), apply various loads to the motor.

Take a large number of readings of corresponding values of the current, terminal voltage, speed, and braking force.

The power supplied to the motor is measured by the product of the current and voltage. If these are measured in amperes and volts, the power is given in *watts* or *joules per second*. The work done by the motor is given by the angular velocity multiplied by the frictional couple exerted by the brake.

If the difference in tension between the ends of the brake-band is $T - T_0$ dynes, and the number of *revolutions per second* is n, the work done per second is

$$2\pi n \, . \, (T - T_0) \, R \text{ ergs},$$

where R is the radius of the pulley round which the brake-band passes. This must be divided by 10^7 to reduce the rate of working

to joules per second, and the **efficiency** of the motor can then be calculated from the expression

$$\epsilon = \frac{2\pi n(T - T_0)R}{CV \times 10^7}.$$

Find the variation of efficiency with load at constant speed, and also find the variation of efficiency with speed at constant load.

This latter variation can best be found by obtaining a series of curves for efficiency and load at different (constant) speeds, and deducing the variation of efficiency with speed at constant load from these curves.

EXPERIMENT WITH A SHUNT-MOTOR

EXPT. 258. **Variation of the Speed of a Shunt-motor with Field Strength.**—Connect the armature of a shunt-motor in series with an ammeter and a variable resistance across the terminals of a battery, and place in series with the *shunt* coils a variable resistance and an ammeter. Connect a voltmeter across the armature brushes.

Couple the armature shaft to a speed indicator and note the variations of speed of the motor as the shunt current is diminished. Note also the reading of the ammeter which is in series with the armature, when the resistance in series with it is altered so as to keep the P.D. across the armature brushes constant.

Note that as the shunt current *diminishes*, the speed *increases*, and that this increase in speed is also associated with an *increase* of the armature current.

Plot curves showing (*a*) speed variation with shunt current; (*b*) armature current variation with shunt current.

NOTE.—**On no account must the shunt current be entirely cut off, otherwise the motor may accelerate to an unsafe speed and the armature may fly to pieces.**

CHAPTER IX

COMPARISON OF CAPACITIES

Methods for the Comparison of Capacities

The **capacity** of a condenser may be defined as the quantity of electricity required to increase the potential difference between the conductors by unity.

The capacity of a condenser is one **farad** when one coulomb of electricity is required to change the P.D. between the plates by one volt. One **microfarad** = 10^{-6} farad = 10^{-15} E.M.U.

When two condensers are charged to the same potential, the quantities of electricity on the condensers are proportional to their capacities. If, therefore, we discharge two such condensers separately through a ballistic galvanometer, observations of the first throws produced will enable us to compare the capacities of the two condensers.

Expt. 259. **Comparison of Capacities—Ballistic Galvanometer Method.**—Connect a secondary cell to a condenser by means of a two-way key, so that in one position of the key the cell is connected to the terminals of the condenser. Connect a galvanometer to the condenser and key, so that in the other position of the key the galvanometer is connected across the condenser terminals and the cell is on open circuit. Special 'condenser' keys or 'discharging' keys are supplied for this purpose, but any quick-acting two-way switch will serve satisfactorily, provided it is insulated well. Sometimes two tapping keys are used. Make the connections as in Fig. 276.

The sudden deflection produced when the key is switched quickly over from position I. to position II. should be observed.

The condenser is then removed, the second condenser being put in its place, and the experiment repeated.

The ratio of the two deflections may be taken as the ratio of the capacities of the two condensers, the deflections being proportional to the quantities of electricity discharged round the galvanometer, to within the limits of accuracy of experiment.

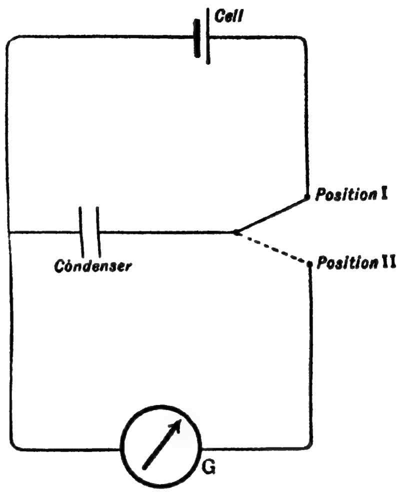

FIG. 276.—Capacity of Condenser.

It is important to change over the condensers as rapidly as possible, in order that there shall be little risk of the E.M.F. of the cell changing. For this reason *two* similar two-way keys may be used, or a double discharging key. The connections in this case would be as represented

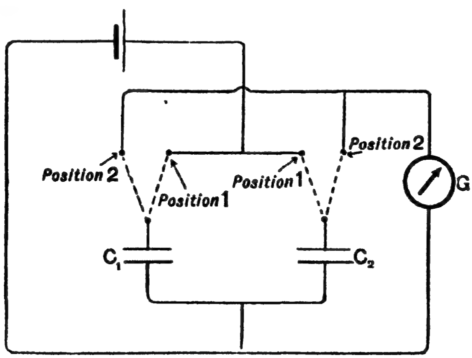

FIG. 277.—Comparison of Capacities.

in Fig. 277. Only one key should be used at a time, the other key not being left in contact with either side if this be possible.

It is frequently possible to facilitate the experimental manipula-

tion required in experiments on current electricity by the use of additional keys, etc., in this way.

Expt. 260. Comparison of Capacities by Wheatstone's Bridge Method.—The capacities to be compared must be

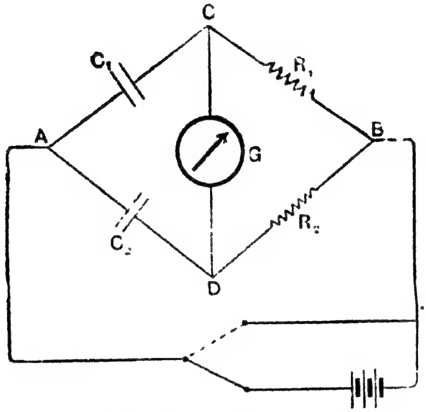

Fig. 278.—Wheatstone's Bridge Method.

connected with a pair of resistances, a galvanometer, a battery and a two-way key, as in Fig. 278.

Adjust the resistances R_1 and R_2 until, on switching the two-way key across in *either* direction, there is no deflection of the galvanometer. Then

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}.$$

For the absence of any deflection means that there is never any difference of potential between C and D, and no current ever flows through the galvanometer. If this is so, the condenser C_1 must be charged up entirely through the resistance R_1, and C_2 entirely through R_2, and they must reach their final potentials simultaneously.

Now the rate at which the condensers are charged up through these respective resistances will be proportional to the reciprocals of the resistances, *i.e.* the charges Q_1 and Q_2 gained in equal times will be proportional to $1/R_1$ and $1/R_2$. But the condensers reach the same final potential together, hence Q_1 and Q_2 are proportional to C_1 and C_2, or—

$$\frac{C_1}{C_2}=\frac{R_2}{R_1}.$$

If either condenser is charged up before the other, a little current will flow through the galvanometer towards the condenser which has not yet obtained its full charge, and hence if the resistances are not adjusted suitably there will be a slight deflection of the galvanometer one way or the other.

NOTE.—This method, although a null method, does not possess any great sensitiveness. The only quantity of electricity which flows through the galvanometer is a small part of the difference of the charges on the condensers when one of them has acquired its full charge. The charges themselves are in general not large, and would be insufficient to cause more than a measurable deflection on the galvanometer. The deflection produced by part of the small difference between the two charges, is therefore very small indeed, and the adjustment of the resistances can be varied usually over a wide range, without causing any appreciable deflection of the galvanometer. The method is most sensitive if R_1 and R_2 are considerable and the galvanometer is of low resistance, but it is unsatisfactory unless the condensers to be compared are large.

EXPT. 261. **Comparison of Capacities — Method of Mixtures.**—Connect up an 8-volt cell in series with two large adjustable resistances (1000 to 10,000 ohms). Arrange the condensers to be compared so that they can first be connected across these resistances, can then be disconnected, their charges mixed, and the residue discharged through a galvanometer.

The method by which this is done is indicated in Fig. 279.

When the double throw-over switch is put over so that A is connected to C and B to D, the condensers C_1 and C_2 are charged to potentials equal to those between the ends of the resistances R_1 and R_2 respectively.

If these are indicated by V_1 and V_2, the charges on the condensers will be C_1V_1 and C_2V_2 respectively.

On switching over so that A and B are connected to E and F respectively, the positive charge on C_1 is mixed with the negative charge on C_2 through the wire PRQ, while the negative charge on C_1 is mixed with the positive on C_2 through the switch. The pairs of plates are connected simultaneously together through the galvanometer, and any residual charge after mixing is dis-

charged through the galvanometer. By suitably adjusting R_1 and R_2 the residual charge can be reduced to zero, and no deflection will be produced in the galvanometer.

When this is the case,

$$C_1V_1 = C_2V_2,$$

but V_1 and V_2 are proportional to R_1 and R_2.

Thus
$$C_1R_1 = C_2R_2,$$

or
$$\frac{C_1}{C_2} = \frac{R_2}{R_1}.$$

This method being a null method is preferable to the method

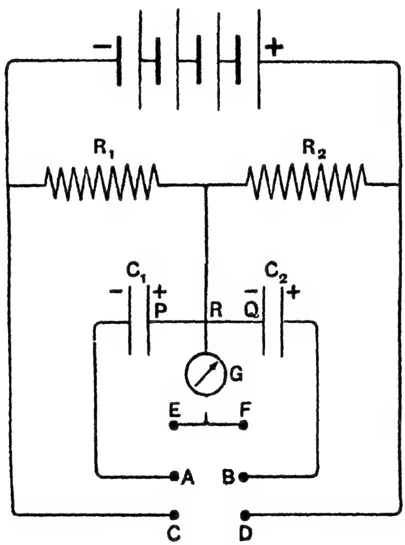

FIG. 279.—Method of Mixture.

using a ballistic galvanometer (Expt. 259). It is also much more sensitive than the Wheatstone's Bridge method, and can be used for quite small capacities.

A high-resistance galvanometer of great sensitiveness is the most suitable type of galvanometer to use for this experiment.

CHAPTER X

NOTES ON ELECTRICAL APPARATUS

§ 1. Tangent Galvanometers

Single Coil Tangent Galvanometer. — The simple type of tangent galvanometer has already been described on p. 459. It consists of a vertical coil of wire placed with its axis east and west, carrying at the centre of the coil a magnetometer box for measuring the intensity of the field due to the current C in the coil.

If F be the strength of the field at the centre of the coil,

$$F = \frac{2\pi nC}{r},$$

where n = the number of turns in the coil,
and r = the radius of the coil.

If the needle be deflected through an angle $\theta°$ from the meridian, we have also $F = H \tan\theta$.

Hence $$\frac{2\pi nC}{r} = H\tan\theta,$$

or $$C = \frac{rH\tan\theta}{2\pi n}.$$

The simple type of tangent galvanometer is usually constructed with one, two, or three coils, all wound on the same framework. These coils have different numbers of turns, and

slightly different radii.[1] By use of one or other of these coils the galvanometer may be made of different sensitivity, so that it is suitable for measuring currents of two or three different orders of magnitude.

Thus if the three coils have respectively 1, 10, and 100 turns and a current of 1 ampere gives a deflection of 45° when flowing in the single turn, that coil will be suitable for use in the measurement of currents from about 0·3 to 3 amperes. The coil with ten turns could be used conveniently for currents ranging from 0·03 to 0·3 ampere. This smaller current circulates ten times and therefore produces the same effect as a current of ten times its magnitude, flowing in the single turn. In the same way the coil with 100 turns would be suitable for the measurement of currents from 0·003 to 0·03 ampere.

General Case.—When the galvanometer is not of this simple type, the equation for the current can be written in the form

$$C = \frac{H}{G} \tan \theta,$$

an equation which holds good for all types of suspended needle galvanometers, however constructed, provided the needle is parallel to the plane of the coil when in its mean position. In this expression H is the strength of field acting on the needle due to any control magnet and the earth, while G is the strength of field due to the coil when unit current passes through the coil.

The Helmholtz Galvanometer.—A special type of tangent galvanometer was devised by the eminent physicist von Helmholtz. In this instrument there are *two* coils so arranged that their distance apart is equal to the radius of either coil.

The magnetometer box is placed midway between the two coils. The axis of the coils is placed east and west. The instrument is used in exactly the same way as the simple type

[1] If the number of turns cannot be counted, and the diameter of the coil cannot be measured, these quantities are, or should be, marked on the base of the instrument by the maker.

of tangent galvanometer, but the magnetic field in which the needle moves is more uniform.

The galvanometer constant G, occurring in the equation

$$C = \frac{H}{G} \tan \theta,$$

is given by $8{\cdot}99N/r$ for C in absolute units, N being the number of turns in *one* coil, and r the radius of the coil.

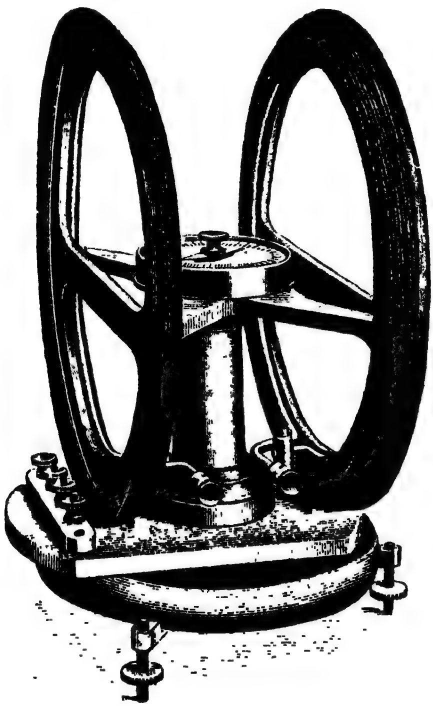

FIG. 280.—Helmholtz Galvanometer.

The magnetic force at a point on the axis of a circular coil of radius r, at a distance x from the centre, is $F = \dfrac{2\pi N r^2 C}{(r^2 + x^2)^{3/2}}$. In this galvanometer $x = \frac{1}{2}r$, and two coils are used. From this the above value of G can be deduced.

ABSOLUTE MEASUREMENT OF CURRENT

The *absolute* value of the current C corresponding with any deflection θ of a galvanometer needle, can be found from the equation

$$C = \frac{H}{G} \tan \theta,$$

provided the quantity G can be calculated from the dimensions and arrangement of the galvanometer coil or coils. If, however, the number of turns in the coil be large, their position cannot be determined accurately, nor can their effect on the needle be calculated exactly.

To obtain a sensitive type of galvanometer, the number of turns has to be made so large that accurate calculation of G is no longer feasible.

It is possible to use sensitive types of galvanometers for the absolute measure of currents of very small magnitude, but a description of the methods of doing so is beyond our scope.

There is no upper limit to the currents for which a tangent galvanometer can be made suitable. The sensitivity of a single-turn galvanometer can be reduced, either by increasing the radius of the coil, or by using the magnetometer box displaced along the axis. In either case there is a quite accurate expression obtainable for the field at the centre, and hence very large currents could be measured with a tangent galvanometer of suitable construction.

§ 2. Sensitive Types of Suspended Needle Galvanometers

SENSITIVITY OF A GALVANOMETER OF THE SUSPENDED NEEDLE TYPE

The sensitivity of a galvanometer may be expressed as the relation between its deflection and the current. If θ/C be large, the galvanometer gives a considerable deflection θ for a small current C.

Now C is proportional to tan θ in all forms of suspended needle galvanometer, provided the needle lies in the plane of

the coil when no current is flowing. Hence θ/C is not constant.

We may, however, take $\tan\theta/C$ as a measure of the sensitiveness of the galvanometer, approximately, for *small deflections*; or

$$\text{Sensitivity} = \frac{\tan\theta}{C}$$

If we wish to increase the sensitivity of the galvanometer, we must therefore increase the ratio of G to H or increase the effect of the field G in some way which does not increase the effect of H, or else diminish the effect of H without altering the effect of G. The methods of increasing the sensitivity may be grouped under the following heads:—

1. Use of an astatic combination for the galvanometer needle.
2. Increase in the actual value of G.
3. Decrease in the value of the controlling field H.

The Principle of the Astatic Combination.—An astatic galvanometer has a compound magnetic system suspended in place of the simple needle used in the tangent galvanometer. In its simplest form the suspended system consists of two light needles fitted in a rigid framework with their magnetic axes in opposite directions.

The two needles are magnetised almost equally and are mounted one *inside* the coil and the other *outside*.

FIG. 281.—Astatic System of Needles.

Effect of the Controlling Field on an Astatic Combination.—If the magnetic moments of the needles be M_1 and M_2, the effect of the controlling field is proportional to $H(M_1 - M_2)$, since the controlling field is practically uniform, and the needles are magnetically opposed.

Effect of the Field due to the Current on an Astatic Combination.—By placing the needles one inside and the other outside the coils, they are in two parts of the coil field which are in *opposite* directions, and as the needles are *also* opposed the couples exerted on the two needles are in the *same* sense. The total couple exerted on the needle by the field of the coil is consequently

proportional to $G(M_1 + M_2)$ very roughly, the approximation being only very rough because the strength of the field outside the coil acting on the needle M_2 is not nearly equal to G the strength of the field inside the coil.

The sensitivity of an instrument of this type is thus greater than that of a single needle instrument with similar coils, by a factor $\frac{M_1 + M_2}{M_1 - M_2}$ (roughly), so that with nearly equal needles an astatic galvanometer may be extremely sensitive.

If M_1 and M_2 are too closely equal, the instrument becomes unstable, and therefore care must be taken to avoid this.

An instrument using a needle of this type is not *absolute* on account of the unknown value of $\left(\frac{M_1 + M_2}{M_1 - M_2}\right)$. It cannot be relied upon to give consistent deflections for the same current from day to day, as a slight change in M_1 or M_2 has a large effect on the denominator of this factor, and consequently on the sensitiveness.

The Control Magnet.—Frequently a control magnet is fitted above the coil of a galvanometer; the height of the magnet above the coil can be adjusted, and the magnet can also be rotated about a vertical axis.

The needle of the galvanometer is then under a resultant field H, due to the field of this magnet and the field of the earth combined. The value of H is thus adjustable over a wide range. For great sensitivity the magnet is adjusted until its field almost completely overcomes the earth's field. If, on the other hand, the galvanometer is required to be insensitive, as when determining its resistance by Thomson's method, the magnet is brought down close to the needle and its field arranged so as to assist the earth's field, H then being very great. A weak control field causes a very slow swing of the needle, a stronger field causing a correspondingly quicker swing, hence for great sensitiveness the magnet must be adjusted to give a very slow swing to the needle; the sensitiveness is proportional to the *square* of the period of swing.

A great advantage of the use of the control magnet is that the controlling field can be directed as desired, by turning the control magnet, or any permanent deflection due to a steady current can be corrected by rotation of the magnet.

Method of obtaining an Increase in the Galvanometer Constant G.—To increase the field due to unit current it is necessary to use a coil of small radius r and with a large number of turns n. These requirements are to a certain extent antagonistic, the radius of the outside turns increases as the number of turns is increased, and there is a limit to the extent to which n may be *usefully* increased.

SIMPLE TYPE OF ASTATIC GALVANOMETER

In the simple type of astatic galvanometer the coils are wound flat so that fairly long needles can be used: the flattening of the coil is equivalent to a reduction of the radius and makes for increased sensitiveness; although if the coil is flattened, G cannot be calculated.

This type of galvanometer is used chiefly to *detect* minute currents in rough experiments with Wheatstone's Bridge.

A useful exercise is to calibrate a galvanometer of this type using a 2-volt cell and a resistance box adjustable to 10,000 ohms. Calculate the current, assuming the cell to give 2 volts, and plot curves giving the variation of C with θ and of C with tan θ.

By combining an Astatic Magnetic System with a control magnet, galvanometers of great sensitiveness can be made. Sometimes the two parts of the Astatic System are placed in separate coils one above the other, wound oppositely so as to give couples in the same sense acting on each of the needles.

The **Figure of Merit** of a galvanometer is usually determined by the current in amperes required to give a deflection of 1 mm. on a scale at a distance of 1 metre, when a lamp and scale method is used for measuring the deflection.

HIGH AND LOW RESISTANCE GALVANOMETERS

A galvanometer may be made with a large constant G, by the use of fine wire in winding the coils. This entails great resistance, but that is no disadvantage for work where a definite quantity of electricity has to be measured by discharging it round the galvanometer. Thus if a condenser is charged and the charge has to be measured, the whole charge will pass round the coil of the galvanometer however large its resistance.

On the other hand, in testing for balance in using a Wheatstone's Bridge, the adjustment has to be made until two points are at the same *potential*, and a galvanometer must be used which will detect the smallest *potential difference* possible. In such a case, the current flowing through a high-resistance galvanometer would be much smaller than that through a low-resistance galvanometer for the same P.D. In the case of the high-resistance galvanometer, the current would be small but would pass through a large number of turns. With the low-resistance galvanometer a much larger current would pass, but through a smaller number of turns. Usually, the galvanometers being similar in design, a smaller deflection would be

obtained with the high than with the low resistance galvanometer in a case of this type, and hence for detecting small potential differences a low-resistance galvanometer should be used.

This may be summed up by saying that a high-resistance galvanometer is extremely *current sensitive*, while a low-resistance galvanometer has a great *potential* sensitiveness. High-resistance galvanometers are used for the *measurement* of relatively large potential differences and for the *detection* of minute currents. Low-resistance galvanometers are used for the measurement of relatively large currents and for detecting small potential differences.

FIG. 282.—Sensitive Galvanometer.

BALLISTIC GALVANOMETERS

When the duration of the current is extremely short, the *quantity* of electricity passing through the coil of the galvanometer may be measured by observing the *throw*, or first swing, of the needle, provided the time which the current lasts is small compared with the time of swing of the needle and provided the damping is slight. A galvanometer of this type is termed a **ballistic galvanometer.**

§ 3. SUSPENDED-COIL GALVANOMETERS

A suspended *needle* galvanometer possesses the great disadvantage that it is susceptible to any variation in the external magnetic field. By use of a suspended *coil* galvanometer, this difficulty is got over entirely: this type of galvanometer also

possesses the advantage that it can be set up facing in any direction desired.

If a wire of length l, carrying a current C absolute units, be placed perpendicular to a field of intensity H, the wire experiences a force of HCl dynes, the direction of the force being perpendicular to the wire and to the magnetic field.

If a rectangular coil be placed so that the plane of the coil

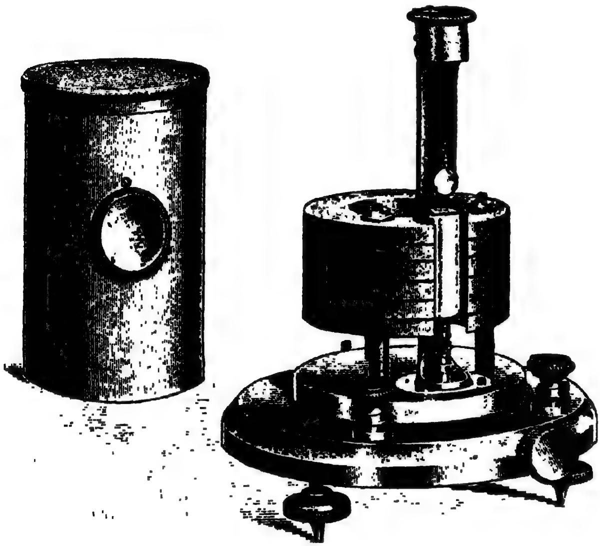

FIG. 283.—Suspended-Coil Galvanometer.

lies in the direction of the field H, a current C flowing through the coil will cause a couple of $HCldn$ to act on the coil, d being the breadth of the coil, l its length, and n the number of turns in the coil.

For a coil of any shape the couple is $HCnA$, where A is the area of the face of the coil.

In a suspended-coil galvanometer, the coil, supported by a fine strip of phosphor-bronze, is suspended between the poles of a very powerful magnet (Fig. 283). The current is led into

the coil by the suspending fibre and leaves by a loosely wound helix attached to the base of the coil.

Any motion of the coil due to the couple $HCnA$ is opposed by a resisting couple exerted by the suspension.

When the coil is deflected through an angle, the couple acting on the coil, if the field is uniform, is $HCnA \cos \theta$, due to the magnetic field, and the restoring couple is $K\theta$, due to the suspension, K being the torsion coefficient of the suspending fibre. The coil comes to rest when $HCnA \cos \theta = K\theta$,

or $$C = \frac{K\theta}{HnA \cos \theta}.$$

For small deflections $\cos \theta$ may be taken as unity, and the sensitiveness is expressed as $\frac{\theta}{C} = \frac{HnA}{K}$.

Hence for sensitiveness we require a coil of large area, and of many turns, hanging in a very strong field H, and suspended by a fibre whose restoring couple for unit twist K is very small.

In some forms a bifilar suspension is used, the current entering and leaving by the two suspending fibres; this type is not very common.

Method adopted in Suspended-Coil Galvanometers to make the Deflection proportional to the Current.—The factor $\cos \theta$, which occurs in the expression for C in the case of a suspended-coil galvanometer with a uniform field, can be got rid of in a simple way.

The pole-pieces of the magnet are ground concave so as to form portions of a cylinder. Between them there is a cylindrical soft-iron core, its axis being coincident with the cylindrical surfaces of the poles. In the annular space between the core and the pole-pieces, the field is very nearly *radial*, and may be considered as *radially symmetrical* over a considerable angle on either side of the mean line. In this annular space the coil moves, and the field is in the same plane as the face of the coil in all positions, provided the coil is not displaced more than 30° from the position where it lies symmetrically across from one pole to

the other. The strength of the field is uniform over the range, and therefore the coil is subjected to a couple HCnA when carrying a current C, irrespective of its position.

If it is deflected by the current through an angle θ from its zero position (which may be anywhere in this range), the deflection will be given by

$$\mathrm{HC}n\mathrm{A} = \mathrm{K}\theta,$$

i.e.

$$\mathrm{C} = \frac{\mathrm{K}\theta}{\mathrm{H}n\mathrm{A}}.$$

'Dead-beat' Type of Galvanometer.—If the coil is wound on a light, conducting frame, or is enclosed in a conducting tube which moves with the coil, the motion of the coil is impeded by currents induced in the frame or tube due to its motion across the lines of force.

A galvanometer of this type moves up to the deflection θ corresponding with the current in the coil, and comes to rest at once. The deflection θ is not affected by the induced currents, as they have a zero value as soon as the motion ceases.

Method of 'damping' the Oscillations of a Suspended-Coil Galvanometer.—If the coil is not mounted on a conducting frame, its oscillations can be reduced by 'shortcircuiting' the galvanometer, when it is required at rest in its zero position. This is done by connecting the terminals to a tapping key; the key is left open during any experiment, but when the galvanometer is to be brought to rest the tapping key is depressed. The E.M.F. in the coil due to its motion across the field can then send an induced current through the coil, this current opposes the motion and the coil comes to rest immediately. The key should only be pressed as the coil is almost in its mean position, otherwise the motion towards the mean position will be very slow and time will be wasted. The coil must be at rest with the tapping key *open* before the next deflection is taken.

In using a P.O. Box it is sufficient usually to depress the galvanometer key alone, no auxiliary tapping key being required.

§ 4. Ammeters and Voltmeters

AMMETERS

An ammeter is a galvanometer graduated in such a way that the current flowing through it can be read off at once in amperes, or fractions of an ampere, by means of a pointer moving over a divided scale.

For large currents a shunt is incorporated in the instrument, so that only a fraction of the total current passes through the galvanometer coil, the shunt being adjusted until currents of the required value give a suitable deflection of the pointer. For a multiple-range ammeter several shunts are provided, so that different fractions of the current flow through the coil.

The following example illustrates the method of calculating the shunt required to convert any kind of galvanometer of a given sensitiveness, into an ammeter reading over a certain range:—

Suppose a current of 0·0002 ampere through the galvanometer coil gives the full deflection on a given instrument, the coil having a resistance of 15 ohms. If it is required to use this as an ammeter reading to 5 amperes a shunt S must be provided, so that when a total current of 5 amperes is flowing, the current in the coil is 0·0002 ampere; the instrument then giving its full deflection for a total current of 5 amperes.

The value of S can be worked out as follows:—

$$\frac{\text{Current through Coil}}{\text{Total Current}} = \frac{\text{Resistance of Shunt}}{\text{Resistance of Shunt} + \text{Resistance of Coil}},$$

i.e.

$$\frac{0{\cdot}0002}{5} = \frac{S}{S+15}.$$

This gives

$$S = \frac{0{\cdot}003}{4{\cdot}998},$$

or

$$S = 0{\cdot}0006 \text{ ohm approximately.}$$

In a similar way the magnitude of the shunt required for any range of current can be calculated.

The shunt is usually calculated approximately, and then is adjusted *after fitting*, till the ammeter reads correctly when a known current is sent through it.

Attracted-Iron Ammeters.—For rough work attracted-iron ammeters are largely used. The current flows round a coil and attracts a piece of iron with a force depending on the current flowing. The iron is attached to a pointer which is moved over a scale as the iron moves, the moving system being pivoted on delicate steel pivots. The motion is controlled by a balance weight and a hair-spring, so that the pointer always takes up the same position for a given current, returning to zero when the current is switched off. The scale of such an instrument is very uneven and must be graduated empirically, *i.e.* by sending known currents through the instrument, and marking the position of the pointer to correspond.

Attracted-iron ammeters may be used for alternating current as well as direct, if a thin piece of very soft iron is used.

Hot-wire Ammeters.—In this type of ammeter the current, or a fraction of the current, flows through a thin wire stretched between two fixed supports. The wire, being heated, expands. Another wire is attached to the middle of the heated wire and passes round a thin spindle on which the pointer is mounted, the spindle being rotated by a hair-spring so as to keep this second wire taut.

When the hot wire expands, its middle point is pulled sideways by the second wire, until all the sag of the hot wire is taken up The rotation of the spindle turns the pointer through a correspond ing angle, and thus the motion is recorded on a scale.

Hot-wire ammeters may be used for either alternating or direct current. The scale is not at all uniform, being much more 'open' for large currents than for small.

Moving-coil Ammeters.—A moving-coil ammeter is constructed exactly like a suspended-coil galvanometer except for the mode

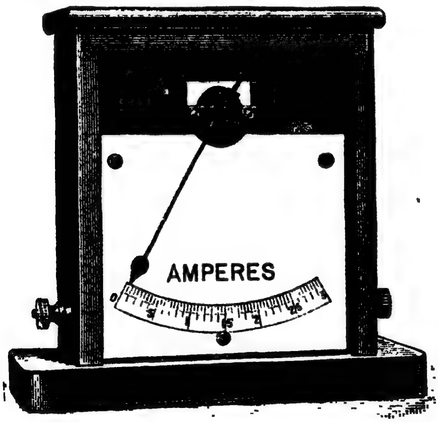

FIG. 284.—Moving-coil Ammeter.

of suspension. The coil is usually mounted on pivots, and its motion is controlled by one or two hair-springs, which also serve to conduct the current up to and away from the coil.

By the use of concave pole pieces and a coaxial cylindrical iron core the deflections are made proportional to the current,

so that the scale is practically uniform, the coil being allowed to move only over the region where the field is radial. A suspended-coil ammeter must be connected so that the current always goes through it in the proper direction; it is only suitable for use with direct current.

The requisites of a good ammeter are (1) accuracy and (2) low resistance. An ammeter must have a low resistance so that it may be inserted in a circuit through which a current is flowing and yet not alter the value of the current in the circuit, *i.e.* it must not introduce any additional resistance into the circuit.

VOLTMETERS

A voltmeter is used to indicate the difference of potential between the two points across which it is connected. It should take no current whatever, otherwise the P.D. between these two points may be changed when the voltmeter is connected across them.

This condition is only satisfied in Electrostatic Voltmeters, the usual type of voltmeter being a high-resistance galvanometer which only approximates to this ideal: *we always assume that the current in the voltmeter is a negligible quantity.* The higher the resistance of the voltmeter, the more accurately will it indicate the P.D. *originally existing* between the points to which it is connected (see example on p. 579).

Any type of galvanometer suitable for use as an ammeter can be adapted for use as a voltmeter. The difference in the construction is, that whereas the galvanometer is *shunted* with a *very low* resistance to make it into an *ammeter*, it has a *very high* resistance in *series* with it when required for use as a *voltmeter.*

Moving-coil Voltmeter.—The usual type of voltmeter is a suspended-coil galvanometer similar to the galvanometer part of the moving-coil ammeter already described. In series with it is a coil whose resistance for a given range can be calculated as follows:—

Suppose the moving-coil galvanometer gives its full deflection for a current of 0·0002 ampere as assumed before (p. 576), and that its re-

sistance is 15 ohms. This can be made into a voltmeter reading up to 5 volts by including in the instrument a resistance R in series with the coil, R being so adjusted that a current of 0·0002 ampere goes through the coil when the P.D. across the terminals of the instrument is 5 volts. Evidently R is given by the equation

$$0\cdot0002=\frac{5}{R+15},$$

i.e.

$$R=24985 \text{ ohms.}$$

If a resistance of this magnitude is connected in series with the moving-coil galvanometer, the whole would be a voltmeter of range 0 to 5 volts.

The value of the resistance required for any other range could be calculated in a similar way.

A moving-coil voltmeter is suitable only for use with direct current.

Hot-wire voltmeters can be constructed in a similar way, the moving system being identical with that of a hot-wire ammeter.

It is of great importance to note that, assuming the graduations to be accurate, the voltmeter reading is the P.D. between its own terminals.

The Effect of the Finite Resistance of the Voltmeter.—This is illustrated in the following example:—

A cell of E.M.F. 2 volts has an internal resistance 20 ohms. Its poles are connected to a voltmeter. What will be the voltmeter reading if the voltmeter resistance is (*a*) 20, (*b*) 200, (*c*) 2000 ohms.

If E is the E.M.F. of the cell and V the P.D. across the voltmeter terminals, we have

$$E=\left\{\frac{R+B}{R}\right\}V,$$

where R is the external resistance and B the internal resistance.

In this case R is the voltmeter resistance.

(*a*) $$V=2\frac{20}{20+20}=1 \text{ volt.}$$

(*b*) $$V=2\frac{200}{200+20}=1\cdot82 \text{ volts.}$$

(*c*) $$V=2\frac{2000}{2020}=1\cdot98 \text{ volts.}$$

These quantities are the voltages which would be registered by the various voltmeters, the errors being 50 per cent, 10 per cent, and 1 per cent in the three cases.

With a voltmeter of higher resistance still, the error would be correspondingly reduced.

With a cell of lower resistance, the accuracy would also be greater.

Cheap voltmeters usually have fairly low resistance, and can only be relied on when the resistances of the conductors between the points to which they are connected, are extremely low.

§ 5. Commutators

A commutator is an arrangement for reversing the direction in which the current flows through a particular piece of apparatus (usually a galvanometer) without disconnecting any wires. A commutator must possess at least four terminals. Of these two must be connected to the apparatus in which it is desired to reverse the current, and the other two to the source of current. The only difficulty is to decide which two are to form a pair. Commutators

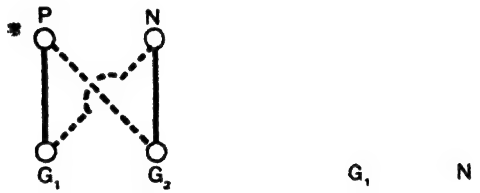

Fig. 285.—Parallel and Diagonal Type Commutators.

may be divided into two types, (1) the parallel type and (2) the diagonal type (Fig. 285).

In the parallel type the battery terminals, P and N, are in a line parallel to the line joining the apparatus terminals. In the diagonal type the battery terminals are diagonally opposite one another. The connections in the first position are shown by continuous lines, and the connections in the second position by dotted lines. In the first position the terminal G_1 becomes positive, G_2 negative; in the second position G_2 becomes positive, G_1 negative. The student should notice that in the first type there is a diagonal connection (PG_2, NG_1), in the second type there is no diagonal connection.

The method of connecting up any commutator can be worked out in the following way:—

Choose one terminal of the commutator and label it P. Note the terminal connected with it in one position of the moving part of the commutator; call this terminal G_1. Then "reverse" the commutator by moving over the switch arm. The terminal to which P is now connected is labelled G_2.

Now find a terminal to which G_2 is connected when P is in the

first position, and call this N; it will generally be found that when P is connected to G_2, N is connected to G_1 at the same time. P and N are used as the battery terminals and G_1 and G_2 are the galvanometer terminals of the commutator.

If only four terminals are fitted, N must obviously be the remaining terminal after P, G_1, and G_2 have been decided on.

If it should happen that N is *not* connected with G_2 when P is connected with G_1, and *vice versa*, it is evident that the wrong terminal has been chosen for P, and the investigation must be recommenced, choosing another terminal as P. This will occur only very rarely, the pairs G_1 and G_2, and P and N being always interchangeable.

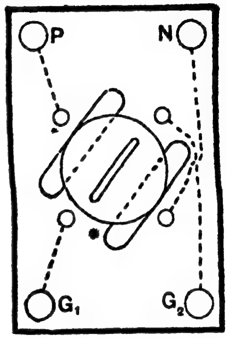

FIG. 286.—Wheatstone Commutator.

This method is not applicable to a plug commutator.

Various forms of commutator are illustrated in Figs. 286-289.

Fig. 286 is a convenient form which appears to be peculiar to the Wheatstone laboratory. The central disk, which can be rotated about a vertical axis, carries two metal strips which make contact with the four metal studs.

Fig. 287 is a double plug switch belonging to the diagonal

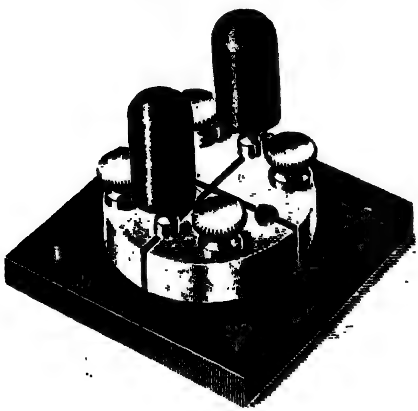

FIG. 287.—Double Plug Switch.

type, and the battery must be connected to terminals which are diagonally opposite one another. The plugs must never be inserted

in *adjacent* holes, but always in holes which are diagonally opposite one another.

Fig. 288 shows a useful type of commutator by R. W. Paul, in which it is merely necessary to push the rod, AB, along its axis in

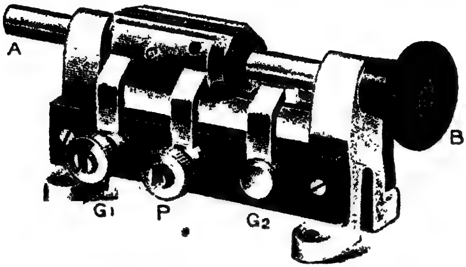

FIG. 288.—Paul's Commutator. (R. W. Paul & Co.)

order to reverse the current. The sliding bar carries two insulated metal plates C, which make contact with the brushes on opposite sides of the switch. The bar has three positions, the central position corresponding with open circuit. As the brushes are highly laminated the contact resistance is extremely low.

In the foregoing diagrams the 'battery terminals' are marked P and N, the 'galvanometer terminals' being G_1 and G_2. The student must verify these indications in each case, and should draw a diagram showing the way in which the current flows in each position of the moving system.

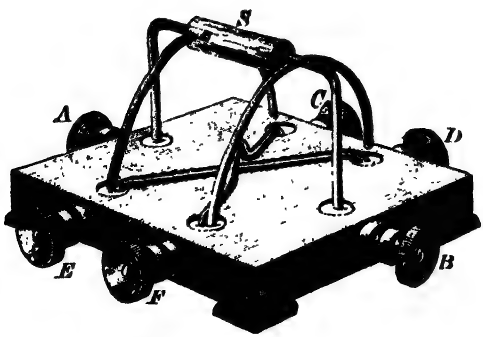

FIG. 289.—Pohl's Commutator.

Fig. 289 is known as Pohl's commutator. A and B are the battery terminals. The apparatus in which the current is to be

reversed is connected either to C and D, or to E and F. The rocking part S dips into mercury cups. This form is not recommended for an elementary laboratory.

A rocking commutator of the Pohl type can be constructed without mercury cups if spring contacts are fitted at the two ends and hinge contacts at the middle.

§ 6. Keys and Switches

Plug Key.—This form is convenient for making a good connection of low resistance when a current has to be maintained for a considerable time.

Tapping Key.—In this key contact is made only when the spring is depressed, the spring automatically breaking contact

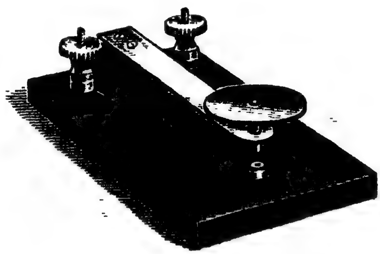

Fig. 290.—Tapping Key.

when the pressure is removed. It is convenient for use whenever a current is only required momentarily, as in damping the oscillations of a suspended-coil galvanometer.

Two-way Switch.—This is a convenient form of switch when

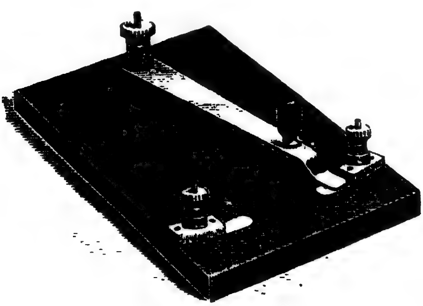

Fig. 291.—Two-way Switch.

it is necessary to change a connection rapidly from one piece of

apparatus to another, as in the case of the Potentiometer (Fig. 237). A hinged arm can be turned so as to make contact with one or other of two metal studs. One terminal is connected to the hinge, and one to each of the studs.

Double Pole Throw-over Switch.—This is a useful type of

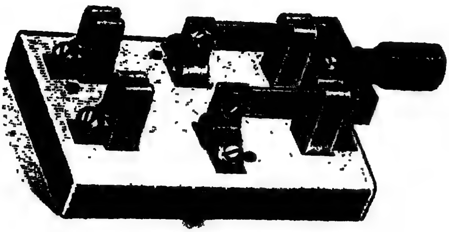

FIG. 292.—Double Pole Throw-over Switch.

switch provided with six terminals, and its construction may be understood at once from the diagrams. In the position shown in

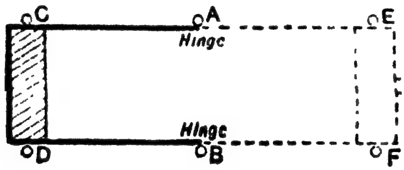

FIG. 293. Double Pole Throw-over Switch.

full line in Fig. 293 the connections are from A to C, and B to D. In the other position, A and E are connected, and B and F.

A Pohl commutator becomes a switch of this type if the cross connections from D to E and C to F are removed (Fig. 289).

§ 7. Resistances and Rheostats.

The simplest form of resistance for laboratory use is a length of bare platinoid or manganin wire. For resistances up to about 1 ohm a length of about 1 metre of wire (No. 22, S.W.G.) serves as an adjustable resistance by fixing one end to a point in the circuit and sliding the free part of the wire under a binding screw till the required resistance is obtained.

Resistance Coils.—Bobbins provided with terminals to which are attached the ends of a coil of silk-covered resistance wire are useful both as known and as unknown resistances. Standard coils are constructed as in Expt. 238, Fig. 254.

Resistance Boxes.—An ordinary resistance box consists of a number of coils constructed to have resistances which are exact multiples and sub-multiples of one ohm. These are wound on small bobbins so as to have as little self-induction as possible (see Expt. 238, Fig. 254), and are thoroughly soaked in paraffin wax.

They are enclosed in a box, usually with an ebonite top, the ends of the coils being brought through the ebonite and connected to thick brass blocks which are mounted on the top of the vulcanite.

Between the blocks of brass, stout brass plugs can be fitted as

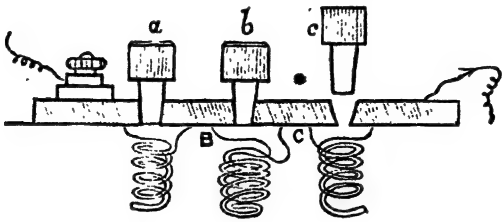

FIG. 294. Coils of Resistance Box.

shown in Fig. 294, the plugs being ground so that they fit half into each block, a connection of extremely low resistance being thereby made between the brass blocks.

If the plug *a* is removed, the current must flow through the resistance coil, the ends of which are attached to A and B, in order to pass from one block to the other. When the plug is inserted, the resistance offered to the passage of the current is negligible. Thus, in using a resistance box, the *total resistance introduced in the circuit by the box is equal to the sum of the resistances indicated at the side of the holes from which the plugs have been removed.*

In using a resistance box, care must be taken to insert and remove the plugs with a slight screwing motion in addition to the pressure or pull required to insert or remove the plug. The plug should always be screwed in a *right-handed direction* even when removing it, otherwise the plug head may be screwed off without removing the plug from the hole. Whenever the plug is removed from between two blocks of the resistance box, the plugs on either side should be pressed firmly into their holes again, as the blocks will have 'sprung' slightly towards the vacant hole and the plugs on either side will be loosened.

Resistance boxes must never be used for heavy currents, as the coils would be overheated and the box 'burnt out.' A box should never be used by a student in conjunction with a secondary cell

unless permission has been received that this may be done. In any case, the resistance in the box must never be reduced to less than 30 ohms when a secondary cell is used.

Sliding Rheostat.—The resistance wire is wound round an insulating cylinder. One end of the wire is attached to one terminal of the rheostat. The second terminal is attached to a sliding contact piece which can be removed parallel to the cylinder, so as to make contact with the resistance wire at some point on a generating line of the cylinder.

Wheatstone's Rheostat.—Two parallel cylinders are mounted side by side, so that each can rotate about its axis. One cylinder is of brass, the other of wood or some insulating material. A screw thread is cut on the latter cylinder, and a length of resistance wire lies at the bottom of the thread. One end of this wire is attached to the metal cylinder so that, when this is rotated, the wire is wound off the wooden cylinder and on to the metal cylinder. The turns of wire on the metal are short-circuited, so that the resistance in use is that of the wire on the wooden cylinder only. The advantage of this arrangement is that it gives a *continuous* adjustment of resistance.

Carbon Resistances.—Adjustable resistances may be constructed by placing a number of circular sheets of carbonised cloth between two metal plates which may be pressed together by means of a nut and a screw. The resistance is altered by altering the pressure between the plates.

In another form solid plates of carbon are pressed together in the same fashion, and the resistance is varied by altering the number of plates or the pressure between them.

Adjustable Resistance Frame.—A convenient form of adjustable resistance in common use is that which consists in a rigid frame on which are stretched a number of spiral coils of wire arranged in zig-zag fashion (Fig. 295). A metal handle connected with one terminal of the rheostat moves over a series of metal studs connected with successive spirals. With the handle in one extreme position the current must pass through *all* the spirals, but as the handle moves from stud to stud the current passes through fewer and fewer spirals, till the other extreme position is reached when the current usually passes direct through the handle to the second terminal of the rheostat. This arrangement is useful as a rough regulating resistance for fairly large currents, say from 1 to 20 amperes.

Such resistance frames are marked usually with their approximate *full* resistance and by the current which they are designed to carry without overheating. This current must not be exceeded.

These resistances must never be used as standards for comparison, the indicated resistances being only approximately correct.

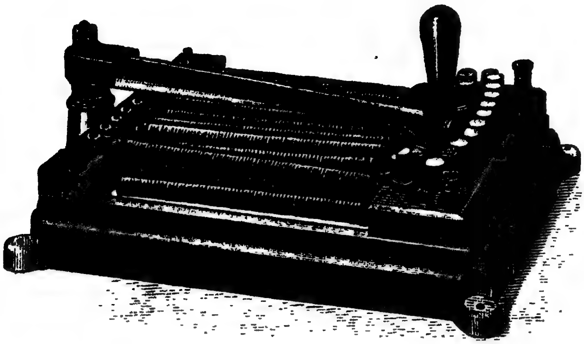

FIG. 295.—Adjustable Resistance Frame.

Rough Fixed Resistances.—A useful type of fixed resistance which is convenient for various purposes where a current has

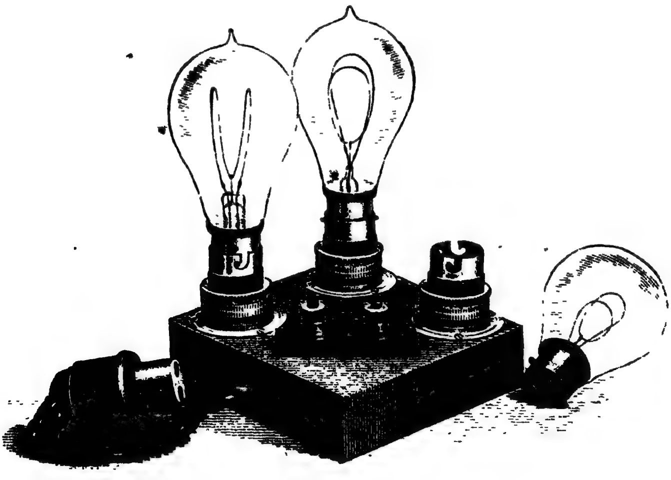

FIG. 296.—Lamp Resistance.

to be reduced to somewhere near a definite amount is the *grid* type. These are made for use on various voltages, and are usually marked with their approximate resistance and the

voltages for which they are designed. From these, the current they are designed to take can be calculated, and this must never be exceeded. These grids will only take this current provided they are well ventilated; if enclosed so that they are not cooled by convection currents, they would overheat rapidly and fuse.

Lamp Resistance.—For many experiments current may be used from the lighting mains if suitable resistances are employed to regulate it. A convenient and cheap apparatus is shown in Fig. 296. This can be made in deal or mahogany, using ordinary *batten* lamp-holders. The connections are as shown in Fig. 297. C, D, and E are lamp-holders screwed to the base and connected in parallel as shown.

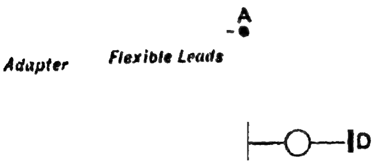

Fig. 297.—Connections of Lamp Resistance.

The current for the experiment is taken from the terminals A and B. The polarity of A and B can be found by pole-finding paper and, by the use of one, two, or three lamps different currents may be obtained for purposes of experiment. A larger number of lamp-holders may be fitted if desired.

§ 8. Tests for Polarity

Pole-finding Paper—testing Polarity of Lighting Mains, etc.—Ordinary litmus paper is quite suitable for finding the polarity of cells, or mains, where direct current is supplied. The paper should be moistened and the two wires placed close together on the paper, but they must not touch each other. The point of contact of the *positive* will show a *red* mark after a moment or two, the *negative* giving a *blue* coloration. When testing the polarity of the lighting mains great care must be taken not to touch the bare wires with the fingers; the wires must not touch each other, nor must either wire come into contact with any metallic fixtures such as gas or water pipes. *Failure to observe these precautions may result in a severe shock to the experimenter,*

and if the wires touch each other, or any metal fixtures, severe burns may result.

Starch paper, *i.e.* paper soaked in a solution of starch and potassium iodide, turns *blue* at the *positive* terminal when two wires are placed in contact with the paper (first moistened).

Methods of determining the polarity of the terminals by means of the magnetic action of the current have been described on p. 449. *In applying these methods to the lighting mains sufficient resistance must be placed in the circuit to prevent the current being excessive.* The lamp resistance described above is suitable for the purpose.

One of the mains of the supply company is usually earthed. In the case of a two-wire system the second main may be either above or below the potential of the earth. In the case of a three-wire system the middle wire is earthed Of the other two, one is above, the other below the potential of the earth. Thus, if the live mains are at + 100 and − 100 volts respectively, lamps or apparatus requiring 200 volts would be connected across these live mains. If only 100 volts is needed for any purpose, the apparatus is connected between *either* live main and the middle (earthed) wire.

PART VI

ADDITIONAL EXERCISES IN ELECTRICITY

1. Test by means of a gold leaf electroscope the sign of the charge produced on rods of glass, ebonite, and sealing-wax when rubbed by fur, flannel, and silk.

2. Find the positive terminal of the given cells, using a compass needle, a straight wire, and a regulating resistance. Verify the result by winding the wire into a rough coil.

3. Plot the lines of force round a long vertical wire carrying a current, and find from your diagram the intensity of the field of the wire at a distance of 15 cm. from the wire. Take the earth's horizontal field = 0·185 C.G.S. units.

4. Plot the lines of force round a circular coil carrying a current, and from your diagram plot a curve showing how the field along the axis varies with the distance from the coil.

5. Connect the given cell by a commutator to a tangent galvanometer—(*a*) directly, (*b*) through a given resistance. Compare the currents in the two cases.

6. Connect two cells—(1) in series, (2) in parallel, (3) in opposition, to a tangent galvanometer. Compare the currents in the three cases.

7. Connect the two cells in series through a commutator to a tangent galvanometer and note the deflections obtained. Invert the connections of one cell and again note the deflections. What result can be deduced from these observations?

8. Compare the number of turns in coil A of the given tangent galvanometer with the number of turns in coil B, given a constant cell and a resistance box.

9. A secondary cell of E.M.F. 2 volts and of negligible resistance is used to send a current through a resistance box and a tangent galvanometer, whose resistance is also negligible. Find what current would produce a deflection of 1°.

10. Plot a curve showing the variation of the *tangent* of the deflection of the given tangent galvanometer, with the value of the resistance placed in series with the galvanometer. Shunt the galvanometer with 5 ohms, and repeat the observations, plotting the curve on the *same* paper as the first curve. Can you estimate *approximately* the resistance of the galvanometer from your results?

11. Connect the three given cells in series with a resistance box and a tangent galvanometer, adjusting the resistance till the deflection is about

55°. Keeping the resistance constant, group the cells in all possible ways, using any number and placing them in series or in parallel. Compare the currents flowing through the galvanometer in the various cases.

12. Measure the strength of the current through the given incandescent lamp by means of a tangent galvanometer. Express the result both in C.G.S. units and in amperes.

13. Wind a length of insulated wire over a glass tube about 20 cm. long and 1 cm. in diameter so as to form a solenoid. Plot a curve showing the relation between the magnetic moment of the solenoid and the current flowing through it, using a magnetometer and a tangent galvanometer.

14. Repeat the previous experiment when the solenoid is provided with a core consisting of a bundle of soft iron wires.

15. Given two coils of wire, a compass needle and a cell, determine which coil has the greater number of turns.

16. Given two coils of thick wire of the same diameter, a compass needle, a resistance box, and a cell, find the ratio of the number of turns in the first coil to the number of turns in the second.

17. Plot a curve showing the relation between the weight that can be lifted by an electromagnet and the current flowing through the coil.

18. Test the accuracy of the readings of the ammeter supplied, using a tangent galvanometer.

19. Find how the deflection of the given galvanometer varies with current.

20. Plot a curve showing the variation of deflection with current for an astatic galvanometer of known resistance, being given a set of resistances and a constant cell of known E.M.F.

21. Find the resistance of the two given coils by connecting them separately, then together, in series with a constant cell, a 30 ohm coil, and a tangent galvanometer, and noting the deflections obtained.

22. Determine the resistance of the given length of wire and calculate its diameter, having been given the specific resistance of the material.

23. Compare the specific resistances of the materials of the two given wires.

24. Find the electrical centre of the slide wire of a metre bridge. (The electrical centre is the point dividing the wire into two parts of equal resistance.)

25. Determine the ratio of the diameters of two wires of the same material by measuring their resistances.

26. Find what length of the wire A would have a resistance of 5 ohms.

27. Equal lengths of the wires A and B, placed in parallel, are to give a resistance of 5 ohms. What must be the length of each wire?

28. Cut off from the given coil a length of wire which shall have a resistance of 1 ohm, allowing 1 cm. at each end for connections. Check the result by direct measurement of its resistance.

29. Set up the resistance boxes supplied to form a Post Office Box, and use it to measure the given resistance coil.

30. Find the length of a tangle of wire, using a P.O. box. Its specific resistance will be given.

31. Find the resistance of the conductor formed by using one, two, three, and four strands of the given wire in parallel, each strand being 20 cm. in length.

32. Compare the resistance of the given coil at 0° C. with its resistance at 100° C.

33. Construct three voltaic cells from the materials supplied and compare their electromotive forces. State which is the positive pole in each case.

34. Determine the resistance that must be connected to the poles of a cell to reduce the potential difference between them to one half. What do you deduce from the result?

35. Plot a curve showing how the potential difference between the poles of a battery varies when different resistances are connected with the poles.

36. Being given a cell (*e.g.* an accumulator), a number of known resistances, and a low-range voltmeter, adjust the circuit so that exactly $\frac{1}{100}$ of an ampere passes through it.

37. Measure the E.M.F. due to polarisation when (*a*) platinum plates, (*b*) lead plates dip in dilute sulphuric acid.

38. Fit up a cell with copper and zinc plates in dilute sulphuric acid. Find how the current from the cell varies with the time.

39. Measure the current in amperes through a coil of known resistance by using a voltmeter.

40. Find the maximum current which can be carried by the fuse wire supplied.

41. Pass a current through a sheet of tin-foil from one marked point to another. Plot the equipotential curves on the sheet by connecting two pins to the terminals of a sensitive galvanometer.

42. Find the horizontal component of the earth's field, using a tangent galvanometer and a copper voltameter. The electrochemical equivalent of copper will be given.

43. Find the deflection of the given galvanometer for a current of 1 ampere, assuming the electrochemical equivalent of copper to be known.

44. Measure the heat produced in the given electric lamp when connected across the lighting mains for a measured time. Hence calculate the current flowing in the lamp, and the resistance of the lamp, being given the difference of potential between its terminals.

45. Find the rate of production of heat in the given coil when a current of one ampere flows through it.

46. The inner of two coils is wound clockwise from A to B as viewed from above. The galvanometer supplied has its north pole deflected east if the current enters it at terminal E. Find the direction of the winding of the outer coil.

47. Find which end of the given magnet is its north pole, given a helix of wire, a sensitive galvanometer, and a voltaic cell.

48. Apply the laws of induced currents to determine the polarity of a magnetised piece of steel with unmarked ends. Test the result with a compass needle brought up to the steel.

49. Apply the laws of induced currents to test the poles of a box of cells and find which is the positive. Check the result with a piece of pole-finding paper.

50. Assuming the E.C.E. of hydrogen to be known, determine the E.C.E. of copper.

APPENDIX

APPENDIX

PHYSICAL CONSTANTS AND MATHEMATICAL TABLES[1]

MATHEMATICAL CONSTANTS

Number.	Logarithm to base 10.
$\pi = 3\cdot1416$	$0\cdot49715$
$\pi^2 = 9\cdot8696$	$0\cdot99430$
$\frac{1}{\pi} = 0\cdot3183$	$\bar{1}\cdot50285$
$\frac{4}{3}\pi = 4\cdot1888$	$0\cdot62209$
$e = 2\cdot7183$	$0\cdot43429$
Frequently occurring—	
2	$0\cdot30103$
3	$0\cdot47712$
$\sqrt{2} = 1\cdot4142$	$0\cdot15052$
$\sqrt{3} = 1\cdot7321$	$0\cdot23856$
981	$2\cdot99167$
$30\cdot48$	$1\cdot48401$
$2\cdot540$	$0\cdot40483$
$453\cdot59$	$2\cdot65666$
760	$2\cdot88081$
273	$2\cdot43616$
$4\cdot2$	$0\cdot62325$

$$\log_e 10 = 2\cdot30258$$

One radian (unit angle, for which the arc equals the radius) $= 57^\circ\cdot2958 = 57^\circ 17' 45'' = 206265''$.

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[1] The values of the constants are in most cases taken from the Smithsonian Physical Tables (1914).

The mathematical tables are reproduced from Mr. F. Castle's *Logarithmic and other Tables for Schools* (Macmillan & Co., Ltd.), price 6d., by kind permission of the author.

FORMULAE IN MENSURATION

Circumference of a circle, radius r	$=2\pi r$
Area of a circle	$=\pi r^2$
Area of an ellipse, semi-axes a and b	$=\pi ab$
Surface of a sphere	$=4\pi r^2$
Volume of a sphere	$=\frac{4}{3}\pi r^3$
Volume of a cylinder	$=\pi r^2\times$ height
Volume of a cone	$=\frac{1}{3}\pi r^2\times$ height
Volume of a pyramid	$=\frac{1}{3}$ area of base $\times$ height
Volume of a prism	$=$area of base $\times$ height

MOMENTS OF INERTIA

Moments of Inertia about an axis of symmetry.

Circular ring or hoop, radius a,

$$I=Ma^2.$$

Rectangular bar about an axis through the centre of gravity perpendicular to the edges of length $2a$ and $2b$,

$$I=M\frac{a^2+b^2}{3}.$$

Elliptic plate, semi axes a and b, about an axis through the centre of gravity perpendicular to its plane,

$$I=M\frac{a^2+b^2}{4}.$$

The circular plate is a particular case, $a=b$ and

$$I=M\frac{a^2}{2}.$$

Solid ellipsoid, semi-axes a, b, c about the axis **c**,

$$I=M\frac{a^2+b^2}{5}.$$

The sphere is a particular case, $a=b=c$,

$$I=\frac{2}{5}Ma^2.$$

These results are summarised in Routh's rule, which states that the moment of inertia, I, about an axis of symmetry is given by—

$$I = \frac{\text{Mass (sum of squares of perpendicular semi-axes)}}{3, 4 \text{ or } 5}.$$

The denominator is to be 3, 4, or 5, according as the body is rectangular, elliptical, or ellipsoidal.

Thus for a *Cylinder*, length $2a$, radius b, about an axis perpendicular to its length, the section parallel to a is rectangular in type while the section parallel to b is elliptical in type, so that

$$I = M\left(\frac{a^2}{3} + \frac{b^2}{4}\right).$$

For a *Circular disk* radius, a, about a diameter,

$$I = M\frac{a^2}{4}.$$

BRITISH IMPERIAL AND METRIC WEIGHTS AND MEASURES

Length

1 inch = 2·5400 cm.
1 foot = 30·480 cm.
1 yard = 91·4399 cm.
1 metre = 39·370 in.

Mass

1 grain = 64·8 milligrams
1 ounce = 28·350 gm.
1 pound = 453·59 gm.
1 kilogram = 2·2046 lbs.

Capacity

1 pint = 0·568 litres.
1 quart = 1·136 litres.
1 gallon = 4·546 litres.

1 fluid ounce = 28·3815 c.c.

ELASTIC MODULI IN DYNES PER SQUARE CM.

Material.	Young's Modulus.	Modulus of Rigidity.
Aluminium	$7 \cdot 2$ to $7 \cdot 5 \times 10^{11}$	$2 \cdot 5$ to $3 \cdot 4 \times 10^{11}$
Brass	$8 \cdot 5$ to $10 \cdot 5 \times 10^{11}$	$3 \cdot 5$ to $3 \cdot 7 \times 10^{11}$
Copper	$10 \cdot 5$ to $13 \cdot 2 \times 10^{11}$	$4 \cdot 2$ to $4 \cdot 8 \times 10^{11}$
Iron	about 20×10^{11}	$5 \cdot 2$ to $8 \cdot 2 \times 10^{11}$
Platinum	15 to 17×10^{11}	$6 \cdot 2$ to $6 \cdot 6 \times 10^{11}$
Silver	$7 \cdot 1$ to $7 \cdot 4 \times 10^{11}$	$2 \cdot 5$ to $3 \cdot 0 \times 10^{11}$
Glass	6 to 8×10^{11}	$2 \cdot 3$ to $2 \cdot 7 \times 10^{11}$

DENSITY OR MASS IN GRAMS PER C.C.

Solids

Elements.		Common Substances.	
Aluminium	2·58	Boxwood	0·95–1·16
Antimony	6·62	Cork	0·22–0·26
Bismuth	9·80	Pitch Pine	0·83–0·85
Copper	8·30–8·95	Yellow Pine	0·37–0·60
Gold	19·3	Mahogany	0·85
Iron	7·5–7·9	Oak	0·60–0·90
Lead	11·3	Walnut	0·64–0·70
Magnesium	1·74	Beeswax	0·96–0·97
Nickel	8·6–8·9	Ebonite	1·15
Osmium	22·5	Glass, common	2·4–2·8
Platinum	21·37	Glass, flint	2·9–5·9
Silver	10·5	Ice	0·917
Tin	7·3	Paraffin wax	0·87–0·91
Zinc	7·1	Brass	8·4–8·7

Liquids

	Grams per c.c.	Temp.
Alcohol, ethyl	0·807	0°
,, methyl	0·810	0°
Anilin	1·035	0°
Carbon disulphide	1·293	0°
Chloroform	1·480	18°
Ether	0·736	0°
Glycerin	1·260	0°
Paraffin	0·878	0°
Petrol	0·873	16°
Mercury	13·596	0°
	13·546	20°

Gases

	Grams per c.c. at 0° C. and 76 cm. pressure.
Air	0·0012928
Aqueous vapour (calculated)	0·000814
Carbon dioxide	0·0019768
Hydrogen	0·00009004
Nitrogen	0·0012514
Oxygen	0·0014292

COEFFICIENTS OF LINEAR EXPANSION

The value quoted is the mean coefficient of linear expansion betwe 0° C. and 100° C.

Aluminium	0·000022	Lead	0·000027
Brass	0·000019	Platinum	0·000009
Copper	0·000017	Silver	0·000019
Glass	0·000008–0·000009	Tin	0·000023
Gold	0·000014	Zinc	0·000029
Iron	0·000011	Fused Quartz	–0·00000

SPECIFIC HEATS

Solids

Aluminium	0·212	Lead	0·031
Brass	0·094	Platinum	0·032
Copper	0·094	Silver	0·056
Glass	0·19	Tin	0·055
Gold	0·0316	Zinc	0·094
Iron	0·115	Quartz	0·191

Liquids

Alcohol (17° C.)	0·58	Mercury	0·033
Anilin	0·514	Paraffin	0·511
Glycerin	0·576	Turpentine	0·43

COEFFICIENTS OF THERMAL CONDUCTIVITY

Aluminium	0·48	Lead	0·08
Brass	(about 0·2	Platinum	0·17
Copper	0·9	Silver	1·0
Glass	about 0·001	Tin	0·15
Iron	0·10–0·14	Zinc	0·26

Pasteboard	(about) 0·0004
Rubber	(about) 0·0003

PRESSURE OF SATURATED AQUEOUS VAPOUR

Pressures in mm. of mercury.

Temp.	Pressure	Temp.	Pressure
0° C.	4·6	13° C.	11·2
1	4·9	14	12·0
2	5·3	15	12·8
3	5·7	16	13·6
4	6·1	17	14·5
5	6·5	18	15·5
6	7·0	19	16·5
7	7·5	20	17·5
8	8·0	21	18·7
9	8·6	22	19·8
10	9·2	23	21·0
11	9·8	24	22·4
12	10·5	25	23·8

REFRACTIVE INDICES FOR SODIUM LIGHT

Temperature.	μ.*
Water (17·5° C.)	1·3332
Alcohol (15·0° C.)	1·3635
Anilin (20·0° C.)	1·5863
Benzene (21·6° C.)	1·5004
Carbon disulphide (20·0° C.)	1·6276
Bromnaphtalin (20·0° C.)	1·6582
Crown glass (ordinary)	1·53
,, (heavy)	1·61
Flint glass (ordinary)	1·65
,, (heavy)	1·74
Quartz (ordinary ray)	1·5442
,, (extraordinary ray)	1·5533

WAVE LENGTHS

Wave lengths are usually expressed in Ångstrom Units (A.U.) or Tenth Metres (10^{-10} metre).

Wave lengths are sometimes expressed in a unit ten times larger, viz. the micromillimetre ($\mu\mu$).

The Solar Spectrum

Atmospheric	A	7604
,,	B	6867
Hydrogen α	C	6562
Sodium	D_1	5895
,,	D_2	5889
Calcium	E	5269
Magnesium	b_1	5184
Hydrogen β	F	4861
Iron	G	4307
Hydrogen	h	4102
Calcium	H	3967
,,	K	3934

Flame Spectra.

Potassium (red)	7865
Lithium (red)	6705
,, (orange)	6102
Sodium (yellow)	5895
,,	5889
Mercury (yellow)	5790 5769
,, (green)	5461
Thallium (green)	5348
Strontium (blue)	4607
Mercury (violet)	4359
Calcium (blue)	4226
Potassium (violet)	4080

RESISTIVITY OR SPECIFIC RESISTANCE

The resistance in ohms of a wire 1 cm. in length, 1 sq. cm. in cross-section.

Elements.	Temp.	Resistivity.	Temp. Coefficient.
Aluminium	(20°)	0·0000028	0·0039
Copper	(20°)	0·0000017	0·0040
Iron (pure)	(0°)	0·0000088	0·0062
,, (piano wire)	(0°)	0·0000118	0·0032
Lead	(0°)	0·0000204	0·0043
Magnesium	(0°)	0·0000044	0·0038
Mercury	(20°)	0·0000957	0·00088
Nickel	(0°)	0·0000069	0·0062

RESISTIVITY OR SPECIFIC RESISTANCE (*contd.*)—

Elements.	Temp.	Resistivity.	Temp. Coefficient.
Platinum	(0°)	0·0000110	0·0037
Silver	(18°)	0·0000016	0·0040
Tin	(0°)	0·0000130	0·0046
Zinc	(0°)	0·0000057	0·0040
Alloys.			
Brass		(about) 0·000007	0·0010
Manganin		(about) 0·000043	0·00002
Platinoid		(about) 0·000034	0·00025
Constantan or Eureka . .		0·000048	±0·00001

ELECTROCHEMICAL EQUIVALENTS

The electrochemical equivalent of silver is here assumed to be 0·001118 gm. per coulomb.

Element	Atomic Weight (1915) (O = 16).	Valency.	E.C.E. (grams per coulomb).
Aluminium	27·1	3	0·0000935
Copper	63·57	(1) 2	0·0003294
Gold	197·2	(1) 3	0·0006809
Hydrogen	1·008	1	0·00001045
Oxygen	16·00	2	0·0000829
Nickel	58·68	2 (3)	0·000304
Silver	107·88	1	0·001118
Zinc	65·37	2	0·0003388

BRITISH STANDARD WIRE GAUGE

S.W.G.	Diameter. Inch.	Diameter. Mm.	S.W.G.	Diameter. Inch.	Diameter. Mm.
0	0·324	8·23	26	0·0180	0·457
2	0·276	7·01	28	0·0148	0·376
4	0·232	5·89	30	0·0124	0·315
6	0·192	4·87	32	0·0108	0·274
8	0·160	4·06	34	0·0092	0·234
10	0·128	3·25	36	0·0076	0·193
12	0·104	2·64	38	0·0060	0·152
14	0·080	2·03	40	0·0048	0·122
16	0·064	1·63	42	0·0040	0·102
18	0·048	1·22	44	0·0032	0·081
20	0·036	0·914	46	0·0024	0·061
22	0·028	0·711	48	0·0016	0·041
24	0·022	0·559	50	0·0010	0·025

MEAN VALUES, FOR THE YEAR 1920, OF THE MAGNETIC ELEMENTS AT OBSERVATORIES WHOSE PUBLICATIONS ARE RECEIVED AT KEW OBSERVATORY.[1]

Place.	Latitude.	Longitude.	Declination.	Inclination.	Force in C.G.S. Units.	
					Horizontal.	Vertical.
	N.			N.		
	° ′	° ′	° ′	° ′	γ	γ
N. Magnetic Pole	70 5	96 45 W.	..	90·0	..	..
Sitka	57 3	135 20 W.	30 28·2 E.	74 22·1	15574	55662
Rude Skov	55 51	12 27 E.	7 57·2 W.	68 59·6	17124	44596
Eskdalemuir	55 19	3 12 W.	16 49·7 W.	69 89·5	16706	45084
Stonyhurst	53 51	2 28 W.	15 52·9 W.	68 43·5	17300	44433
Potsdam	52 23	13 4 E.	7 29·4 W.	66 33·5	18606	42912
Seddin	52 17	13 1 E.	7 31·2 W.	66 30·6	18645	42899
De Bilt (Utrecht)	52 5	5 11 E.	11 24·2 W.	66 51·8	18397	43056
Valencia (Ireland)	51 56	10 15 W.	19 17·9 W.	68 5·8	17840	44353
Kew (Richmond)	51 28	0 19 W.	14 31·0 W.	66 57·9	18410	43297
Greenwich	51 28	0 0	14 8·6 W.	66 53·6	18454	43249
Val Joyeux (near Paris)	48 49	2 1 E.	12 53·0 W.	64 41·6	19666	41591
Munich	48 9	11 37 E.	8 3·8 W.	..	..	..
Agincourt (Toronto)	43 47	79 16 W.	6 45·4 W.	74 44·6	15865	58166
Tortosa	40 49	0 30 E.	11 59·3 W.	57 39·4	23291	36781
Coimbra	40 12	8 25 W.	15 21·5 W.	58 22·8	23087	37496
Cheltenham (Maryland)	38 44	76 50 W.	6 18·5 W.	70 55·4	19118	55285
Tsingtau	36 4	120 19 E.	4 12·9 W.	52 7·0	30817	39610
Tucson (Arizona)	32 15	110 50 W.	13 48·0 E.	59 27·6	26910	45610
Lu-ka pang	31 19	121 2 E.	3 21·4 W.	45 30·7	33175	33773
Dehra Dun	30 19	78 3 E.	1 52·0 E.	44 59·9	32951	32949
Hongkong	22 18	114 10 E.	0 20·8 W.	30 46·4	37174	22137
Honolulu (Hawaii)	21 19	158 4 W.	9 53·2 E.	39 25·1	28847	23711
Toungoo	18 56	96 27 E.	0 23·7 W.	23 7·7	39114	16707
Alibag (Bombay)	18 39	72 52 E.	0 20·3 E.	24 54·7	36922	17147
Vieques (Porto Rico)	18 9	65 26 W.	3 46·1 W.	51 22·7	27827	34832
Antipolo	14 36	121 10 E.	0 85·9 E.	16 11·7	38100	11065
Kodai-Kanal	10 14	77 28 E.	1 49·9 W.	4 36·1	37787	03042
	S.			S.		
Mauritius	20 6	57 33 E.	10 20·3 W.	52 40·1	23093	30278
Christchurch (N.Z.)	43 32	172 37 E.	17 1·7 E.	68 9·2	22261	55525
S. Magnetic Pole	72 25	154 0 E.	..	90·0	..	..

[1] By permission of the Controller of the Stationery Office, from "Meteorological Office. Hourly Values from Autographic Records, Geophysical Section."

1γ corresponds to 1×10^{-5} C.G.S. unit.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
5	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
6	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
7	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·8	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·9	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
0	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
1	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
2	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
3	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
4	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
5	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

NATURAL SINES.

Degrees.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences. 1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES.

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
45	·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	·7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	·9998	9999	9999	9999	9999	1·000	1·000	1·000	1·000	1·000	0	0	0	0	0
90	1·000														

NATURAL COSINES.

Degrees.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences. 1	2	3	4	5
0	1·000	1·000	1·000	1·000	1·000	1·000	·9999	9999	9999	9999	0	0	0	0	0
1	·9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	·9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	·9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	·9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	·9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	·9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	·9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	·9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	·9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	·9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	·9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	·9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	·9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	·9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	·9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	·9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	·9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
18	·9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	·9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	·9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	·9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	·9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	·9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
24	·9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	·9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	·8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	·8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	·8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	·8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	·8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	·8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	·8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	·8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	·8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	·8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	·8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
37	·7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	·7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	·7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	·7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	·7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	·7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	·7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	·7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
45	·7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	·6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	·6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	·6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	·6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50	·6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	·6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	·6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	·6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	·5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	·5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	·5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	·5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	·5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	·5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	·5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
61	·4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	·4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	·4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	·4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	·4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	·4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
67	·3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	14
68	·3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	·3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	·3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	·3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	·3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	·2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	·2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	·2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	·2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	·2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	·1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
80	·1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	·1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	·1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
83	·1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	·1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	·0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	15
86	·0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	·0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	·0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	·0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15
90	·0000														

NATURAL TANGENTS.

Degrees.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences. 1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences.				
											1	2	3	4	5
45	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	2·0057	2·0145	2·0233	2·0323	2·0413	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2·9042	9208	9375	9544	9714	9887	3·0061	3·0237	3·0415	3·0595	29	58	87	116	145
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4·7046	7453	7867	8288	8716	9152	9594	5·0045	5·0504	5·0970	Mean differences cease to be sufficiently accurate.				
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5·6713	7297	7894	8502	9124	9758	6·0405	6·1066	6·1742	6·2432					
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	7·0264					
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	8·0285					
83	8·1443	2636	3863	5126	6427	7769	9152	9·0579	9·2052	9·3572					
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0					
90	∞														

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THE END

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